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Disk planet interaction during the formation of extrasolar planets

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Abstract

Radial velocity observations of extrasolar planets have shown that many planets with a mass comparable to that of Jupiter have orbits close, less than 1.5 AU, to their stars. The solar nebula model predicts that the core of this type of planets form outside the so-called snowline, which lies roughly at 3-5 AU from the star. This means that the planets have to move radially inwards in order to end up at their observed locations. This radial motion of the planets is called orbital migration and is due to interactions between the planet and a disc of gas or dust particles in the infancy of a planetary system.

We have examined the dependance of the exchange in the specific angular momentum, derived through an impulse approximation, on the distance from the planet. This has given the result that the change of specific angular momentum of the co-rotating particles, which are situated inside the separatrices and affects the planet with co-rotation resonance torques (CRT), are roughly one order of magnitude larger than that of the circulating particles, which are situated outside the separatrices and affects the planet with Lindblad resonance torques (LRT), in accordance with the results of Ida et. al. (2000). The behaviour of the exchange of specific angular momentum is also very similar to the results of Ida et. al (2000). The dependance of the particles which affects the planet with CRTs is linear. The largest contribution to the planet migration of the particles which affects the planet with LRTs come from very close to the separatrices and this is in accordance to the theory, which predicts that the exchange of angular momentum depend on the particle-planet distance to a power of -5. The contribution from the particles which affects the planet with LRTs very close to the separatrices are a factor of several larger than the contribution of the LRT particles more than roughly 2 Roche lobes, r_L, from the separatrices.

The effect of an initial surface density power-law disk distribution on the radial migration of a Saturn-mass planet on a circular orbit has been examined. The method is based on the prescription by Masset (2002) with the additional assumptions that the interactions are instantaneous and that we have point mass particles.

The initial distributions are divided into two types: two-sided disks, which mean that there are particles both outside and inside the planet's initial radius, for which there is very little migration (less than 5 percent of the initial planet radius) and one-sided disks, where particles are only on one side, either outside or inside, of the planets initial radius, for which there is large migration (more than 10-20 percent of the initial planet radius). An intresting exception for the two-sided disk is a run with β=1.5 and q$\geq$ 0.004 which results in large outward migration. The situation here, though, is more similar to a one-sided disk, with almost all mass outside the planets initial radius.

In all the simulations the migration is in the direction of higher surface density of particles in the corotation region (CR). The temporal evolution of the migration of a planet in a two-sided disk looks like a damped sinusoidal curve, as it adjusts itself to the initial disk distribution and strife for an "equilibrium point" where the co-rotational torques, or the effects of the close encounters, on each side of the planet equals out. For inner one-sided disks the migration is inward toward the central star while for the outer one-sided disks the migration is directed outwards away from the central star.

The large migration for the one-sided disks is due to a situation where the

particles in the CR region interacts with the planet, and thus cause it to migrate, and undergo a jump over the CR region but after a libration timescale when the particles again should interact with the planet, due to a large migration, they now are outside the CR region, in the LR region, and instead performs a distant encounter. This causes the planet to feel particles in the CR region only on one side and we have a further enhancement of the migration. This situation is similar to the theory of runaway migration by Masset and Papaloizou (2003) and the results are also very similar.

The implications for planet formation are that the interactions and the initial distribution of the CR particles are more important than earlier theories have asssumed and predicted. It is now necessary to include these types of torques in the complete early evolution of planetary system formation. Another implication for planetary formation is that the migration, earlier approximated to occur on timescales of 10^3 initial periods, is possibly even faster, on timescales of 100 initial periods. A third implication for the planetary system formation is the importance of the initial conditions, most importantly in the CR region, in the disk. If initially there is a large imbalance between the torques of the corotating particles "inside and behind" and "outside and in front of" the planet there will be a large migration. This phase of migration will occur as long as the imbalance is maintained. If instead, the torques of the corotating particles "inside and behind" and "outside and in front" of the planet more or less balance, either types of migration due to Lindblad resonances may take over.

1 Introduction

Radial velocity observations of extrasolar planets have shown that a lot of planets with a mass comparable to that of Jupiters have orbits close, less than 1.5 AU, to their stars [6]. The solar nebula model predicts that the core of this type of planets form outside the so called snowline, which lies roughly at 3-5 AU from the star. This means that the planets have to move radially inwards in order to end up at their observed locations. This radial motion of the planets is called orbital migration and is due to interactions between the planet and a disc of gas or dust particles in the infancy of a planetary system.

The field of astronomy that models and simulate the disc-planet interactions is very young. In sections 1, 2 and 3 we give the major ideas of the formation of planetary systems over the last 30 years, mainly short and concise but in detail when this is essential to this work. The purpose of this report is to examine the effect of the initial disk-structure on the planetary migration and examine the possibilities of a runaway migration regime and outward migration. Here we present a crude equal mass particulate model instead of the hydrodynamical approach normally used.

In section 1.1 we start off with describing how the planetary system formed from an interstellar cloud to form a protostar and a disk rotating this protostar. In sections 1.2 and 1.3 we describe how this disk evolves to planetesimals and later planetary embryos which scatter or accrete all gas in the disk so that the biggest planetary embryos are the only objects left rotating the protostar. In section 2 we consider the physics behind the dynamical evolution of the disc-planet interactions and in section 3 we consider how this disk-planet interaction affects the planet giving rise to a movement in the radial direction, called migration. In section 4 we go through the basic ideas behind the simulations, the method and present the results of the simulations. In section 5 we discuss the results and finally in section 6 we give a conclusion and pin point the main results of this work.

1.1 Star System Formation

A star is born from a cold dense giant molecular cloud that collapse under its own gravity. This collapse makes the cloud rotate faster and faster in order to conserve angular momentum. Particles close to the core of the cloud has its gravity well overtaking the centripetal force of the collapsing cloud thus accelerating radially towards the core and hitting it. This produces an accreting core that fast builds up. This core will eventually form a star. The particles initially not so close to the core still collapse but the radial gravity-force of these particles are balanced by centripetal forces while the vertical forces are not and the particles strife to end up in the equatorial plane of the initial cold gas cloud. This produces a central core, that eventually will be a star, that accretes particles and gain mass and a disc rotating around the core. The process producing a disc also increases the possibility to produce a smaller core, a planetesimal, that accretes matter and eventually forms a planet, that rotates around the bigger core. The situation of a planet, or several planets, initially embedded in a disc orbiting a central star is the situation of this work.

The giant molecular cloud would collapse into one supermassive star if there was not anything to prevent it, that is if there was no inhomogeneities in the cloud. What can cause these inhomogeneities? First of all if the cloud had any angular momentum $|\vec{L}| = |\vec{r} \times m\,\vec{v}|$ around the central region this angular momentum is conserved as the cloud contracts. As the contraction proceeds the rotation speed of the cloud increases. This increases the cenrifugal force so much that the radial component of the centrifugal force eventually balances the gravitational force and further contraction of the cloud is prevented.

As the collapse of the clouds proceeds the angular momentum is conserved and so it rotates faster, just as in the giant molecular cloud and the subclouds. This increase in rotation prevents the clouds to contract into a single spherical ball that contains all the material. Instead the clouds collapses to a central accumulation of material and a disk of gas and dust since the rotation keeps the material from moving closer to the axis of rotation, i.e it hinder further collapse radially, but it does not hinder it from collapsing parallell to this axis. This forces particles to form a disk around the soon to be protostar. It is also possible that the clouds with not so much angular momentum collapse homologously to two or more central acumulations thus creating binaries or multiple systems.

The second and a very important factor in triggering or preventing star formation is the magnetic field. For partially ionized clouds gravitational forces have to be larger than the forces due to the pressure from frozen in magnetic fields. This causes the cloud to contract only parallell to the magnetic field lines and a disk is formed. For a warmer cloud we get more free electrons and the effect is greater but for a cloud with T=10 K there is almost no free electrons and the magnetic fields isn't necessarilly frozen in, but decoupled, and we can neglect the effect of this process.

A third facor is the the large scale turbulence of matter within a galaxy. As giant molecular clouds move through the spiral arms of a galaxy matter may be compressed to dense regions. This may trigger vigorous star formation inside the densest parts of this cloud and O and B stars forms and starts sequential star formation. Most stars within an open cluster forms almost simultaneously but if a very massive star, O or B type, were formed and went through it's entire lifecycle and became a supernova before the less massive protostars had evolved to the main sequence, they might even still be accreting a main fraction of their clouds mass, it might blow the less massive protostars clouds apart hindering the protostars further evolution. This supernova explosion may in turn produce strong density fluctuations and a second generation of stars may be born. The question is whether the high density regions may be cool enough to be gravitationally unstable or if the supernova outbursts or other heating mechanisms heat these regions too much. This process produces the typical sequential star formation pattern of OB-associations where the massive and shortlived first generation gives rise to a higher metallicity in the second generation stars. These O and B stars have large stellar winds which may be sufficient to get a giant molecular cloud dense enough to trigger star formation.

A fourth factor may be a collision of two giant molecular clouds which makes the density increase enough.

1.2 Planet System Formation - The Solar Nebula Theory

When astronomers first tried to model extra-solar planets they assumed they could use the standard model of solar system formation. In this model planetary systems forms from gas and dust in the proto-stellar accretion disc (as described above) to planetesimals that further accretes into planetary cores, or planetary embryos, and later to proto-planets [2]. Planetesimals, formed from gas and dust particles, then accumulates via binary collisions and planets form in less than roughly 1 Myr and in less than roughly 100 Myr terrestrial planets has been assembled in the inner disc. Outside the radius where it was cold enough to form water ice, called the ice condensation boundary, there were more ice available for the planets to accrete and the planets formed outside this boundary could grow larger and quicker.

The gas and dust particles in the disk are too small to sufficiently attract each other gravitationally. As these gas and dust particles orbit the proto-sun, the particles move turbulent enough to collide sufficiently often to ensure the continuing growth by "contact welding" or electrostatic forces. Planetesimals, bodies of roughly the size of 1-10 km, form from these processes within 100 kyr assuming a surface density of 100 kg/m^2 [3].

When the particles reach the size of 1-10 km, particles also called planetesimals, they are large enough to affect each other gravitationally. This makes collisions more probable and thus increases the rate of collisions. Now the planetesimals can continue to grow and grow even faster to even larger bodies. The biggest bodies have the largest accretion (since more massive bodies may attract more gravitationally than smaller bodies) and thus grows faster. This is called runaway accretion since the biggest body in the region accretes the most of the mass in the region. The accretion of the largest proto-planet in the region of smaller planetesimals tends to average out orbital properties of the proto-planet. The proto-planet ends up in a nearly circular orbit around the proto-sun thus decreases the ellipticity $e \approx 0$. The runaway accretion phase takes roughly 1 Myr and gives as an end result a disk almost emptied of gas and many moon to mars sized $(0.01 - 0.1 m_e)$ planetary objects called planetary embryos.

When the objects get moon to mars sized, reach the planetary embryo phase, they can gravitationally perturb other embryos orbits. This gives the planetary embryos quickly a less circular, more elliptic, orbit and the embryos might be so much perturbed that they cross the orbits of other embryos so they may collide or they may even be scattered out of the planetary system. This phase ends when there are no more embryos to scatter or accrete. This normally takes roughly 10-100 Myr.

Models of the structure of the outer planets indicate that they have a high-density planetary core of 10-20 m_e, envelopes of metallic hydrogen (for Jupiter and Saturn) and an atmosphere of hydrogen, helium and methane (for Neptune and Uranus) [3]. This means that besides the rocky material accreted by the inner terrestrial planets the outer planets also have to accrete gas particles from the solar nebula. Jupiter and Saturn have abundances of the elements that matches the solar nebulas while Neptune and Uranus have a depletion of elements compared to the solar nebula. This means that Jupiter and Saturn need to accrete gas from the disk very early very fast while Neptune and Uranus need to take longer time before accreting the diskgas [3].

For planets of masses less than 10-20 m_e the gas in the atmosphere are not gravitationally bound but for planets with masses of 10-20 m_e the gas bounds and can form an envelope of gas [3]. Putting all this together gives a theory for the formation of the gaseous giant planets. First the rocky inner planetary core forms in the same way as the terrestrial planets (see above). This accretion process gives proto-planets, the proto-planetary cores, that are 10-20 m_e. When the core reaches 10-20 m_e it also starts to trap, or bind, gas particles. All this occurs under a phase of star formation called the T-Tauri phase and observations of T-Tauri stars indicate that the gas in the disk would all be lost due to outflow of gas within 10^7 years. This means that the planetary cores of Jupiter, Saturn, Neptune and Uranus must have been formed before this time. Jupiter, 318 m_e, and Saturn, 95 m_e, need to have reached the runaway accretion phase, i.e reached 10-20 m_e, quite early in order to accrete so much gas (compare to 10-20 m_e for the rocky core) while Neptune, 14.5 m_e, and Uranus, 17.1 m_e, need to have reached 10-20 m_e quite late [3].

1.3 Planet System Formation - Extrasolar Planets

The migration scenario applies to some of the extra-solar planets: the 51 Peg-type planets (for properties see [4]). They all have masses of the order of Jupiter mass and orbit their stars very closely with periods of only a few days. According to the solar nebula theory these planets all have formed several AU from their stars so they must have migrated to their present position. After a substantial orbital distance decrease the migration eventually was stopped by some processes. This process has not really been determined yet.

With very careful measurements of a stars apparent position, a method called astrometry, one may see a star accompanied by a giant planet (in order to see any effect on the orbit of the star it is necessary to have a giant planet), wobble around the centre of mass of the system due to the changing direction of the gravitational attraction the giant planet gives the star [5]. This has been searched for in observations of stars in about 30 years but not given any conclusive evidence for planets orbiting a star.

Radial velocity observations, that are based on the Doppler effect, of stars gave the first conclusive evidence of an extra-solar planet orbiting the star 51 Peg (51 Peg means star number 51 in the constellation of Pegasus) [6]. This technique uses light from the star that pass through a reference gas contained in a tube. A spectrometer then measures the wavelengths of the spectral lines of the star with high spectral resolution and high precision relative to the absorption lines from the gas in the tube [5] [7]. If a planet orbit the star the lines, when compared to the lines of the gas, changes with time but if the star don't have any companion then the lines are not. When the interaction between the star and the planet make the star move away from us the wavelengths of the spectrum are shifted to longer wavelengths relative to the lines of the gas and when the star move towards us it shifts the wavelengths to shorter wavelengths relative to the lines of the gas. Thus the motion of the star will give a periodic shift of the stars absorption lines from shorter to longer wavelengths relative to the absorption lines of the reference gas [5] [7].

In a number of subsequent radial velocity surveys more extra-solar planets were found [8] [9] [11] orbiting solar-type stars. Extra-solar planets were even found orbiting solar-type stars to an extent of roughly 5 percent [12].

If the star-planet system is inclined relative to us there is today no way to tell by how much. A star-planet system might be face on or edge on or something in between. As long as the star-planet system is not face on a planet might come in front of the star in our line of sight. In this case there might be an observable change in the radiation flux from the star, which we see as a slight dip in the radiation flux of the star. This method is called the transit method. The G-star

HD 209458 has been found to have a planet orbiting it from the transit method just described [13].

How do one get the properties of the observed extra-solar planets? By determining the changes in velocity, actually the line of sight component of the velocity, of the star one may deduce the mass of the planet [5] [7]. The stars total velocity is proportional to the mass of the planet M_p. This makes the change of the line of sight component of the stars velocity also proportional to the planet mass. The amplitude of the line of sight velocity curve is K where $K = M_p sin\, i$, where i is the inclination of the star-planet system plane to the line of sight. A more massive planet give a higher amplitude of the velocity curve of the star since at a certain distance from the star a more massive planet give rise to a higher gravitational force on the star and thus also larger velocity changes. The mass determined though is only the minimum mass of the planet, since a slight inclination of the star-planet system relative to the line of sight makes the line of sight component, which is the only component that can be observed, of the stars velocity smaller than the stars actual velocity, so the mass may in fact be larger [5] [7]. The factor $sin\, i$ is between 0 and 1 and depends on whether we have a face on system (no inclination 0) or an edge on system (maximum inclination 90) respectively.

By observing the periods P of the stars relative Doppler shift one can get the period of the orbit of the planet and if the mass is determined one may use Keplers third law to get the average star-planet distance, the semi major axis, a [5] [7].

By observing the change in the line of sight component of the stars velocity one also may determine the eccentricity of the planet orbiting the observed star [5]. A planet in a circular orbit have a rate of change in the velocity that is the same all the time since the distance between the star and the planet is constant and thus has more or less a sinusoidal pattern. A more eccentric orbit though gives a less smooth velocity change. The line of sight component of the stars velocity changes faster when the planet is closer to the star since the force on the star is larger and the line of sight component changes not as fast when the star planet distance is further away [5].

Almost all of the observed extra-solar planets today orbit solar-type stars. One may divide the observed extra-solar planetary systems into three groups depending on their observed properties, see figure 1. These are the 51 Peg like, also called Hot Jupiters, with masses of roughly Saturn to Jupiter mass and roughly circular orbits very close, less than 0.1 AU, to their stars and planets more distant to the star but with high eccentricities, roughly 0.2 or more, and the 55 Cancri like (also more like our own planetary system) systems with more than one planet. The last system, 55 Cancri, has a Jupiter mass planet at 5.9 AU and two slightly smaller but still Jupiter sized planets in close orbits to the star all moving in more or less circular orbits. The latter systems are also the type of systems most suitable to find earth type planets.

Why do all these new observed properties of the extrasolar planets change the view of formation theory? The picture of planet formation according to the solar nebula theory, described above in section 1.2, could not explain why the first extra-solar planets were found [6] [12] in very close orbits ($<$0.1 AU from the star) around their parent stars. These planets were of roughly Jupiter size and that type of planets were expected to be formed outside the ice-condensation boundary of roughly several AU. But how does these planets end up in these

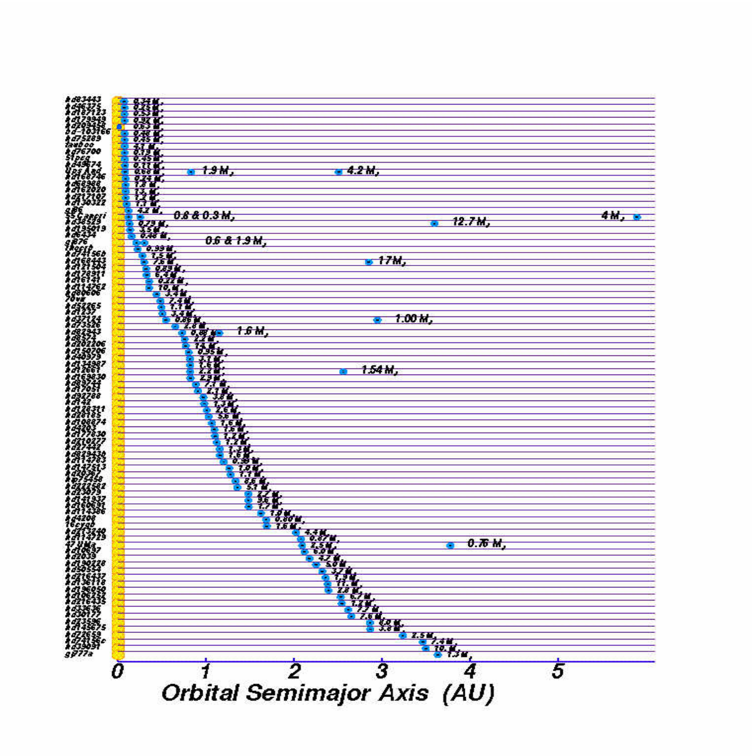

Figure 1: Here is a picture of the observed planets so far. On the x axis is the radial distance from the central star. The picture has been taken from http://www.exoplanets.org for further information.

close orbits? The most probable explanation of this is that the planets actually have formed outside the ice-condensation boundary and then migrated, travelled radially, inwards and somehow stopped at the present orbit. This produces the questions: what are the processes that makes the planet migrate and what processes makes the planet stop, i.e. why does the planet not migrate all the way in and hit the star? To answer this we need some sort of torques affecting the planet and the particles in the disk around the planet. There are mainly three processes used to get these torques: viscosity of the disk, gravitational tidal torques launched at the Lindblad resonances and torques due to the difference between the massflow and libration components, also called a surface density gradient, of the corotating particles.

2 Theory

In order to examine the interaction between a planet and a gaseous disk mostly the hydrodynamical approach has been used. A gaseous component of the disk need to follow the continuity equation 1 and the force equation 2

$$\frac{\delta\rho}{\delta t} + \nabla\cdot(\rho\,\vec{u}) = 0 \tag{1}$$

$$\rho\,\frac{D\,\vec{u}}{D\,t} = \rho\left[\frac{\delta}{\delta t} + (\vec{u}\cdot\nabla)\right]\vec{u} = -\rho\,\nabla\cdot\Psi - \nabla\cdot P + \nabla\cdot\vec{\Pi} + \vec{f} \tag{2}$$

where ρ, $\vec{u}$, $\Psi = -\frac{GM}{\sqrt{R^2+z^2}}$, P, $\vec{\Pi}$ and $\vec{f}$ are the density, the three dimensional velocity, the gravitational potential, the pressure of the disk, the stress tensor and a possible additional specific force.

The complete three-dimensional problem of solving the continuity equation 1 and the force equation 2 is most often reduced to a two-dimesional problem by assuming that there are no vertical motions [14], [15], [16], [17], [18]. This assumption, together with the assumption of hydrostatic equilibrium in the vertical direction, makes it possible to average the continuity equation and the force equation over the vertical direction. Now assume we only work with quantities of the system that is vertically averaged. This is possible since in most accretion disks the vertical thickness are small in comparison with the distance from the center. The vertically averaged continuity equation is

$$\frac{\delta\Sigma}{\delta t} + \nabla\,\cdot\,(\Sigma\,\vec{v}) = 0 \tag{3}$$

where $\Sigma \equiv \int \rho\,dz$ is the vertically averaged two dimensional surface density and $\vec{v}$ is the vertically averaged two dimensional velocity. The radial and azimuthal components of the vertically integrated force equations, i.e. the radial and azimuthal components of the Navier-Stokes equation is

$$\frac{\delta(\Sigma\,v_r)}{\delta t} + \nabla\cdot(v_r\,\Sigma\,\vec{v}) = \frac{\Sigma\,v_\theta^2}{r} - \frac{\delta P}{\delta r} - \Sigma\,\frac{\delta\Psi}{\delta r} + \Pi_r \tag{4}$$

and

$$\frac{\delta(\Sigma\,v_\theta)}{\delta t} + \nabla\cdot(v_\theta\,\Sigma\,\vec{v}) = -\frac{\Sigma\,v_r\,v_\theta}{r} - \frac{1}{r}\,\frac{\delta P}{\delta\theta} - \Sigma\,\frac{1}{r}\,\frac{\delta\Psi}{\delta\theta} + \Pi_\theta \tag{5}$$

where $\nabla\cdot P$, $\nabla\cdot\Psi$ and Π_i are the pressure gradient, the gradient of the gravitational potential and components of the viscous stress tensor per unit area. The third, vertical, component of the Navier Stokes equation now becomes

$$0 = -\rho\,\frac{\delta\Psi}{\delta z} - \frac{\delta P}{\delta z} + f_z. \tag{6}$$

One way to solve the three equations, 3, 4 and 5, in a two-dimensional (axisymmetric or not axisymmetric) disk, is linearization [19] [20] [21] [14] [22] [23]. This is mostly done by fourier transforming the equations [24] [15] [16] [17] [25] [26]. The result is a set of linearized equations which give the evolution of the system. Another way of solving the equations is transforming them to the canonical form [27] [28].

2.1 Vertical structure of the disk

In a planetary system it is viable to use a cylindrical coordinate system and for an axisymmetric disk, in Keplerian motion around its central star in the equatorial plane, with pressure gradients only in the vertical direction, we get a distribution of particles that follows

$$\frac{\delta \Sigma}{\delta t} = \frac{3}{r}\,\frac{\delta}{\delta r}\left[r^{\frac{1}{2}}\,\frac{\delta}{\delta r}\,(\Sigma\,\nu\,r^{\frac{1}{2}}) - \frac{1}{3\,\pi\,\sqrt{GM}}\,r^{\frac{1}{2}}\,T(r)\right] \tag{7}$$

in the r-θ plane, where z=0, and follows equation 6

$$0 = -\rho\,\frac{\delta \Psi}{\delta z} - \frac{\delta P}{\delta z} + f_z$$

in the vertical direction. Here ν and $T(r)$ are the kinematic viscosity and a possible linear torque density that depends on how the planet affects the disk. Equation 7 will be explained in the next section while we focus on equation 6 in this section.

For a disk in Hydro Static Equilibrium in the vertical direction, the specific force f_z is zero so equation 6 becomes

$$0 = -\rho\,\frac{\delta \Psi}{\delta z} - \frac{\delta P}{\delta z} \tag{8}$$

and

$$\begin{aligned} 0 &= -\frac{\delta}{\delta z}\,\Psi - \frac{\delta}{\delta z}\,\frac{P}{\rho} \\ 0 &= -\frac{\delta}{\delta z}\,(\Psi + \eta) \end{aligned}$$

which give the relation

$$\eta = -\Psi \tag{9}$$

between the enthalpy, $\eta \equiv \int \frac{dP}{\rho}$, of the system and the gravitational potential Ψ. For a vertically isothermal disk, we have $P = c_s^2(r)\,\rho$ and the enthalpy can be calculated

$$\eta \equiv \int \frac{dP}{\rho} = \int \frac{c_s^2(r) d\rho}{\rho} = c_s^2(r)\,\ln\frac{\rho}{\rho_0}$$

and thus

$$\eta = c_s^2(r)\,\ln\frac{\rho}{\rho_0} = -\Psi = -\left(-\frac{GM}{\sqrt{R^2+z^2}}\right) = \frac{GM}{R\sqrt{1+\left(\frac{z}{R}\right)^2}}$$

and if we expand the term $\left(1+\left(\frac{z}{R}\right)^2\right)^{-\frac{1}{2}}$ with a Taylorexpansion to the first order in $\frac{z}{R}$ we get (see appendix section A.1)

$$c_s^2(r)\,\ln\frac{\rho}{\rho_0} \approx \frac{GM}{R}\left(1 - \frac{z^2}{2\,R^2}\right) = \frac{GM}{R^3}\left(R^2 - \frac{z^2}{2}\right).$$

This means that we may use $\Omega_K^2(R) = \frac{GM}{R^3}$ to get

$$ln \frac{\rho}{\rho_0} = \frac{GM}{R^3} \frac{R^2}{c_s^2(R)} - \frac{GM}{R^3} \frac{z^2}{2\, c_s^2(R)} = R^2 \frac{\Omega_K^2(R)}{c_s^2(R)} - z^2 \frac{\Omega_K^2(R)}{2\, c_s^2(R)}$$

and we may define the vertical scale height as $z_0 = H \equiv \frac{c_s(R)}{\Omega_K(R)}$ which gives us the vertical density distribution as

$$\begin{aligned} \rho(z) &= \rho_0\, e^{R^2 \frac{\Omega_K^2(R)}{c_s^2(R)} - z^2 \frac{\Omega_K^2(R)}{2\, c_s^2(R)}} = \rho_0\, e^{\frac{\Omega_K^2(R)}{c_s^2(R)} R^2}\, e^{-\frac{\Omega_K^2(R)}{2\, c_s^2(R)} z^2} = \\ &= \rho_0\, e^{\frac{v_K^2}{c_s^2(r)}}\, e^{-\frac{z^2}{2\, H^2}} = \rho(R)\, e^{-\frac{z^2}{2\, H^2}} \qquad (10) \end{aligned}$$

which depends on the radial density distribution, for amplitude, and the disk vertical scale height. The assumption of a disk in vertical hydrostatic equilibrium is most often reliable and we have the possibility of splitting the cylindrical system into one vertical component and one disk assumed to lie in the equatorial plane. The most important thing in this section for further reading this work is the definition of the vertical scale height and the possibility of splitting the three dimensional cylindrical system into one vertical component balanced by the assumption of hydrostatic equilibrium and a two dimensional planar system in the equatorial plane. In this work we shall further examine the evolution of this planar system and will not focus any more on the vertical structure of the disk. What from now on is called the disk is this planar system.

For typical models of the solar nebula the vertical scale height of the nebula is roughly 0.05-0.1 times the distance to the star [29]. This scale height is comparable to the Roche lobe radius from proto-planets with mass in the range 10^{-3}-10^{-4} solar masses. A gap that forms are expected to be of the order of one scale height wide[14].

2.2 Viscosity of the Disc

There have been several mechanisms proposed as the source of angular momentum transfer and energy damping. There have been some linear mechanisms proposed. For example viscosity in the disk [23], radiative damping of the tidal perturbations [30], magneto-hydrodynamic turbulence driven by magneto-rotational instability [31] and convection [29]. The magneto-hydrodynamic turbulence mechanism probably do not occur in the cold and poorly ionised environment of proto-planetary nebulae [32]. Convection is probably not sufficient enough on transporting angular momentum and thus probably not good to use [33] [34]. Radiative transports efficiency of transporting angular momentum and energy is strongly reduced by dust opacity that leads to high optical depths and thus low radiative losses [35]. Instead probably the best damping processes are viscosity, which is linear, and shock formation, which is non linear [36].

If the viscosity of the disk is high enough $\alpha > 10^{-4}$ it could be a good mechanism [23] [37] but it is difficult to find what the strong source of viscosity in the disk is. The viscosity transport angular momentum outwards, from small radii to large radii, causing matter to flow, migrate, inwards [38] [39]. Actually the angular momentum of the disk is concentrated onto a small fraction of the mass that spiral, or migrates, outwards and the rest of the mass spiral, or migrates, inwards [40].

If one assume that we have an axi-symmetric disk [16], no specific force and average the quantities in the equations 3 and 5 over the azimuthal direction we get a diffusion equation

$$\frac{\delta \Sigma}{\delta t} = \frac{3}{r}\,\frac{\delta}{\delta r}\left[r^{\frac{1}{2}}\,\frac{\delta}{\delta r}\,(r^{\frac{1}{2}}\,\nu\,\Sigma)\right] \tag{11}$$

which tends to equal out any density perturbations in the disk. If one use the azimuthal component of the force equation together with the radial component of the continuity equation one get for a viscous disk equations 7 and 11 for disks with a specific force in the radial direction or without any specific force respectively. This transforms the problem from a two-dimesional to a one-dimensional problem.

In figure 2, we have the evolution of an initial gap in a disk with no planet at roughly 1.4-2.5 AU. Here equation 11 has been solved numerically. Each plot in the figure displays the disks vertically and azimuthally averaged surface density evolution, as a function of radius, for ten different times, each with the timestep given in the plots. The disk tends to equal out the gap within a time roughly equal to that of a viscous timescale. Using an α-prescription introduced by Shakura and Sunyaev 1973 [38]

$$\nu = \alpha\, c_s\, H = \alpha\,\left(\frac{H}{r}\right)^2\, r^2 \Omega_K(r) \tag{12}$$

this viscous timescale is

$$t_\nu \approx \frac{r^2}{\nu} = \frac{1}{2\pi\,\alpha\,\left(\frac{H}{r}\right)^2}\,P(r). \tag{13}$$

where $\alpha = 10^{-4} - 10^{-2}$ is a viscous dimensionless constant and $\frac{H}{r} = 0.03 - 0.10$ is the aspect ratio determined by the vertical scale height H and $P(r)$ is the period of a particle at radius r. This means that for low α values the viscous timescale is longer while for high α values the viscous timescale is shorter. One can also see that the normal viscous timescales are roughly $t_\nu \approx 10^3 - 10^6\, P(r)$.

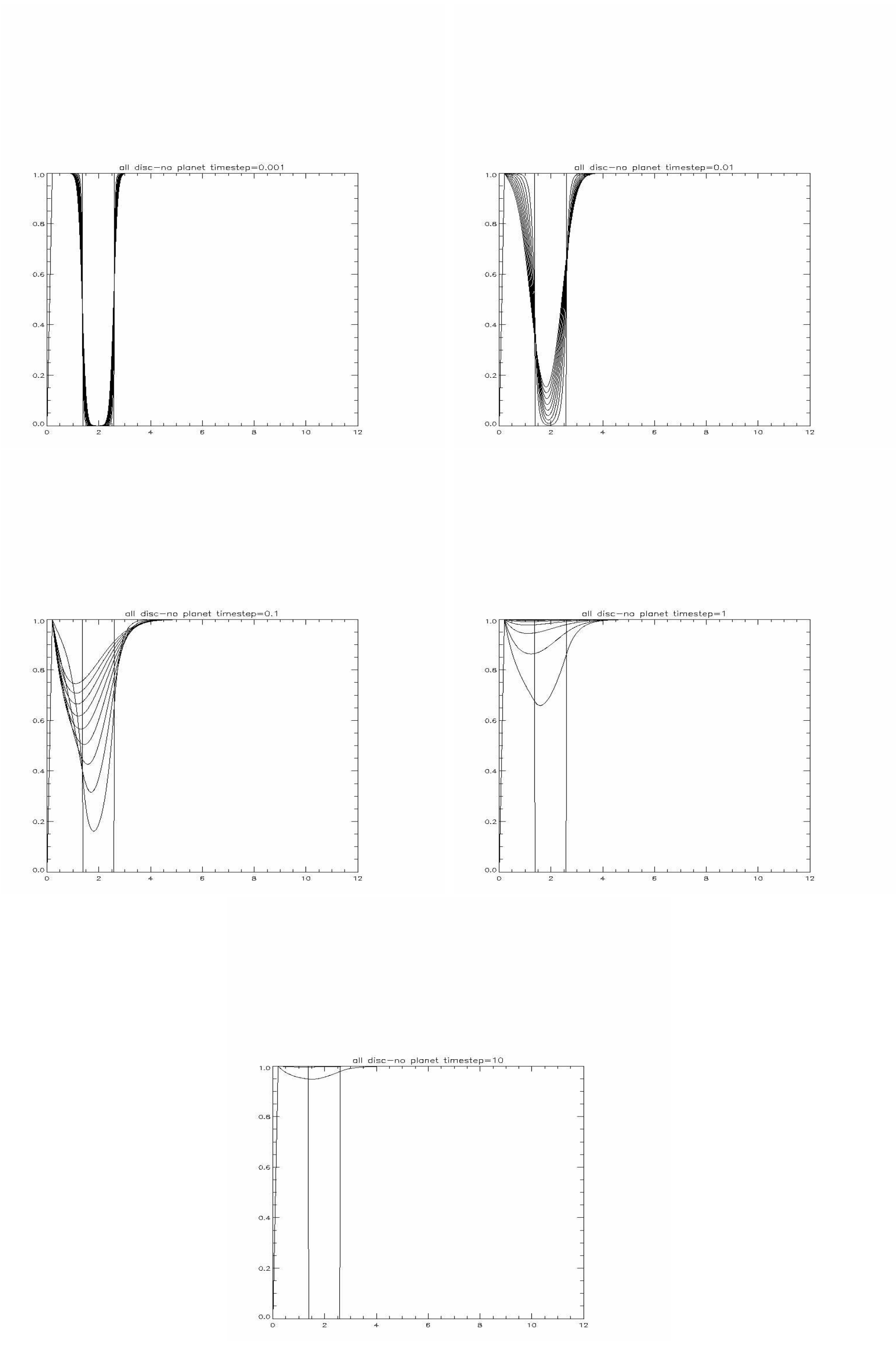

Figure 2: The viscous evolution of only a disk, no planet, with an initial gap. The vertical lines in the figures are the position of the initial gap. All timesteps are normalized to the viscous timescale, t_ν. Each new plot shows that the gap or dip gets more shallow and more broad as the disk try to even out any irregularities in the disk. We can see that after a time of the order of the viscous timescale the gap has dissappeared.

2.3 Lindblad resonance torques

In a system with a planet and a disk that orbit a central star, specific angular momentum

$$l = |\vec{l}| = |\vec{r} \times \vec{v}| = \sqrt{GM\, a\, (1 - e^2)} \tag{14}$$

are exchanged between the disk and the planet, due to either Lindblad resonance torques (LRT) or co-rotation resonance torques (CRT), at locations that are called Lindblad resonances (LR) and co-rotational resonances (CR) respectively[41]. For planets embedded in a disk we have an outer and an inner disk. The LRs in the inner disk is called the inner Lindblad resonances, ILR, and the LRs in the outer disk is called the outer Lindblad resonances, OLR. Conventionally there have not yet been necessary to divide the CRs into different regions but in this work we do: inner co-rotation resonances, ICR, and outer co-rotation resonances, OCR. In section 2.3.1 we derive the Lindblad resonance torque through an impulse approximation and in section 2.4 we derive the impulse approximation of the co-rotation resonance torque.

A planet embedded in a gas disk affects a volume element, or in this work the particle, of the outer (inner) disc gravitationally at the OLR (ILR). This gravitational disturbance produces density waves in the disc which carries angular momentum. When the disc dissipates, or absorb, and damps these waves the disc gains (looses) angular momentum. As a result of too weak dissipation of the waves it is possible that the density is enhanced at the LRs[27]. The process that makes the disc dissipate the waves is not yet really determined. There are several processes proposed, where viscosity (see section 2.2) and shock formation [42] are the most probable.

A particles radial distance to the planet determine wether it has a high (LRT-distant encounters) or low (CRT-close encounters) relative velocity compared to the planet. The intersection between these two domains are called the separatrix, b_s, and lies within roughly $1.3\, r_L < b_s < 3.1\, r_L$ [43] from the planet where $r_L = \left(\frac{\mu}{3}\right)^{\frac{1}{3}} r_p$ is called the Roche lobe radius. Here b_s is of the order of b, the impact parameter (see further down). In this work we use $b_s \approx 2.3\, r_L$.

For particles with $b > b_s$ the locations of the OLR (ILR) are distant enough to the planet (in the radial direction) so the particles have a relative velocity, compared to the planet, that is fast enough (even though in oppposite directions) to make the time of interaction, between the planet and the particle, short enough to only be able to give a slight disturbance and thus give the particles a disturbance that repells it from the planets orbit. These types of particles are called circulating particles since in a co-rotating frame they seem to rotate fast. The circulating particles performs distant encounters or LRTs. For particles with $b < b_s$ (see section 2.4) the locations of the CR are close enough to the planet (in the radial direction) so the particles have a relative velocity, compared to the planet, that is slow enough to make the time of interaction, between the planet and the particle, long enough to be able to give such a significant disturbance that it makes the particles cross the planets orbit. These particles are called co-rotating particles since they are more or less co-rotating with the planet in a co-rotating frame. Corotating particles performs close encounters or CRTs.

In 1993 Pawel Artymowicz [22] came to the result, from linear calculations, that the LRT from an annulus, with size comparable to a disk scale height (see above), from the planet, but within the CR, does not contribute significantly to the total LRT between the planet and the disk. This is caused by a mild, power law, and a sharp, exponential, decrease in the torque due to shifting locations where the resonances acts from. This decrease in the torque is called a cut-off. This cut off means that for CR regions with sizes comparable to a disk scale height, the effect of LRT comes from outside the separatrices and there is a possibility to divide the disk into two regions: where there are CRT but no LRT (inside the separatrices) and where there are LRT but no CRT (outside the sparatrices).

The particles in the inner disc near the ILR are disturbed, decelerated, by the perturbing planet, loose orbital speed and spiral inward towards the star. These particles end up in an orbit closer to the star and thus looses angular momentum. The particles in the outer disc near the OLR are also disturbed, but accelerated, by the perturbing planet and increase their orbital speed and spiral outward away from the star. These particles end up in an orbit further away from the star and thus gain angular momentum.

These periodic gravitational disturbances is manifested through what is called Tidal Wave torques or Lindblad resonance torques. The net effect of these type of torques is that the inner disk looses angular momentum to the planet while the outer disc gains angular momentum from the planet[24]. The direction of the Lindblad resonance torques is toward the central star in the Inner Disc and outwards from the star in the Outer Disc [20]. For the planet the net effect of the Lindblad resonance torques is that it gains angular momentum from the Inner Disc and looses it to the Outer Disc [24].

2.3.1 Impulse approximation

In deriving an expression for the Lindblad resonance torques one may use an impulse approximation. The derivation is similar to what Lin and Papaloizou did 1979 [21], see equation 25, and it works as following: First the total system is a star, a planetary core (from now on called the planet) rotating this star and a number of planetesimals (from now on called particles) also rotating this star. The star is placed at origo of the inertial system thus leaving a system of a planet and a number of particles rotating the origo. Both the planet and the particles are then assumed to be rotating with Keplerian velocities,

$$v_K(r) = r \cdot \Omega_K(r) = r \cdot \sqrt{\frac{GM}{r^3}} = \sqrt{\frac{GM}{r}} \tag{15}$$

in nearly circular orbits ($e \approx 0$) around this star, or origo, as long as the particles and the planet are not sufficiently close to each other. Sufficiently close means in this work that the planet and the particles are in conjunction where the particles and the planet interact. G is the normal gravitational constant and M is the mass of the star.

Now, one can derive a particle's speed relative to the planet in a frame co-rotating with the planet, i.e rotating with the same speed as the planet by using

$$v_{rel}(r) = r \cdot \Omega_K(r) - r \cdot \Omega_K(r_p) = r \cdot (\Omega_K(r) - \Omega_K(r_p)). \tag{16}$$

In this co-rotating frame one may then focus on a very narrow part concentrated immediately around the planet(since we are interested in the relative velocities of the particles when they are in conjunctiuon with the planet) where particles are passing the planet, that is not moving, in straight lines at a distance of $b = |x| = |r - r_p|$ from the planet, i.e on both sides of the planets orbit in order to represent the LR's, with speed $v_{rel}(r)$. This narrow part is called the local co-rotating frame and the distance b is called the impact parameter and is much shorter than the distance from the planet to the star $b << r_p$. Using this expression for the impact parameter b one get the particles radius $r = r_p + b$ for OLR and $r = r_p - b$ for ILR. If we insert these radii for outer (and inner) disc particles and use a Taylor expansion to the first order of $\frac{b}{r_p}$ one get the relative velocity of the particles in the co-rotating frame. This is (see appendix section A.2)

$$v_{rel}(outer) = -\frac{3}{2} \cdot b \cdot \Omega_K(r_p)$$

while

$$v_{rel}(inner) = \frac{3}{2} \cdot b \cdot \Omega_K(r_p)$$

so that the inner particles rotates relatively faster than the planet and the planet is caught up by these particles while outer particles rotates slower than the planet and the planet catches up with the these particles. Important to understand is that in this co-rotating frame the planet is not moving azimuthally at all since both the planet and the co-rotating frame have the same azimuthal, angular, velocity in the inertial system and thus the planet cannot move azimuthally in the co-rotating frame.

From this relative speed in the co-rotating frame one proceeds as follows: Each time the particle get close enough to the planet, i.e when the planet and the particle are in conjunction, there might be a gravitational interaction between the planet and the particle. This gravitational interaction takes place as long as the particle is within the local co-rotating frame, which is equal to the time it takes for the particle to travel the distance $2b$, so $\Delta t_{interaction} = \frac{2\,b}{v_{rel}(r)}$.

One may divide the velocity of a particle in the local co-rotating frame into two components: one parallell to the overall circular motion of the particles and one perpendicular, in the radial direction, to the overall angular velocity. When the interaction starts the particle is moving in circular orbits outside or inside the planets orbit with velocity $v_\theta = v_\parallel = v_{rel}(r)$. This sets the amplitude of the total velocity, which is constant during the interaction, but we also have a gravitational interaction taking place changing the particles radial component as

$$\begin{aligned}\Delta v_r = \Delta v_\perp &= \frac{GM_p}{b^2}\,\Delta t_{interaction} = \frac{GM\mu}{b^2}\,\frac{2\,b}{v_{rel}(r)} = \\ &= \frac{2\,GM\mu}{\mp\frac{3}{2}\,b^2\Omega_K(r_p)} = \mp\frac{4}{3}\,\frac{GM\mu}{b^2\Omega_K(r_p)}. \qquad (17)\end{aligned}$$

This means that the interaction forces the particles orbital velocity, the parallel component, to accelerate or decelerate according to

$$\Delta v_\theta = \Delta v_\parallel = \sqrt{v_{rel}^2(r) - \Delta v_\perp^2} - v_{rel}(r) \qquad (18)$$

and if one use a Taylor expansion of the first order in $\frac{\Delta v_\perp}{v_{rel}(r)}$ and that $G^2\,M^2 = r_p^6\,\Omega_K^4(r_p)$ one get (see appendix section A.2)

$$\begin{aligned}\Delta v_\parallel &\approx -\frac{\Delta v_\perp^2}{2\,v_{rel}(r)} = -\frac{16}{9}\,\frac{G^2 M^2 \mu^2}{b^4\,\Omega_K^2}\,\frac{1}{2}\,\frac{1}{\mp\frac{3}{2}\,b\,\Omega_K(r)} = \\ &= \pm\frac{16}{27}\,\mu^2\,r_p\,\Omega_K(r_p)\left(\frac{b}{r_p}\right)^{-5}. \end{aligned}\tag{19}$$

The specific angular momentum is per definition the crossproduct between the lever arm, i.e the radii of the particle, times its velocity $l \equiv |\vec{r} \times \vec{v}|$ and to the first order in b the only non-zero component of this crossproduct is

$$\Delta l \approx r_p\,\Delta v_\parallel \approx \pm\frac{16}{27}\,\mu^2\,(r_p^2\,\Omega_K(r_p))\left(\frac{b}{r_p}\right)^{-5}. \tag{20}$$

For the Outer Disk we get

$$\Delta l_i \approx +\frac{16}{27}\,\mu^2\,(r_p^2\,\Omega_K(r_p))\left(\frac{b_i}{r_p}\right)^{-5} \tag{21}$$

$$\Delta L_i \approx +m_i\,\frac{16}{27}\,\mu^2\,(r_p^2\,\Omega_K(r_p))\left(\frac{b_i}{r_p}\right)^{-5} \tag{22}$$

and for the Inner Disk we get

$$\Delta l_i \approx -\frac{16}{27}\,\mu^2\,(r_p^2\,\Omega_K(r_p))\left(\frac{b_i}{r_p}\right)^{-5} \tag{23}$$

$$\Delta L_i \approx -m_i\,\frac{16}{27}\,\mu^2\,(r_p^2\,\Omega_K(r_p))\left(\frac{b_i}{r_p}\right)^{-5} \tag{24}$$

where index i means the change in angular momentum of particle i.

In 1979 Lin and Papaloizou [21] introduces an impulse approximation which gives the rate of angular momentum transfer as

$$\frac{dL_T}{dt} = \int_0^{\dot{m}} \Delta l\,d\dot{m} = sign(\Delta_0)\,\frac{8}{27}\,\mu^2\,(\Sigma_0\,r_p^2)\,(r_p\,\omega)^2\left(\frac{\Delta_0}{r_p}\right)^{-3} \tag{25}$$

where $d\dot{m} = \frac{\Sigma\,r\,dr\,d\theta}{dt}$ is the flux of matter within each unit annulus db that interacts with the planet, giving Δl during a time dt. Here Δ_0 is exactly the impact parameter, or the distance between the particle and the planet, b. This Δ_0 is often approximated by the vertical scale height H. In a corotating frame $r\,d\theta = v_{rel}\,dt$ and we have $d\dot{m} = \Sigma\,v_{rel}\,db$. If we integrate equation 20 and this $d\dot{m}$ we get exactly 25, which indicates that the only difference in the derivations between 22, 24 and 25 is that we use point mass particles with mass M_i and translate this into the exchange of angular momentum each interaction instead of using the $d\dot{m}$ to get the differential torque and the total torque on the planet of the disk. In 1984 Lin and Papaloizou [14] further examines the effects of poly-tropic disk models on the impulse approximation of LRTs. This gives a slight change in the constant of the expression

$$\frac{dL_T}{dt} = \int_0^{\dot{m}} \Delta l\,d\dot{m} = sign(\Delta_0)\,0,23\,\mu^2\,(\Sigma_0\,r_p^2)\,(r_p\,\omega)^2\left(\frac{\Delta_0}{r_p}\right)^{-3} \tag{26}$$

for the LRT.

Another way of deriving an expression for the rate of angular momentum transfer of the circulating region is to assume that the particles, when they are close enough to the planet, performs a distant encounter and similarly for the librating region, when the particles are close enough to the planet, they perform a close encounter. This was done for Neptune and trans-neptunian planetesimal objects, by Ida et al. 2000 [43], which got

$$\dot{L}_{DISTANT}(\sigma = 0) = 2\pi\, \Sigma_z\, a\, r_L \int_{3,1}^{\infty} \frac{\Delta l_D(b_R)}{P_{syn}(b_R)}\, db_R \approx 0,3\, \Sigma_z\, a\, \Omega^2\, r_L^3 \tag{27}$$

$$\dot{L}_{CLOSE}(\sigma = 0) = 2\pi\, \Sigma_z\, a\, r_L \int_{0}^{3,1} \frac{\Delta l_D(b_R)}{P_{syn}(b_R)}\, db_R \approx -4,7\, \Sigma_z\, a\, \Omega^2\, r_L^3. \tag{28}$$

From this one can see that the LRT is not necessarily significantly larger than CRT as earlier theories have assumed in order to be able to neglect the CRT. In fact, LRT and CRT are of same order [20] or the effect of a close encounter (CRT) are one order of magnitude larger than the distant encounter (LRT) [43]. It is one of the questions of this work, to see what implications the initial conditions (wether the disk is a one-sided or two-sided disk) have on the relation between CRT and LRT. We will examine the behaviour of the exchange in specific angular momentum as a function of the impact parameter scaled to the roche lobe, r_L and compare this to the results of Ida et al. 2000 [43]. A hypothesis is that the exchange of specific angular momentum due to LRT is largest near the separatrices and follows a power law with index of -5 according to equations 22 and 24 while the exchange of specific angular momentum due to CRT (see below) is linear according to equations 34 and 36.

2.4 Co-rotational torques

In order to derive the impulse approximation expression of the co-rotational torques we need to consider a three body relation. This works as follows: we have a system of a central star, a planet and a third particle. We assume that both the planet and the particle has a Keplerian circular velocity in the undisturbed situation. Now we assume we sit on the planet rotating the central star and the particle is in an orbit lying slightly outside (in radial direction) the planets orbit. In fact the particle lie at $r = r_p + b$, where $b = r - r_p << r_p$ is called the impact parameter. For particles with $b < b_s$ (see section 2.3) the locations of the co-rotational resonances, CR, are close enough to the planet (in the radial direction) so the particles have a relative velocity, compared to the planet, that is slow enough to make the time of interaction, between the planet and the particle, long enough to be able to give such a significant disturbance that it makes the particles cross the planets orbit. These particles are called co-rotating particles since they are more or less co-rotating with the planet in the co-rotating frame.

If we sit on the planet, i.e. the planet doesn't move relative to us, we are in the coordinate frame co-rotating with the planet. In this system we see particles moving with a speed v_{rel} relative to the planets velocity. As the particles performs a hole orbit relative to us on the planet, so the particle starts near the planet and ends the orbit near the planet, it has performed a half libration orbit. Due to the Keplerian velocity distribution of the system the particle, lying slightly outside the planets orbit, is rotating the central star slightly slower than the planet, and also the planet frame, and thus lags behind the planet. This means that after some time the particle has lagged behind so much relative to the planet that it get close enough to the planet. The particle then enters, after it has performed a half libration orbit, a domain of the orbit that isn't entirely dominated by the stars gravity but the particle also gets disturbed by the gravitational forces from the planet. The particle moves relatively slow for this process to take enough time, but still fast enough compared to a librating period (see further down), to make the particle change its angular velocity sufficiently to end up in an orbit inside the planets orbit. The dominating effect of the gravitational interaction is that the planet gets decelerated by outer material while the outer material gets slightly accelerated.

When it gets to an inner orbit the particle, due to the Keplerian velocity distribution starts to move with an angular velocity that is greater than the planets and the particle thus moves away relative to the planet. This makes the gravitational effect of the planet less important again and the particles velocity gets completely dominated by the stars gravity again and thus gets no disturbance. The result of this gravitational interaction is that the particle has moved from an orbit very close to but slightly outside the planets orbit, $r = r_p + b$, to an orbit slightly close to but inside the planet orbit, $r = r_p - b$.

When the particle has performed this first interaction it starts to keep up relative to the planet instead, since it has a higher angular velocity than the planet. When it again reaches the planet, after a half libration orbit, it again enters this domain of gravitational disturbance from the planet, on an orbit slightly close to but inside the planets orbit, $r = r_p - b$, and now the planet particle interaction forces the particle to end up in an orbit slightly farther out than the planet, $r = r_p + b$, where it again starts to lag behind the planet, due to the Keplerian angular velocity distribution and everything starts again. The total effect of this gravitational interaction is that the planet gets accelerated by the inner lying particles while the particles inside the planets orbit gets slightly decelerated. In a frame co-rotating with the disturbing planet this gravitational interaction between the particle and the planet makes the particle perform what is called a horseshoe orbit, since it looks like a horseshoe in this frame. The particles that are moving in a horseshoe orbit are in fact librating.

As long as the interaction between the particle and the planet takes less time than half an horseshoe orbit of the particle, which is the time between two different interactions, it is possible to derive an expression for the change in specific angular momentum in this interaction [44]. The specific angular momentum of co-rotating particles in nearly circular Keplerian motion of radius r, in the equatorial plane is

$$l \approx \sqrt{GMr} \tag{29}$$

The particles that transform their jump gets a change in angular momentum that is

$$\Delta l = l_1 - l_0 \approx \sqrt{GMr_1} - \sqrt{GMr_0} \tag{30}$$

For particles in the outer disk we have

$$\begin{aligned} r_0 &= r_p + b \\ r_1 &= r_p - b \end{aligned}$$

$$\Delta l_{OD} = \sqrt{GM(r_p - b)} - \sqrt{GM(r_p + b)} \tag{31}$$

while for particles in the inner disk we get

$$\begin{aligned} r_0 &= r_p - b \\ r_1 &= r_p + b \end{aligned}$$

and

$$\Delta l_{ID} = \sqrt{GM(r_p + b)} - \sqrt{GM(r_p - b)} \tag{32}$$

Using a Taylor expansion to the first order of (b/r_p) we get for the outer disk (see appedix section A.3

$$\Delta l_i \approx -r_p\, \Omega_K(r_p)\, b_i \tag{33}$$

$$\Delta L_i \approx -m_i\, (r_p^2\, \Omega_K(r_p))\, \frac{b_i}{r_p} \tag{34}$$

and for the inner disk

$$\Delta l_i \approx +r_p\, \Omega_K(r_p)\, b_i \tag{35}$$

$$\Delta L_i \approx +m_i\, (r_p^2\, \Omega_K(r_p))\, \frac{b_i}{r_p} \tag{36}$$

where index i means the change in angular momentum of particle i. The main difference between this derivation and the derivation of Ward [44] is that instead of using a hydro-dynamical fluid element $dm = \Sigma\, r\, dr\, d\theta$ we use a pointmass particle m_i and the main difference between this derivation and the derivation of Masset [41] is that the jump performed by the particle is equal to $2b_i$.

For low mass planets the disturbance of the disk is linear and there still haven't been so much disturbance in the surface density profile of the disk so the disk don't need to shock to be able to dissipate the angular momentum and energy transferred to it. The evolution of these types of disk-planet systems follows the linearized Eulerian equations (see above). This type of migration is called the type I migration mode. Instead, when the mass of a planet is higher than a critical mass it enters a mode where the disk structure is strongly disturbed and shocks in order to dissipate the angular momentum and energy that is transferred to it. This shock of the disk material opens up a gap and the gap acts to lock the planet into a mode of migration that is slower and ruled by the viscous evolution of the disk, i.e. the disk material need to flow to the LRs (since biggest contributions of LRT comes from LRs) and this happens due to viscous diffusion mass-flows. Most of the theories of this non-linear evolution of the disk is based on the balance between the flow of material away from the planet due to the Lindblad resonance torques and the viscous evolution mainly of the outer disc, thus giving a planet migration directed inwards toward the star. This type of migration is called the type II migration mode [45]. These two types of migration has been examined extensively[46].

Between these two migration modes, when the planet mass is just massive enough to make the disk slightly disturbed, i.e. the gas starts to shock, and thus is not linear anymore but the mass of the planet isn't massive enough to create a clear gap, there is a new mode of migration where it is possible to get a runaway migration mode. What is prominent about this new mode is that the co-rotation torque is dominating or at least comparable to the Lindblad resonance torques or viscous torques. Estimates have shown that in this mode the co-rotational torques are more than one magnitude larger than the Lindblad resonance torques and the viscous torques [2]. This give implications that the end result of the planet formation process depends on the initial conditions, i.e the initial surface density distribution, since the co-rotation torque depends on the specific vorticity gradient [20] and thus by the surface density profile [46].

In 1992 Ward [44] derived an impulse approximation of the co-orbiting material and got the expression of the CRT for the disk from one side. He used the equation of the change in specific angular momentum and related this change to the Oorts constant $B_p = \frac{1}{2\,r_p}\,\frac{d(r_p^2\,\Omega_K(r_p)}{dr}$ as

$$\Delta h = r_o^2\,\Omega_o - r_i^2\,\Omega_i \approx 2\,r_p\,B_p\,(r_o - r_i) = 4r_p\,B_p\,b \tag{37}$$

and the amount of mass that interact during a time dt

$$dm = \Sigma\,r\,d\theta\,dr = \Sigma\,r\,|\Omega_K(r) - \Omega_K(r_p)|\,dt\,dr \tag{38}$$

and got the following expressions

$$T \approx 4\,\Sigma\,|A_p|\,B_p\,b^4\,\frac{dln\frac{\Sigma}{B}}{dlnr} \tag{39}$$

$$T = \frac{3}{4}\left(\frac{3}{2} - s\right)\,\Sigma\,b^4\,\Omega^2 \tag{40}$$

for the co-rotational torques where

$$A_p \equiv \frac{1}{2}\,r\,\frac{d\Omega}{dr}. \tag{41}$$

Here $b = (r_o - r_p)$ is the distance between the fluid element and the planet. We can see that there is a derivative of the surface density in the radial direction. This derivative is called the surface density gradient.

In 1993 Korycansky and Pollack derived the net torque on the planet from numerical linear calculations and showed that the co-rotational torque is also effective in the net torque [26]. This give us a hint that the CRT should not be neglected in the evolution of the planets migration. The expression for the co-rotational torque they derived in the linear case is

$$N_{c,m,0} = \frac{m\,\pi^2}{2}\left[\frac{|\eta_{m,0} + \phi_{p,m,0}|^2}{\frac{d\Omega}{dr}}\,\frac{d\left(\frac{\Sigma}{B}\right)}{dr}\right]_{r_c}. \tag{42}$$

In 2001 Masset examines the CRT on planets in a fixed circular orbit for both inviscid and viscous disks [41]. He derives an expression

$$N_C = \frac{9}{2}\,x_s^4\,\Omega_K(r_p)\,\Sigma_\infty\,\mathcal{F}(z_s) + \left(\frac{x_s}{r_p}\right)\,\mathcal{G}(x_s)\,N_{LRT} \tag{43}$$

where

$$\mathcal{F}(z_s) = \frac{1}{z_s^3} - \frac{g(z_s)}{z_s^4\,g'(z_s)} \tag{44}$$

$$\frac{g(z_s)}{g'(z_s)} = z_s - \frac{1}{4}\,z_s^4 + \mathcal{O}(z_s^4) \tag{45}$$

$$z_s = \left(\frac{\Omega_K(r_p)}{2\pi\,\nu\,r_p}\right)^{\frac{1}{3}}\,x_s \tag{46}$$

that relates the CRT to the viscosity of the disk and a differential LRT, that may be saturated [41].

In 2003 Masset and Papaloizou examine the CRT on already migrating planets [46]. They divide this CRT into two components: a massflow component and a librating component. For the massflow component they assume that the particles very close to the separatrix interact through CRT only one time with the planet, as they execute one u turn, before ending up in the circulating region (particles that interacts with LRT). This mass flow component is dependent on the disks surface density or rather the surface density gradient. These particles contributes a CRT that is directed in the same direction as the migration [46]. The material trapped inside the CR region, not very close to the separatrix, librates and thus performs a CRT that is in the opposite direction as the migration[46].

For a planet that moves in a circular pattern there are material in front of and behind the planet. In a frame corotating with the planet this material moves circular but with a shear equal to v_{rel} (see above). The particles initially inside the planets orbit keeps up with the planet and the particles initially outside the planets orbit lags behind (or the planet keeps up with the particle). In the initial moment there are particles inside the planets orbit and behind the planet and there are also particles outside the planets orbit and in front of the planet. The difference between the CRT from the particles outside and in front of the planet and the CRT form the particles inside and behind the planet gives an azimuthal gradient of the CRT. In the runaway migration mode the planets initial direction is determined by this difference or gradient.

The particles giving the dominating contribution (from either the particles inside and behind the planet or the particles outside and in front of the planet) of this gradient will behave like the mass flow component. The other particles will behave like the librating component. For planets in disks with components that equals out there will be no migration due to CRT. However the contributions to the CRT from these two components do not necessarily cancel out if the librating region has lower surface density than the circulating region, in which case the dominating part is the mass flow component [46]. This gives an extra push (i.e. an extra angular momentum surplus or deficit depending on the direction of the migration) to the planet in the same direction as the migration. This gives a runaway migration mode where the planets migration accelerates, assuming that the surface density gradient can be maintained. If this gradient all of a sudden changes direction so should the migration and the runaway migration mode is halted or reversed.

The difference between the contributions of the massflow and librating components has been derived by Masset and Papaloizou as a comparison between the CRT due to a mass of the CR region that one assumes have a constant surface density equal to the surface density at the separatrix and subtract the real mass of the CR region. This is expressed as

$$\delta m = 4\,\pi\, r_p\, B_p \left[x_s\, \frac{\Sigma_R(-x_s)}{B(-x_s)} - \int_{-x_s}^{0} \frac{\Sigma_R(x)}{B(x)}\, dx \right] \tag{47}$$

and is called the vorticity weighted mass deficit[46]. If this δm is larger or of the same order than the planet then there is a runaway migration mode.

This process may be stopped by density gradients since migration due to CRT is toward high surface density (since CRT has the same sign as the radial gradient of vorticity per unit surface density at the CR [20]) which means that a gap, where surface density is low, may change sign of migration [2]. This means that for low viscosities (Masset and Papaloizou also show that) outward migration is possible [41] [46]. In this work we use low viscosity in all simulations so we can not examine wether the outward migration that we get do depend on the low viscosity.

2.5 Accretion onto the Planet

If a planet is modelled as a massless sinkhole, that accretes all matter that falls onto it, the surface density of the disk near the planet decreases fast and opens up a gap. The disks evolution starts to follow the diffusion of a viscous disk and thus the planets migration and the accretion rate onto the planet occurs with the viscous diffusion timescale[48]. If instead the planet is modelled as a sinkhole, that accretes all matter that falls onto it, with mass and thus as a planet that disturbs the disk, this gives a torque on the disk which in turn decreases the accretion rate compared to the first model[48]. If one uses polytropic or isothermal equations of state and as long as the roche lobe r_L is larger than the disk scale height H and the mass, μ, is greater than $40/\mathcal{R}$ a clean gap forms[48]. These three models indicate that sufficiently massive planets can make the accretion onto planets ineffective over disk lifetime timescales[48]. The most recent estimates of the upper limit of a proto-planet mass after accretion is roughly 5 M_J for a solar type nebula[18].

3 The Back-Action of the torque - Planet Migration

For a planet on a nearly circular (the ellipticity $e \approx 0$) orbit at radius r_p the angular momentum is defined as

$$L_p = M_p \sqrt{GM\, r_p} = M_p\, r_p^2\, \Omega_K(r_p) \tag{48}$$

If we take the time derivative of this and assume $\dot{M}_p = 0$ and also assume that $r_p = r_p(t)$ we get

$$\begin{aligned} N_{D\rightarrow P} &= \frac{dL_p}{dt} = \frac{d(M_p \sqrt{GM\, r_p})}{dt} = M_p \sqrt{GM}\, \frac{d(r_p^{\frac{1}{2}})}{dt} = \\ &= M_p \sqrt{GM}\, (\frac{1}{2})\, r_p^{-\frac{1}{2}}\, \frac{dr_p}{dt} \end{aligned} \tag{49}$$

and from rearranging we get the planets radial velocity $v_p = \frac{dr_p}{dt}$

$$v_p = \frac{dr_p}{dt} = \frac{2\, r_p^{\frac{1}{2}}}{M_p \sqrt{GM}}\, \frac{dL_p}{dt} = \frac{2}{(M_p\, r_p\, \Omega(r_p))}\, \frac{dL_p}{dt} \tag{50}$$

where $N_{d-p} = \frac{dL_p}{dt}$ is the torque on the planet produced by the disk and N_{p-d} is the torque on the disk produced by the planet that results from the action-reaction law. The reaction, here meant the disturbance of the disk on the planet, is from now on called the back-action. Planetary orbital migration occurs because the torques on the disc from the planet has a back-action from the disk which manifests itself by producing a torque on the planet. Assuming these are the only torques on the planet and in order to conserve angular momentum we need $N_{d-p} + N_{p-d} = 0$ so $N_{d-p} = -N_{p-d}$ and:

$$\frac{dr_p}{dt} = \frac{-2}{(M_p \cdot r_p \cdot \Omega(r_p))}\, N_{p-d} \tag{51}$$

so by determining the disturbance the planet gives a fluid element, or particle, of the disk one can get the migration rate of the planet $\frac{dr_p}{dt}$.

3.1 The Response of Low Mass Planets - Type I Migration

If the contributions to the torque of the inner and outer disk are equally large (symmetric) but opposite, and thus equal out, the planet does not migrate (a non-linear disturbation may still cause the disk to open up a gap [24] [27] though) but if there is a slight asymmetry of torques the planet do migrate. When the net co-rotational resonance torque is negligible to the net Lindblad resonance torque and the perturbation on the disk from the planet is small enough to be treated linearly the outer disk torque contribution is most often bigger (due to that the OLR are slightly closer to the planet than ILR) than the inner one and thus this migration is mostly inwards [17] [24] [49]. This type of migration is called type I migration and is characterized as linear [45].

It corresponds to a fast migration rate and occurs only for small mass planets. When the mass of a planet is small (i.e. when the Hill radius is smaller than the disk thickness $r_H << H$), the migration rate has been shown to be proportional to the planet mass $M_p = \mu\, M$ and the local disk surface density $\Sigma_0 = \frac{q\, M}{\pi\, r_p^2}$ at the planet radius and inversely proportional to the square of the disk aspect ratio $\frac{H}{r}$ [18] [49]

$$\frac{dr_p}{dt} \approx -\frac{M_p}{M}\left(\frac{\Sigma_0\, r_p^2}{M}\right)\left(\frac{H}{r}\right)^{-3}(r_p\, \Omega_p). \tag{52}$$

If one defines the timescale of the migration as

$$t_{PM} = \frac{-r_p}{\frac{dr_p}{dt}} \tag{53}$$

then

$$t_{typeI} \approx \frac{1}{2\pi\, \mu\, q}\left(\frac{H}{r}\right)^3 P(r_p) \tag{54}$$

and the time it takes for the planet to migrate inwards its initial radius to the central star is roughly $t_{typeI} \approx 10^2 - 10^5$ years. The timescales for this migration is much shorter than the disk lifetimes, roughly $10^6 - 10^7$ years, and planetary formation timescales [50].

When a planet becomes more massive, due to accretion, it perturbs the disk so that it becomes non-linear, i.e. it shocks in order to be able to dissipate the density waves in a faster rate, and the disk gives a back-action on the planet. This leads to an increase in the type I migration rate of the planet, see equation 52. In this regime the planet pushes the material in the disk ahead of it in its radial drift. This lead to a disturbance in the disk profile with leading or trailing density waves in the disk. This disturbance of the disk-structure opposes the radial drift of the planet and when the planet gets massive enough to significantly disturb the mass distribution of the disk, i.e. when it opens up a gap in the disk, and thus produce a perturbation that is strongly non-linear, the back-action is again stopped altogether.

The viscosity of the disk tends to smooth out perturbations of the density profile as it distributes the angular momentum of the disk (see above). This decreases the back-action on the planet and thus the larger viscosity the disk has the larger is the mass (see section 3.2) of the planet where the type I migration regime stops [45]. This mass has been derived by assuming that the viscous diffusion of the disk balance the Lindblad resonance torque and thus force the planet to follow the viscous evolution of the disk [15]. When the disk and the planet has reached this balance it has reached a new type of migration called type II migration.

3.2 The Response of Massive Planets - Type II Migration

In 1986 Lin and Papaloizou [15] derives an analytical expression of the critical mass necessary to open a gap. This expression is based on the balance between the rate of angular momentum transfer due to Lindblad resonances, which try to open the gap, and the rate of angular momentum transfer due to a steady state viscosity, which tries to equal out any perturbations in the disk [14] [17]. If

$$\frac{M_p}{M} \geq \frac{40\, \nu}{r_p^2\, \Omega_K(r_p)} = 40\, \alpha \left(\frac{H}{r}\right)^2 \tag{55}$$

a gap opens [15]. In our case we use $\alpha = 10^{-4}$ and $\frac{H}{r} = 0.05$ so in order for us to get a gap we need

$$\frac{M_p}{M} \geq 10^{-5} \tag{56}$$

and since we examine the situation for a Saturn mass planet with $\frac{M_p}{M} = 3 \cdot 10^{-4}$ this criterion is always met. This expression gives that a planet can open a gap if its mass is larger than roughly 2 and 15 m_e depending on the disk properties [42].

The largest exchange in angular momentum due to LRT occurs for particles very close to the separatrices. When an interaction due to LRT, very close to the separatrices, has occured we get a gap since the material in the disk has moved slightly radially, Δr, away from the planet. Once this gap has formed, i.e the particles that interacts are slightly further away from the separatrices, the interaction due to LRT near the separatrices decreases. Now the viscous diffusion of the disk material replenish the gap with particles and therefore the gap narrows. After a while the gap has found an equilibrium where the amount of particles replenishing the gap, due to viscosity, and the amount of particles interating via LRT, and opening a gap, balances. Now the planet enters a type II migration determined by the viscous diffusion of the disk. An estimate of the type II migration rate is

$$\frac{dr_p}{dt} \approx \frac{3\,\nu}{2\,r_p}. \tag{57}$$

Typical timescales for this type II migration, using equations 53 and 57 for an α-prescription viscosity with $\alpha \approx 10^{-4} - 10^{-2}$ and $\frac{H}{r} \approx 0.02 - 0.10$, is roughly $t_{typeII} \approx 10^3 - 10^7$ years.

Planet migration due to LRT can decrease the tendency for opening a gap in the disk [17]. This gives us a second criterion that need to be met. This criterion comes from the fact that, in order to open up a gap around the planet, the planet need to migrate sufficiently slow, compared to the interacting particles, so that the particles interacting via a LRT has the time to move away from the planet. Otherwise the separatrix in the migration direction and the gap around the planet are replenished due to the migration. If this is the case, the effect of LRT is opposing the migration instead of promoting it, since the effect of the LRT now should be bigger for the particles near the separatrix in the migration direction, which LRT is in the opposite direction. This criterion has been derived by Takeuchi et. al. 1996 [23] by assuming the time it takes for the gap to open is smaller than the time it takes for the planet to migrate the gap size. The criterion for the planet migration not to inhibit gap formation is

$$\mu \leq 0.42\, C\, \frac{\Sigma_0\, r_p^2}{M} \left(\frac{H}{r}\right) \alpha^{-\frac{1}{4}}. \tag{58}$$

Planets with masses higher than this value gets a planet migration that inhibits gap formation. On the other hand if the migration is even faster, the particles replensihing the gap might enter the CR region and we get the massflow component of the CRT. In this case we have a runaway migration situation (see next section). In our case, as mentioned before, $q = 0.002 - 0.005$, $\frac{H}{r} = 0.05$ and $\alpha = 10^{-4}$ so in order for the migration not to inhibit gap formation we need $\mu \leq 4.2\,10^{-4} - 10^{-3}$. We have a Saturn, with $\mu = 3\,10^{-4}$, so it is always fulfilled even if it is a boarder case for the low mass disks. This means that as soon as the migration due to the CRT is small enough, and the LRT takes over, a gap forms. This can also be seen in section 4.2.4.

When the mass of a planetary core, embryo, is sufficiently large, it can open a gap in the disk, which is then divided into an inner and an outer disk. The planet is then drifting along inwards with the disks viscous evolution and is not so much affected by the other torques. This type of migration is called type II migration [49]. This type of migration is characterized as a non-linear regime of migration (compare type I) since the planet is massive enough to give perturbations of the disc that gives a solution of the equations of continuity and conservation of angular momentum that is not posssible to linearize but has to be solved in some other way.

3.3 Runaway regime

The co-rotation torque due to particles either in a libration or massflow component is dependant on the surface density gradient of the disk. As long as this gradient produces equally large but opposite directed torques the planet migration is controlled by LRT or viscous torques. Now this gradient does not necessarily equal out and then the planet migration is dominated by the CRTs. As mentioned above we divide the CRT into two components: the librating particles and the massflow particles. The massflow component only manifests itself if the planet migrates fast enough. For a planet that migrates fast enough inwards (the dominating domain are the massflow component inside and behind the planet which due to the migration and the existence of particles in the inner disk always is replensihed) the interacting particles ends up in the outer CR region and before the particles have finished half a librating period (the time between two interactions [44]) the planet has migrated over half of the CR region and the particles ends up outside the CR region. The particles now ends up among the outer circulating material instead. If the migration is not fast enough for this to happend (i.e. the planet migration is not high enough for the interacting particles to end up outside the CR region after half an librating period) then the particle give a CRT that is in the opposite direction and thus tends to decrease the initial planet migration. The exchange of angular momentum is largest for the particles close to the separatrices (see equations 34 and 36).

This situation has been examined by Masset and Papaloiou 2003 [46]. For a very large migration the massflow component will exactly be the particles very close to the separatrices that enters the separatrices, and thus the CR region, due to the migration and only interacts with the planet once, while the librating component is the particles that constantly drifts along with the planet in the dip or the gap and thus interacts many times with the planet before they leave the CR region (if at all). The massflow component of the CRT opposes a librating component of the CRT that is in the opposite direction. They assume that the migration already has a direction and derives the difference in mass between the massflow component and the librating component. This difference is exactly equation 47.

Now what happends in the situations where we do not have any initial dip or gap but instead we have a power law surface density distribution in the CR region? This is one of the questions of this work. We examine the planet migration that occurs due to such a distribution. For one-sided disks, for which there are practically no librating component, we get a similar result as the theory of Masset and Papaloizou 2003 [46].

There have not yet been any analytical estimates of the migration time-scales due to runaway migration but in simulations Pawel Artymowicz 2000 [2] and Masset and Papaloizou 2003 [46] get typical timescales of the runaway migration that is roughly 100 orbital periods. In our simulations we also get results that is similar. This means that the migration is even faster than the migration due to LRTs.

4 Simulations and Results

The theory so far means that for small mass planets the migration is of type I and for planets sufficiently massive a gap in the disk is cleared why the planet drifts with the disks viscous diffusion and the migration is of type II. Both these types of migration are assumed to have a CRT that balances, i.e. the CRT from the inner disk is equal but opposite to the CRT of the outer disk, and thus the total effect of the CRT on the planet is negligible. The evolution is then dominated by the LRT. Artymovicz [2] and Masset and Papaloizou 2003 [46] have examined situations with a runaway migration regime - type III migration - in which the CRT is dominating and the surface density gradient is such that it increases the migration speed. The purpose of this work is to examine the effect of the initial disk distribution on the planetary migration via the interaction of Lindblad resonance torques, LRT, and co-rotational torques, CRT, and examine wether it is possible to get a runaway migration based on this type of method.

4.1 Simulations

4.1.1 Method - Code Description

The method of these simulations for disk-planet interactions is based on a co-rotation torque prescription described in Masset 2001 [41] based on streamline topology. Added to this description for the co-rotation torque is a similar prescription for the Lindblad resonance torques and the assumption that there is no accretion onto the planet. These prescriptions are as follow:

1. The planet acts on a particle at θ=θ_p (modulo 2π) only, i.e. the action takes place when the particle and the planet are at conjunction.

2. If the distance $|x|$ of the particle to the planet orbit, when in conjunction, is smaller than some threshold value x_s (x_s is called the separatrix distance which is roughly the half-width of the Horseshoe-region) then the particle is reflected with respect to the planet orbit from the coordinate $(r_p \pm |x|; \theta)$ to the coordinate $(r_p \mp |x|; \theta)$ in order to mimic the horse-shoe orbits and the CRT. If the distance of the particle to the planet is larger than the threshold value, when in conjunction, the particle is shifted $(\Delta r^{LRT})_i$ radially. This is done in order to mimic the Lindblad resonance torque action instead.

3. The shifts for both the particles and the planet are instantaneous.

4. The particle velocity is assumed everywhere to be the velocity in the unperturbed, nearly circular, disk. The planets angular velocity is also nearly circular Keplerian velocity and the interactions between the particles and the planet do not change this except for the change in radius which translates into a new Keplerian velocity next timestep, i.e. the planet have no eccentricity excitation nor damping (since roughly zero) since the damping term is normally slightly larger [22].

5. The particles are assumed to be point mass particles with equal mass, M_i.

6. Both the Lindblad resonance torque and co-rotational torque action between the point mass particles and the planet are based on impulse approximations of the torques derived in sections 2.3 and 2.4 respectively.

7. There is no accretion.

Assume we define two variables as $\Delta\theta_{new}(t) = \theta_p(t) - \theta_i(t)$ and $\Delta\theta_{old}(t) =$

$\Delta\theta_{new}(t_0) = \theta_p(t_0) - \theta_i(t_0)$, where $\theta_p(t)$ and $\theta_i(t)$ are the new values of the azimuthal coordinate of the planet and the particle respectively at the timestep under consideration and $\theta_p(t_0)$, $\theta_i(t_0)$ are the azimuthal coordinates of the planet and the particle respectively during the timestep just before. If one of the two variables are 0 it means that $\theta_p = \theta_i$ and thus the particle and the planet are at conjunction. Now assume $\Delta\theta_{new}(t)$ and $\Delta\theta_{old}(t)$ are both positive or both negative, i.e. the particle does not pass the planet during the timestep, then the product of these two variables is positive but if one variable is positive and the other is negative, i.e. the particle pass the planets azimuthal coordinate, then the product is negative.

If both the particle and the planet are represented by an azimuthal coordinate between 0 and 2π (or $-\pi$ and π) modulus 2π, then it is possible for a particle to jump from e.g. 2π to an azimuthal coordinate slightly above 0 (or vice versa), which in turn also may produce a change in the sign of the new $\Delta\theta$ variable compared to the old variable above. In a try to mimic the particles correct behaviour in the azimuthal direction this has to be considered and thus we need a coupling constant, here called the particleshift(t), which is -1 whenever a particle is performing a jump across the 2π-0 (or $-\pi$ - π) edge and 1 whenever else, in order to be able to take into account the effect of the variables.

In a non-corotating frame a coupling constant is also necessary to mimic the 2π-0 (or $-\pi$ - π) edge jump of the planet, which also may give a wrong sign of the new $\Delta\theta$ variable compared to the old variable. This coupling constant, here called the planetshift(t), is also -1 when the planet jumps over the edge and 1 whenever else. Another effect is also necessary to take into account for non-corotating frames. This is the possibility of an CRT interaction which causes the new $\Delta\theta$ variable change sign when it should not. In this work we use a corotating frame and so we leave the discussion of non-coroating frames with these remarks.

If the product of the 3 (5 for non-corotating frames) variables is negative it means that a particle is passing the conjunction and thus has been in conjunction and an interaction should take place. If instead the product is non-negative nothing happends and no interaction do take place.

When the action between a particle and the planet takes place, this action gives a back action on the planet (see section 3) which, in turn, give the planet a slight disturbance in the radial direction, called Planet migration. Here in this work the planet do not move radially within one timestep (i.e. after each encounter) but sum all contributions of the interacting particles that occurs during a timestep. This sum then translates into a planet jump Δr_p which give the new planet radius at the end of each timestep. This is possible since we assume that the interactions between the planet and the particles are instant.

In this work we assume that the co-rotational region is separated by a separatrix which lies roughly $|x_s|$=2.3 r_L from the planet radius (which is roughly in the middle of the interval that Ida et al. 2000[43] got). Both the outer and the inner separatrices are equally distant. The interactions is discussed in more detail in sections 2.1-2.5 and 4.3-4.4.

Each particle moves all the time with a Keplerian velocity. In a corotating frame this speed is adjusted with the planets velocity too so that it produces a velocity shear which is roughly equal to the $v_{rel}(r)$ used in section 2.3.

All particles have the same mass, M_i, which is determined by the total mass, M_D, of the disk. In turn, this total disk mass is determined by the value of the diskmass parameter $q = \frac{\pi\,\Sigma_0\,r_{p0}^2}{M}$, where Σ_0 is the local surface density near the planets initial radius, and the distribution of the disk (here determined by the index β since we assume both the disks initial inner radius and the initial outer radius to be the same for all runs and only varies for one- and two-sided disks respectively: $R_{IN} = 0.4\,r_{p0}$ and $R_{OUT} = 2.5\,r_{p0}$ for two-sided disks while $R_{IN} = 0.4\,r_{p0}$ and $R_{OUT} = 1.0\,r_{p0}$ for the inner one-sided disks and $R_{IN} = 1.0\,r_{p0}$ and $R_{OUT} = 2.5\,r_{p0}$ for the outer one-sided disks). In this work both q and β are free parameters when the other is held constant. The value of the diskmass parameter $q \approx 0.002 - 0.005$. This has been determined by observations and thus we use these values in this work.

There is no accretion that may empty the co-rotation region so the only mechanism that may empty the co-rotation region is the migration itself along with some small viscous effect.

4.1.2 Initial Disk Distribution

For a disk with an inner disk radius R_{IN} and an outer disk radius R_{OUT} the total disk mass is

$$M_D = \int_0^{2\pi} \int_{R_{IN}}^{R_{OUT}} \Sigma\, r dr d\theta \tag{59}$$

where $\Sigma = \int_{-\infty}^{\infty} \rho\, dz$ is the disks vertically averaged surface density. In this work we assume the initial distribution of the surface density to be a radial power law distribution according to

$$\Sigma = \Sigma_0 \left(\frac{r}{r_{p0}}\right)^{\beta} \tag{60}$$

where Σ_0 is the local surface density at the radius of the initial planet position, r_{p0}. This Σ_0 is determined by the diskmass parameter q which we, together with the power law index β, use as a free parameter in this work, according to

$$q = \frac{\pi\, r_{p0}^2\, \Sigma_0}{M} \tag{61}$$

where M is the mass of the central star.

The particle mass is derived from the total disk mass $M_D = N_{TOT}\, M_i$ defined as eq. 59 so

$$M_i = \frac{M_D}{N_{TOT}}$$

and normalized to the inner disk mass calculated from the constant Σ_0 the particle mass is

$$\frac{M_i}{q\,M} = \frac{\int_0^{2\pi} \int_{R_{IN}}^{R_{OUT}} \Sigma_0 \left(\frac{r}{r_{p0}}\right)^{\beta} r\, dr d\theta}{\int_0^{2\pi} \int_{R_{IN}}^{r_{p0}} \Sigma_0 \left(\frac{r}{r_{p0}}\right)^{\beta} r\, dr d\theta} = \frac{\left[\left(\frac{R_{OUT}}{r_{p0}}\right)^{\beta+2} - \left(\frac{R_{IN}}{r_{p0}}\right)^{\beta+2}\right]}{\left[1 - \left(\frac{R_{IN}}{r_{p0}}\right)^{\beta+2}\right]} \tag{62}$$

The probability p of finding a certain mass within a certain radius R is

$$p = \frac{\int_0^{2\pi} \int_{R_{IN}}^{R} \Sigma\, r dr d\theta}{\int_0^{2\pi} \int_{R_{IN}}^{R_{OUT}} \Sigma\, r dr d\theta} \tag{63}$$

where the value of p is a number between 0 and 1.

When we initially distribute the particles we do it randomly by giving p a random number between 0 and 1 and from this we get the initial radius R of the particle through the equation

$$\frac{R}{r_{p0}} = \left[p \left(\frac{R_{OUT}}{r_{p0}} \right)^{\beta+2} + (1-p) \left(\frac{R_{IN}}{r_{p0}} \right)^{\beta+2} \right]^{\frac{1}{\beta+2}} . \tag{64}$$

This randomly distributes the particles initial radius according to the power law distribution corresponding to the value of β. The initial particles distribution in the azimuthal direction is derived similarly

$$\theta = \left[p \left(\theta_{OUT} \right)^{\gamma+1} + (1-p) \left(\theta_{IN} \right)^{\gamma+1} \right]^{\frac{1}{\gamma+2}} . \tag{65}$$

but in this work we assume the initial azimuthal distribution to be axi-symmetric and thus always use $\gamma = 0$ and $\theta_{IN} = -\pi$ and $\theta_{OUT} = \pi$ so we get an axi-symmetric azimuthal distribution.

In this work we examine the distributions β=-1.5, -1, -0.5, 0, 0.5, 1 and 1.5 and the diskmass parameter q=0.002-0.005. Distributions with negative β have higher number (surface) density closer to the central star while positive βs give a number distribution that is higher farther away from the central star. The solar nebula has a distribution that is similar to β=-1.5 why the planets behaviour in this type of distributions is particularly intresting.

4.1.3 Dynamics of Particles

For systems with a planet and particles rotating a central star the equation of motion for the particles is

$$M_i \frac{d^2\vec{r}}{dt^2} = -\frac{GMM_i}{r^3}\vec{r} + \vec{F}_{RAD} + \vec{F} \tag{66}$$

where in this work the radiative force $\vec{F}_{RAD}$ is assumed neglible and $\vec{F} = -\frac{GM_pM_i}{(r_p-r)^3}(\vec{r}_p - \vec{r})$ is an additional gravitational force due to the planet. But in this work this additional force is assumed to act only when the particle and the planet are in conjunction, from assumption 1, giving rise to a change in the angular momentum and this additional force thus does not affect the equation of motion when not in conjunction (see the prescriptions above). Therefor we have for the particles not in conjunction

$$M_i \frac{d^2\vec{r}_i}{dt^2} = \frac{GMM_i}{r^3}\vec{r} \tag{67}$$

and from this we get the normal Keplerian velocity under the assumption that the particles travel in nearly circular orbits, from assumption 4. On the other hand, for the particles in conjunction the planets disturbance cannot be neglected and we have

$$M_i \frac{d^2\vec{r}_i}{dt^2} = \frac{GMM_i}{r^3}\vec{r} + \frac{GM_pM_i}{|\vec{r}_p - \vec{r}|^3}(\vec{r}_p - \vec{r}) \tag{68}$$

where $\frac{GM_p}{|\vec{r}_p-\vec{r}|^3}(\vec{r}_p - \vec{r})$ is exactly the term $\frac{GM_p}{b^2}$ used in equation 25 (if $b = r_p - r$), which gives the slight change in the particles orbit, Δv_r. This then leads to the change in angular momentum (and radius) described in more detail in sections 2.3 and 2.4.

For a particle in a circular Keplerian disk we have

$$\theta_{t_0+\Delta t} = \theta_{t_0} + \Omega_K(r)\,\Delta t \tag{69}$$

and for a particle in a circular Keplerian disk in the frame corotating with the planet at $r_p(t_0)$ we have

$$\theta_{t_0+\Delta t} = \theta_{t_0} + (\Omega_K(r) - \Omega_K(r_p(t_0)))\,\Delta t\,. \tag{70}$$

The change in specific angular momentum for a particle in nearly circular motion is defined as

$$\Delta l^{LRT} = \sqrt{GMr} - \sqrt{GMr_0} \tag{71}$$

where r_0 is the old radial coordinate and r is the new radial coordinate. From this we get (see appendix section A.4)

$$\Delta r_i^{LRT} = \frac{1}{GM}\,(\Delta l_i^{LRT})^2 + 2\sqrt{\frac{r_0}{GM}}\,\Delta l_i^{LRT} \tag{72}$$

where $\Delta r_i^{LRT} = r - r_0$ and

$$\Delta l_i^{LRT} = \pm\frac{16}{27}\,\mu^2\,(r_p^2\,\Omega_K(r_p))\left(\frac{b_i}{r_p}\right)^{-5} \tag{73}$$

so to the first order of specific angular momentum

$$\Delta r_i^{LRT} \approx \pm\frac{32}{27}\,\mu^2\,\sqrt{r_0\,r_p}\left(\frac{b_i}{r_p}\right)^{-5} \tag{74}$$

and (due to the jump over the CR-region)

$$\Delta r_i^{CRT} \approx \mp 2\,b_i \tag{75}$$

for outer (top) and inner (bottom) disc particles respectively.

We also need a viscosity of the disk. In this work we derive this from the definition of the viscous timescale. The viscous timescale is defined through

$$t_\nu = \frac{r^2}{\nu} \tag{76}$$

so during a timestep Δt a particle has moved, due to an assumed constant α-prescription viscosity ($\nu = \alpha\,H\,c_s(r)$ where $c_s(r) = H\,\Omega_K(r)$ is the sound speed of the disk at radius r) introduced by Shakura-Sunyaev 1973 [38],

$$\begin{aligned}\Delta r &= \sqrt{\nu\,\Delta t} \approx \sqrt{\alpha\,c_s(r)\,H\,\Delta t} = \sqrt{\alpha\,\frac{c_s(r)}{\Omega_K(r)}\,H\Omega_K(r)\,\frac{r^2}{r^2}\,\Delta t} = \\ &= \sqrt{\alpha\left(\frac{H}{r}\right)^2 (r^2\,\Omega_K(r))\;\Delta t} \approx (\alpha\left(\frac{H}{r}\right)^2\;\Delta\tau)^{\frac{1}{2}}\left(\frac{r}{r_{p0}}\right)^{\frac{1}{4}} r_{p0}.\end{aligned} \tag{77}$$

This is the shift in radius of a particle due to the viscosity. As one can see from the definition, eq. 76, the shift is depending on the time so it is not strange that the shift here is depending on the timestep $\Delta\tau$. One can also see that the shift depends on the radius to the power of one fourth. In this work we use a low viscosity, with $\alpha = 10^{-4}$ and the aspect ratio $\frac{H}{r} = 0.05$ so the shift in each timestep is very small.

4.1.4 Dynamics of the Planet

Assumption 1 and 2 in the code description gives that when particles are in conjunction with the planet interactions between the planet and the particles takes place whereby the planets radius changes, due to the back-action of the interactions (see section 3.1), according to

$$r_p(t_1) = r_p(t_0) + \Delta r_p \tag{78}$$

each timestep. Here

$$\Delta r_p = \sum_{i=1}^{N_{TOT}} (\Delta r_p)_i \tag{79}$$

is the sum of the contributions to the planet migration from all interacting particles during a timestep and

$$(\Delta r_p)_i = \frac{-2}{M_p\, r_p\, \Omega_K(r_p)}\, (\Delta L_{P\to D})_i \tag{80}$$

where $(\Delta L)_i = (\Delta L^{CRT})_i$ or $(\Delta L)_i = (\Delta L^{LRT})_i$ (see above in sections 2.3 and 2.4) depending on wether the particles, corresponding to the interactions, lies within the corotating region or not (from assumption 2).

From the assumption 4 in the code description we have the situation that the planet travel around the central star in nearly circular Keplerian orbits and

$$\theta_p(t_0 + \Delta t) = \theta_{p0}(t_0) + \Omega_K(r_p(t_0))\, \Delta t \tag{81}$$

and if we instead watches the situation in the coordinate frame corotating with the planet we have

$$\theta_p(t_0 + \Delta t) = \theta_{p0}(t_0) + (\Omega_K(r_p(t_0)) - \Omega_K(r_p(t_0)))\, \Delta t = \theta_{p0}(t_0) \tag{82}$$

where $\theta_{p0}(t_0)$=0 is the initial planet position.

In this work we want to examine wether it is possible to get a runaway migration mode. Masset and Papaloizou 2003 [46] have achieved a result that show that the planet mass that most easily get a runaway migration is a Saturn mass planet. Therefor we chose such a planet in this work.

4.1.5 Units and Setup

The unit of planet and particle radius is r_{p0}, which is the initial radius of the planet, e.g. 1.0 AU for an earth and 5.2 AU for a Jupiter. The unit of time $\Delta t = \frac{1}{\Omega(r_{p0})}\, \Delta\tau = \frac{\Delta\tau}{2\pi}\, P(r_{p0})$, which is equal to the period of the planet with the initial radius r_{p0}. This period is for an earth 1 year and for a Jupiter roughly 12 years. All masses are given as a fraction of the central star, both the planet mass μ and the diskmass parameter q. Here we mostly examine planets around a Sun which have $M \approx 2\,10^{30}$ kg.

4.1.6 The Computer Program

In this work we used Fortran 90 to make a program that produced dumps of data. After the simulations was done, these dumps of data were read by an IDL program that plot the graphics of these simulations.

4.2 Results

First we examine the effects of different initial disk distributions and different diskmass parameters on the migration of planets in two-sided disks. Then we compare the migration of a planet in a two-sided disk to the migration of a planet in an one-sided disk, i.e. a planet that lie either inside an outer disk or outside an inner disk respectively. We also examine the dependence of the exchange of specific angular momentum on the distance to the planet in order to see what type of torque, LRT or CRT, is the largest. Last we present a sequence of surface density pictures for both the two-sided and one-sided disks.

4.2.1 Planet inside a two-sided disk - runaway migration or adjustment to the initial distribution?

Assume we have a disk with an initial surface density profile with a power law distribution, then the planet may lie within an initial surface density gradient. How does a planet interact with such a disk? It is this works outset to use a simple particulate model to examine what happends with the planet when it interacts with such a disk. In sections 2.1-2.5 and 4.1 we have described the theory and the model used in this work. Here we present the results of the simulations.

First we examine what happends with the planets migration when we have disks with the same initial disk distribution, i.e. when we have a constant β, and change the diskmass parameter q. Figure 3 and 4 show the evolution of a planet inside a two-sided disk with β=-1.5, -1.0, -0.5, 0.0, 0.5, 1.0 and 1.5. We have 6 or 7 runs in each graph which each represents a value of q=0.0020, 0.0025, 0.0030, 0.0035, 0.0040, 0.0045 or 0.0050. On the x-axis we have the time in the planets initial period and on the y-axis we have the planets distance to the central star in the planets initial radius why the planet starts at 1. The planet moves radially inwards a bit for negative β and moves radially outwards a bit for positive β in a pattern similar to that of a damped sinusoidal curve as it adjusts itself to the initial disk distribution. For the more massive runs the amplitude of the migration (the maximum migrated distance from the initial planet position) are somewhat bigger than for the less massive runs. The difference though, is almost neglible except for the run in figure 4 with β=1.5 and q=0.0040 where the mass (surface density) gradient is so steep that it almost looks more like the situation with the planet inside an outer disk (see below) than a two-sided disk. It seems like the value of q have very little effect on the migration for the distributions -1$\geq \beta \geq$1.

Further, for the planets inside a two-sided disk there are not much migration (the planet always lies within 0.90-1.10 r_{p0}) over 200 - 500 periods at all. The planet migrate toward the larger surface density due to the fact that the effect of CRT is one order of magnitude larger than the effect of LRT (see below). Since the planet are very mobile, sensitive to surface density gradients, it feels fast how to migrate and when the surface density profile changes direction so do the planets migration. In less then roughly 200 orbits it has more or less reached the disks "equilibrium point", where the CRT from the outer disk balances the CRT due to the inner disk. Now there are no steep gradients (or rather a gradient that causes the CRTs from both CR regions to be equal but opposite) and the migration due to CRT is reduced significantly. The planet should now more or less settle radially and the effect of LRT should be more important.

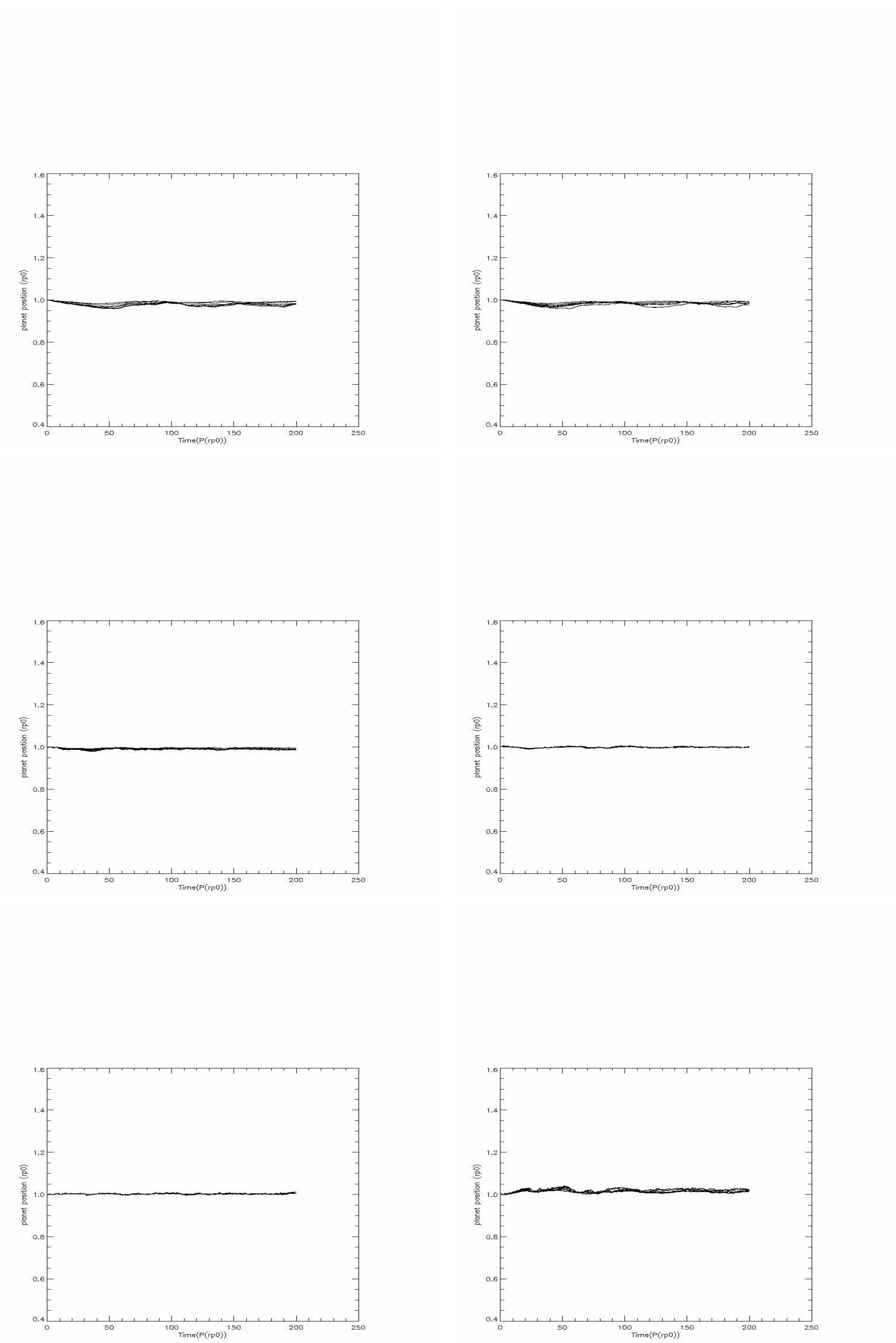

Figure 3: The temporal evolution of a planet inside a two-sided disk with β=-1.5 (top left), -1.0 (top right), -0.5, 0.0, 0.5 (bottom left), 1.0 (bottom right). We have 7 or 6 runs in each graph which each represents a value of q=0.0020, 0.0025, 0.0030, 0.0035, 0.0040, 0.0045 or 0.0050. On the x-axis we have time in initial planet period and on the y-axis we have the planets distance to the central star normalized to the planets initial orbit.

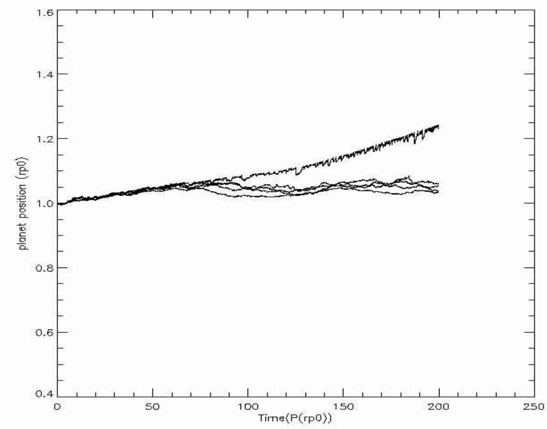

Figure 4: The temporal evolution of a planet inside a two-sided disk with β=1.5. We have 5 runs which represents a value of q=0.0020, 0.0025, 0.0030, 0.0035 and 0.0040 respectively. On the x-axis we have time in initial planet period and on the y-axis we have the planets distance to the central star normalized to the planets initial orbit.

In figures 5 and 6 we instead show the evolution of a planets migration when the diskmass parameter q is held constant and the initial disk distribution changes, β. On the x-axis we have the time in the planets initial period and on the y-axis we have the planets distance to the central star in the planets initial radius. We let q=0.0020 (top left in figure 5), 0.0025, 0.0030, 0.0035, 0.0040, 0.0045 or 0.0050 (figure 6) respectively and change in each run the value of β= -1.5, -1.0, -0.5, 0.0, 0.5, 1.0, 1.5. There were quite a big difference, a factor of several, in the planets migration for disks with different initial disk distributions. Actually there were more or less no migration, less than 2 percent of the initial planet radius, for the planets in disks with distributions of $-1 \leq \beta \leq 1$ while there were a migration of up to 5 percent for planets in the more steep disk distributions with β=-1.5 and β=1.5. Another interresting result is the difference in direction of the migration between β=-1.5 and β=1.5. We get inward migration for β=-1.5 and outward migration for β=1.5. This means that outward migration is possible for power law distributions with non-negative index β.

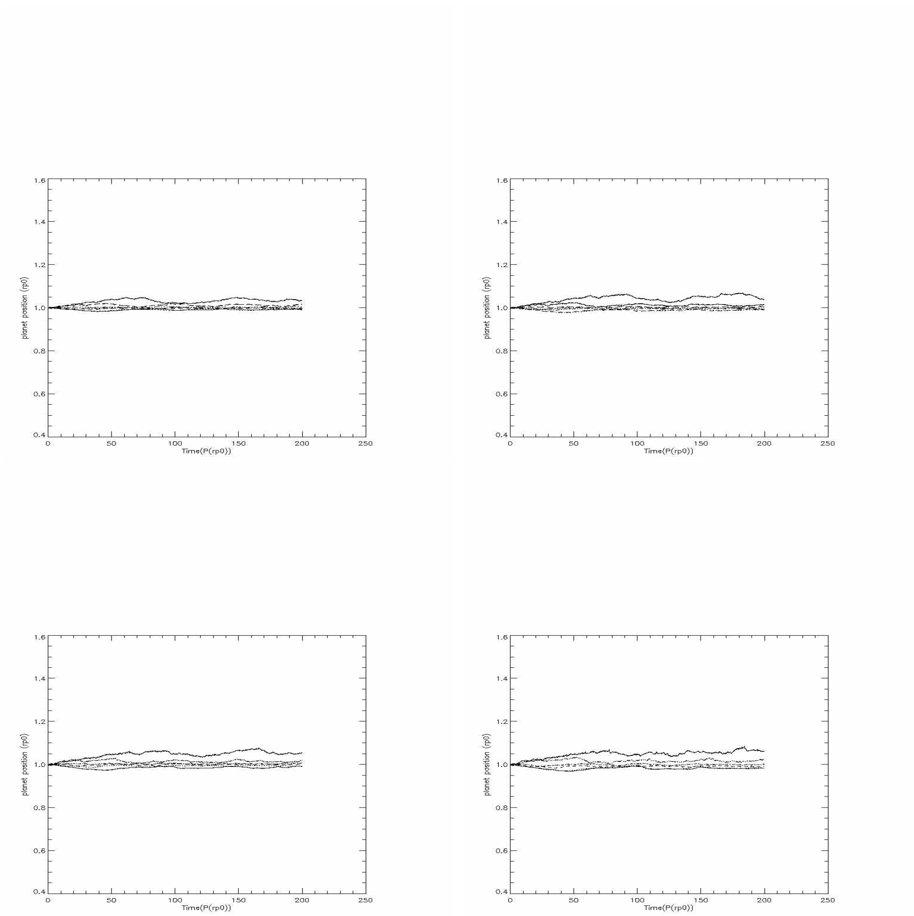

Figure 5: The temporal evolution of a planet inside a two-sided disk, when the diskmass parameter q is held constant while changing the initial disk distribution, β. Here we have held q= 0.0020 (top left), 0.0025, 0.0030 or 0.0035 respectively and changed in each run the value of the initial disk distribution β= -1.5, -1.0, -0.5, 0.0, 0.5, 1.0, 1.5.

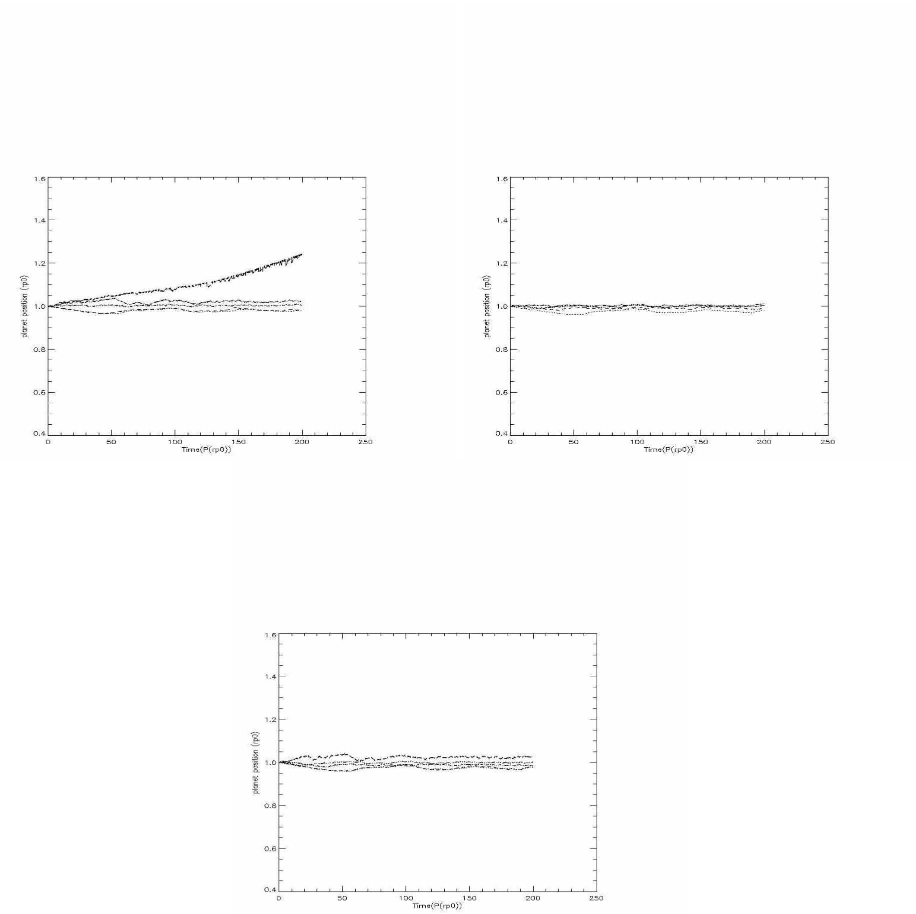

Figure 6: The temporal evolution of a planet inside a two-sided disk, when the diskmass parameter q is held constant while changing the initial disk distribution, β. Here we have held q= 0.0040 (top left), 0.0045 or 0.0050 respectively and changed in each run the value of β= -1.5, -1.0, -0.5, 0.0, 0.5, 1.0, 1.5.

4.2.2 Planet inside or outside a one-sided disk - runaway migration?

Now we assume that we have a planet initially outside an inner disk and that there is no outer disk. In figure 7 we present the results of the simulations for the constant disk mass parameters q=0.0020 (top left), 0.0035 and 0.0050 respectively and examine the migration of the planet under the most extreme initial disk distributions β=-1.5, 0 and 1.5. On the x-axis we have the time in the planets initial period and on the y-axis we have the planets distance to the central star in the planets initial radius.

For inner disks with the same diskmass parameter q the two disk distributions β=0 and β=1.5 gave a very large migration inward ($r_p < 0.9\, r_{p0}$) first but then gave a result similar to that of a two-sided disk and did not migrate much more. For β=-1.5 instead we got a runaway migration all the way to $r_p < 0.6\, r_{p0}$ within 300 periods (see below) before the planet settles and also behaves like a two-sided disk.

For a planet that lies in a β=-1.5 distribution the least massive disk makes the migration of the planet act like the planet was in a two-sided disk. If one increases the value of the disk mass parameter the planet feels a surface density gradient which promotes more migration, i.e. the number (surface) density increases in the same direction as the planets migration, and we get a fast "runaway" migration (of roughly 25-35 percent of the initial planet radius within 200 orbits). How it continues to migrate over 500 periods is presented in section 4.2.4.

The other two distributions, β= 0 and β= 1.5, would normally promote roughly no migration or outward migration (i.e. their number, surface, density increases in the opposite direction as the planet migration). Now on the other hand the planet only feels particles in the inner CR region and thus promotes inward migration anyway until it reaches the "equilibrium point" and we get a behaviour of the planets migration that is similar to the behaviour of a planet in a two-sided disk, though with a slightly translated (shifted) "equilibrium point", due to the initial migration phase. The planet now more or less stops.

When we examine inner one-sided disks with the same initial disk distribution but different diskmass parameters q, figure 8, there are similar patterns of migration for all values of the diskmass parameter, except for the runs with β=-1.5 and q$\geq$0.035 for which there are a runaway migration. The planets in the other runs migrate fast to an "equilibrium point" within roughly 0.9 r_{p0} to 0.8 r_{p0} and stops and behaves similar to a planet in a two-sided disk. We also have larger migration for larger mass even though the difference is small as for two-sided disks. The migration is inward toward the star.

In figure 9 we place a planet initially inside an outer disk and asssume there is no inner disk. On the x-axis we have the time in the planets initial period and on the y-axis we have the planets distance to the central star in the planets initial radius. We present the results of the simulations for the constant disk mass parameters q = 0.0020, 0.0035 and 0.0045 (for q= 0.0050 the total diskmass is too big, $M_D \geq 0.10$ M, and this does not make sense) respectively and examine the migration of the planet under the most extreme initial disk distributions β= -1.5, 0 and 1.5 just as above.

One can see that the evolution of a planet initially inside an outer disk is similar to that outside an inner disk but give outward migration instead. Again it is the evolution of the β= -1.5 distribution that deviates from the other two distributions and behaves like a planet in a two-sided disk. For both β= 0 and β= 1.5 there is a fast "runaway" migration (of between 20-40 percent of the initial planet radius) within 200 orbits.

When we examine outer one-sided disks with the same initial disk distribution but with different values of the diskmass parameters q, figure 10, there are similar patterns as for inner one-sided disks but here it is reversed so that the distribution β=-1.5 do not migrate much but for the other distributions there are runaway migration. There are slightly more migration for larger diskmasses (could be difficult to see in the figures since we have such large fluctuations for β=1.5) but also here the difference is very little. Also the direction of the migration is outward and thus opposite to inner one-sided disks as suggested by theory.

In the outer disk simulations there are large fluctuations. These probably occur due to the fact that each particles back-action on the planet during a

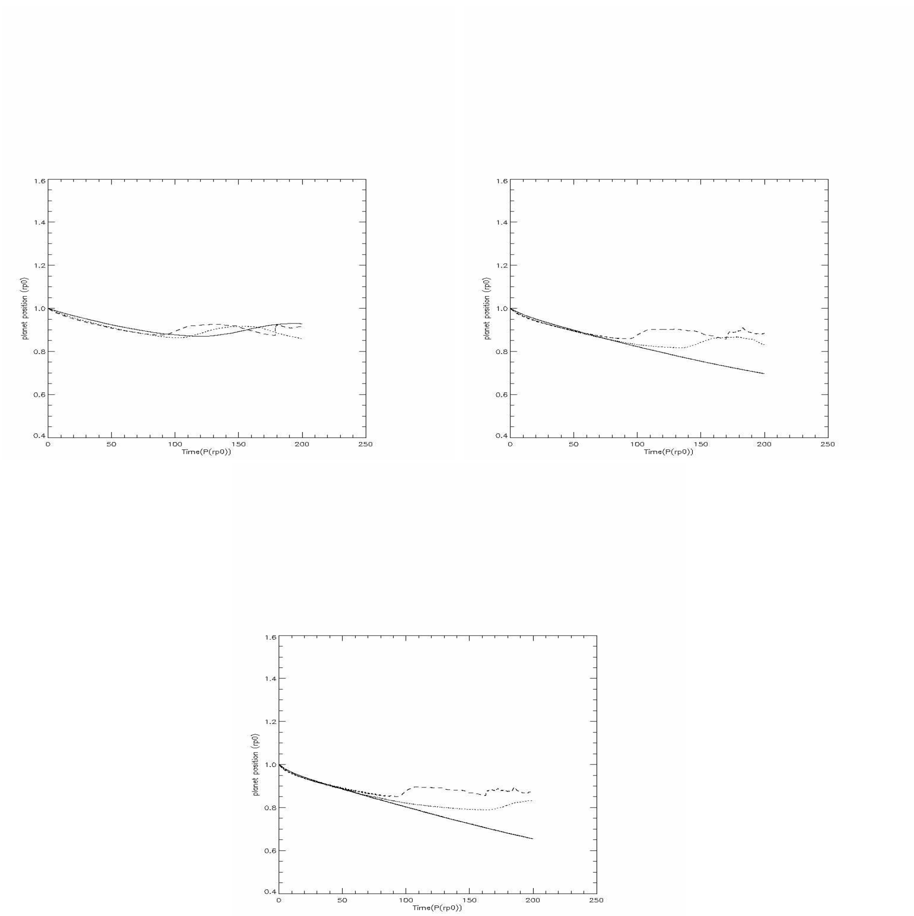

Figure 7: The temporal evolution of a planet only outside an inner disk and no outer disk for disk mass parameters q = 0.0020 (top left), 0.0035 and 0.0050 and with the initial disk distributions β = -1.5 (solid line), 0 (dotted line) and 1.5 (dashed line).

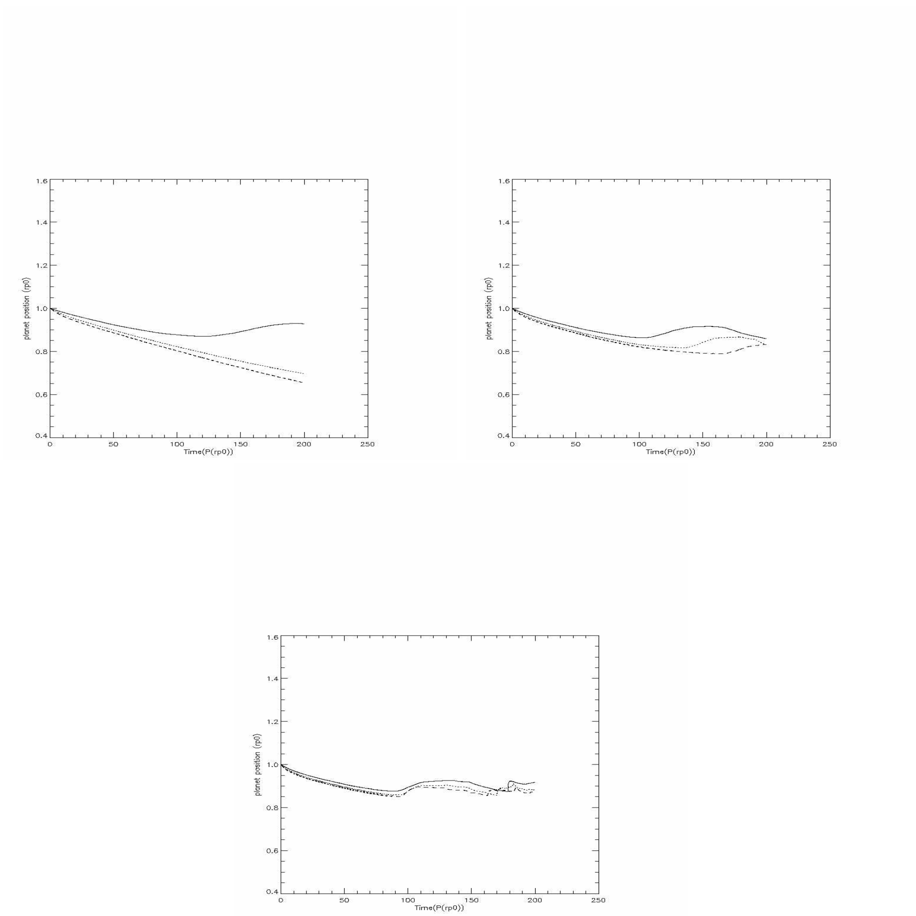

Figure 8: Temporal evolution of a planet only outside an inner disk and no outer disk for the initial disk distributions β = -1.5 (top left), 0 and 1.5 and with the disk mass parameters q= 0.0020 (solid line), 0.0035 (dotted line) and 0.0050 (dashed line).

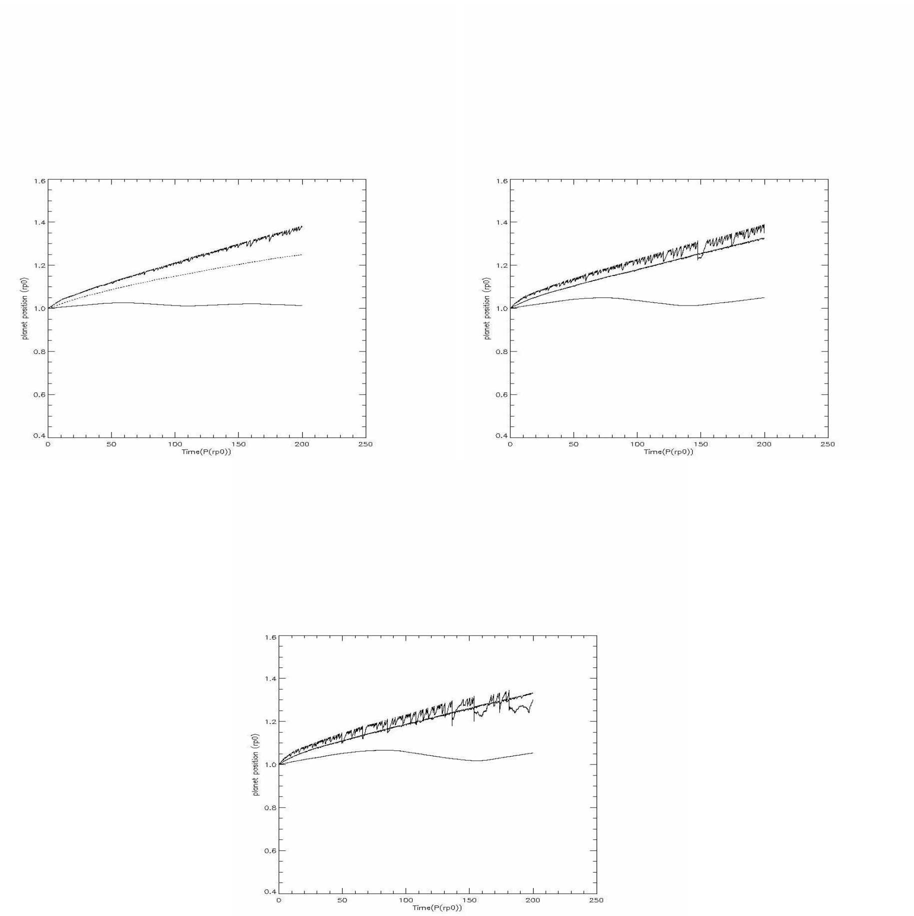

Figure 9: Temporal evolution of a planet only inside an outer disk and no inner disk for disk mass parameters q = 0.0020 (top left), 0.0035 and 0.0045 and with the initial disk distributions β= -1.5 (solid line), 0 (dotted line) and 1.5 (dashed line).

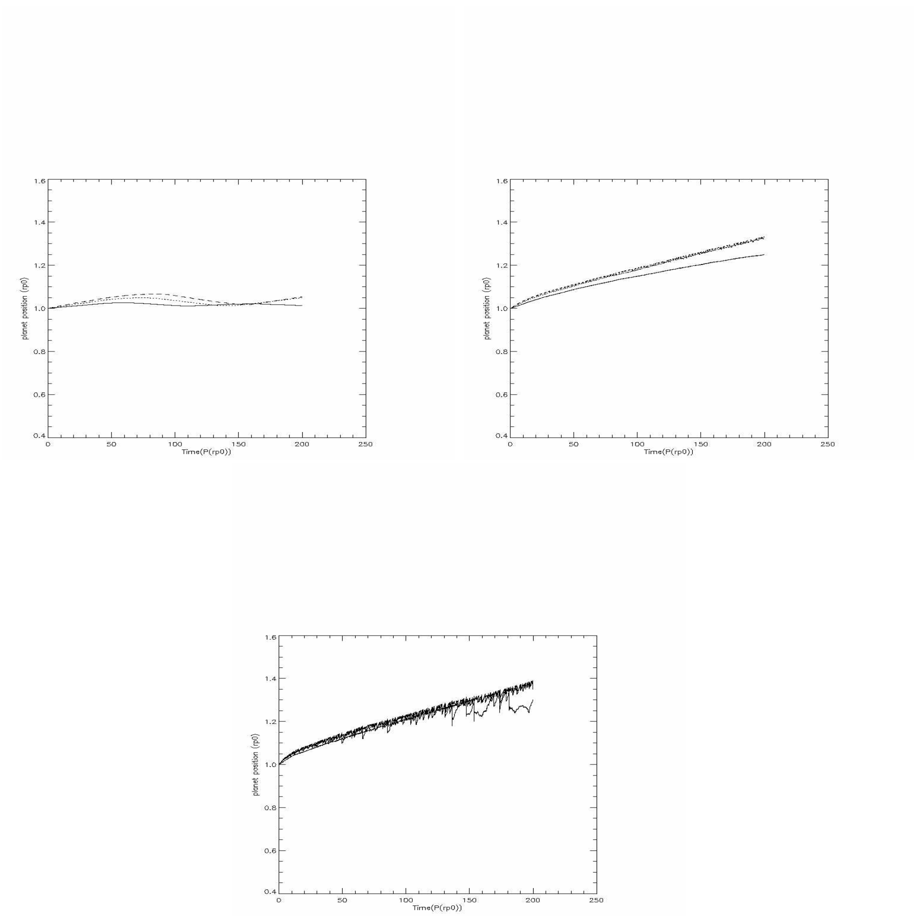

Figure 10: Temporal evolution of a planet only inside an outer disk and no inner disk for the initial disk distributions β = -1.5 (top left), 0 and 1.5 and with the disk mass parameters q= 0.0020 (solid line), 0.0035 (dotted line) and 0.0045 (dashed line).

timestep are added each timestep and affect the planet after each timestep instead of each interaction affect the planets migration directly but this is for practical reasons in the computerprogram.

The big difference in planet migration between one-sided disks and two-sided disks is that the migration due to close encounters (or CRT) are small for two-sided disks and the migration for one-sided disks is dominated by close encounters (or CRT) and enters a very fast runaway migration mode. This situation may be maintained by the migration itself as long as it is large enough to always send the interacting particles in the CR region outside the CR region, or outside the separatrix on the opposite side of the planets radius during a half a libration period, and thus maintain the azimuthal assymetry of the CR region, i.e. the difference between the interacting CR particles outside and in front of the planet and the interacting CR particles inside and behind the planet. This means that for planets in one-sided disks the runaway migration decreases or stops as the CR region on both sides of the planet get filled and the planet lies within a two-sided disk. These particles in the CR region may be distributed so that a surface density gradient that promote migration still exist in the CR region but migration due to this gradient is not big and the planet more or less stops at an "equilibrium radius". Still the planet may migrate due to a LRT, and enter a type I or type II migration mode. The important feature of the runaway migration mode seems to be the relative emptiness of one side of the CR region relative to the other.

When a planet is placed outside an inner disk the planet migrates fast inward and when the planet is placed inside an outer disk the planet migrates fast outward no matter the distribution. For the distributions that do not "promote" fast migration, i.e. distributions in which the number (surface) density increases in the opposite direction as the planets migration, the planet migrates less and reaches a state which is similar to a two-sided disk much earlier than disk distributions which "promotes" migration, i.e. distributions in which the number (surface) density increases in the same direction as the planet migration.

4.2.3 The dependence of specific CRT and LRT on radius

Figures 11 and 12 show the dependance of the exchange in angular momentum on the distance to the planet when a particle in the disk interacts with a planet. On the x-axis we have the radius from the planet in roche lobes $\frac{b_i}{r_L}$ and on the y-axis we have the normalized exchange in specific angular momentum $\frac{\Delta l_i}{r_p\,\Omega_K(r_p)\,r_L}$. Δl_i was taken from eq. 21, 23, 33 and 35. The interaction is due to both close encounters (CRTs), which occurs when the particle is inside the separatrices (see assumption 2 in the method, section 4.1.1), and distant encounters (LRTs), which occurs when the particle is outside the separatrices. The both separatrices lies at $x_s^- \approx r_p - 2.3\,r_L$ and $x_s^+ \approx r_p + 2.3\,r_L$. This can also be seen in both figures. Each figure is divided into two graphs in order to show the behaviour of both the close and distant encounters despite the difference in scale. Interesting in all these figures are that the distant encounters and the close encounters on the same side of the planet have different signs as the theory in sections 2.3 and 2.4 suggest. Another intresting result is that the exchange in angular momentum of the close encounters is linear also as suggested by theory.

One can also see that the exchange in angular momentum of the close encounters are more than one order of magnitude larger than the exchange of

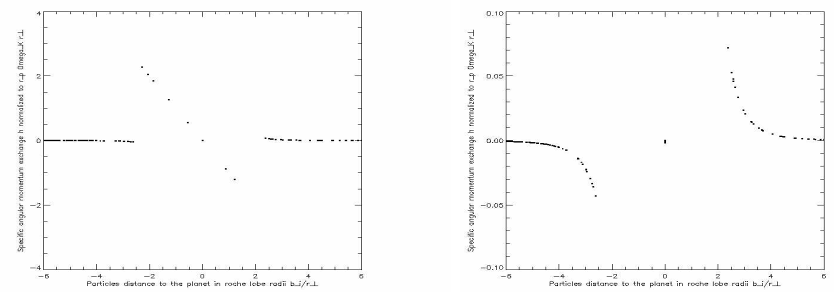

Figure 11: The dependance of specific angular momentum exchange Δl as a function of the distance from the planet $\frac{b}{r_L}$. One can see that the exchange is linear and more than one order of magnitude larger for the close encounters inside the CR-region than for the distant encounters outside the CR region. Here we have the situation for a two-sided disk and we can see that the effect of CRT are roughly balancing, even though the inner CRT is slighly larger, each other why we only have a small migration inwards. Planet migration is directed opposite to the direction of the specific angular momentum exchange of the particles due to equation 51.

angular momentum of the distant encounters. For the distant encounters one can also notice that most of the effect comes from very close to the separatrices. As a matter of fact the exchange of angular momentum drops a factor of several up to an order of magnitude within $2r_L$ from the separatrices. The results is similar to figure 5 in Ida et al. 2000 [43].

In figure 11 we have a two-sided disk and in figure 12 we have a one-sided disk. The biggest difference between the two figures is that in the first there is close and distant encounters on both sides of the planet which counteracts each other and thus the total effect on the planet is not as great as it is in the latter situation where the close and distant encounters are not counteracted. It is this difference in the gradient in the CR region that feeds the fast migration. As long as there are close encounters only on one side the planet migrates fast but as soon as there are close encounters on both sides of the CR region the fast migration slows down and when the close encounters from both the inner CR region and the outer CR region balance and equal out there is no migration due to close encounters. There might still be a migration due to an imbalance between the inner and outer LRT and the planet may enter either type I or type II migration. This has not been examined here but in many earlier works.

4.2.4 Surface density simulations?

In figure 13 we have the long term (over 500 $P(r_{p0})$) behaviour of a Saturn mass planet in a two-sided disk. On the x-axis we have time in initial periods

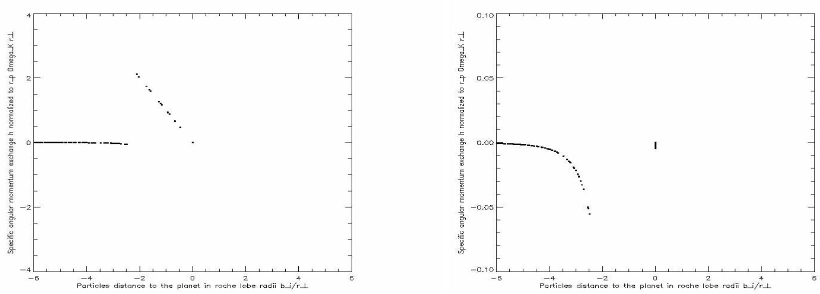

Figure 12: The dependance of specific angular momentum exchange Δl as a function of the distance from the planet $\frac{b}{r_L}$. One can see that the exchange is linear and more than one order of magnitude larger for the close encounters inside the CR-region than for the distant encounters outside the CR region. Here we have the situation for an inner one-sided disk and we still notice that the CRT is more than one order of magnitude larger than the LRT. Since we have an inner one-sided disk there are no CRT in the outer CR region and thus the most dominant angular momentum exchange are the inner CRT and the planet migrates fast inwards. Planet migration is directed opposite to the direction of the specific angular momentum exchange of the particles due to equation 51.

$P(r_{p0})$ and on the y-axis we have planet radius in initial planet radius. The behaviour over 500 $P(r_{p0})$ is not that different than over 200 $P(r_{p0})$. There still is a clear sinusoidal pattern of the planets position as it adjusts itself to the initial conditions.

In figures 14, 15, 16 and 17 we see a two-sided disk with β= -1.5 and q= 0.0050 and how it interacts with a Saturn mass planet initially at r_{p0}. On the x-axis we have the radius from the central star and on the y-axis we have the azimuth. Each point on the plot represents a particle, so darker area means more dense area. The different surface density plots are taken at the times t= 4 (top left), 8, 16, 24, 32 and 40 (bottom right) periods at the planets initial position $P(r_{p0})$ for figure 14, t= 60 (top left), 80, 150 and 180 $P(r_{p0})$ for figure 15, t= 215 (top left), 250, 285, 320, 355 and 390 $P(r_{p0})$ for figure 16 and t= 425 (top left), 460, 495 $P(r_{p0})$ for figure 17.

Since the surface density in the CR region are more or less symmetric and the effect on the planet migration due to close encounters from the inner CRT balances those by the outer CRT there is very little migration. Now the effect on the particles outside the CR region is more visible and one can see the slow formation of a gap. The particles performing distant encounters are moving away from the planets radius. This produces features, white lines where there are no particles, in the surface density plots since there are very little migration of the planet. A large planet migration destroys easily these features, as we will see further down. As the time goes these features moves with the relative velocity relative to the planet and when these features have completed an orbit, a libration orbit, with this relative velocity a gap has opened.

One can see in figure 14 and from the initial disk distribution β=-1.5 that the number density is slightly higher closer to the star. This means that there are more particles in the inner CR region than in the outer CR region. The number of particles that are performing close encounters are therefor more in the inner CR region than in the outer CR region. Thus we should have an inward migration. The inward migration though is too small to cause the particles to end up outside the outer separatrix and after some time, a libration period, the number of particles in both CR regions are equal and thus the effects of the inner and outer CRTs is balancing each other.

In figure 18 we see the long term (over 500 $P(r_{p0})$) behaviour of a one-sided inner disk with β= -1.5 and q=0.0050 and how it interacts with a Saturn mass planet initially at r_{p0}. On the x-axis we have time in initial periods $P(r_{p0})$ and on the y-axis we have planet radius in initial planet radius. One can see, in sharp contrast to the two-sided disk, a fast migration to roughly 0.6 r_{p0} over 300 initial periods and then it get a damped sinusoidal pattern similar to that of a two-sided disk.

In figures 19, 20, 21 and 22 we have the surface density plots of a one-sided inner disk with β= -1.5 and q=0.0050 and how it interacts with a Saturn mass planet initially at r_{p0}. The surface density plots are taken at the same times as for the two-sided disk. Each point on the plot represents a particle, so darker area means a more dense area. One can see, in sharp contrast to the two-sided disk, a fast migration, indicated by the spiral pattern or leaning lines of particles, which has had a close encounter with the planet. This is in accordance to theory [46] and indicate that it is this feature of the one-sided disk that gives the runaway migration mode.

One can also see that as time pass and as the migration occurs the surface

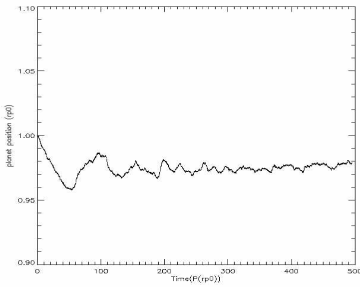

Figure 13: Planet migration of a planet in a two-sided disk with β=-1.5 and q=0.0050. On the x-axis we have time in initial periods $P(r_{p0})$ and on the y-axis we have planet radius in initial planet radius r_{p0}. We see very little migration, less than 5 percent of the initial planet radius r_{p0}, over 500 periods. The pattern of the migration speed is that of a damped sinusoidal curve as the planet and the disk tries to settle to the initial conditions.

density gradient gets pushed more and more towards the separatrices and the steepness of the gradient decreases, which causes the speed of the migration to decrease so that the massflow component decrease. These particles then spreads on both sides and the planet feels a situation similar to a two-sided disks. Also worth to notice is that as the gradient gets low enough and thus the migration gets low enough some effect causes the co-rotating particles (particles at the same radius as the planet) to interact with the planet. Perhaps this has something to do with the change in the direction of the planet migration. This has not been examined here but just noted. This planet now stays at the radius it has migrated to and opens up a gap. Perhaps we now enter a situation similar to that of a type II migration. This has not been examined here but would be intresting to know.

In figure 21 one can see that for the plot at t= 285 $P(r_{p0})$ there is a runaway migration inward. The only particles in the CR region in this plot is in the inner disk, the co-rotating particles inside and behind the planet. In the next plot t= 320 $P(r_{p0})$ we have a situation that is the opposite. Here we have particles only in the outer CR region, co-rotating particles that are outside and in front of the planet, and thus we have an outward migration. Somewhere between t= 285 $P(r_{p0})$ and t= 320 $P(r_{p0})$ the planet has to come across a number, surface, density gradient that alters the direction of the migration. This we also see happends in in figure 18.

The gradient that causes the outward migration, though, is too small to give a fast enough migration to cause the particles interacting via close encounters to end up outside the inner CR region. Instead the particles fills up the entire CR region, both inner and outer. This in turn give a situation very similar to a planet in a two-sided disk. Now what happends if the change in direction of

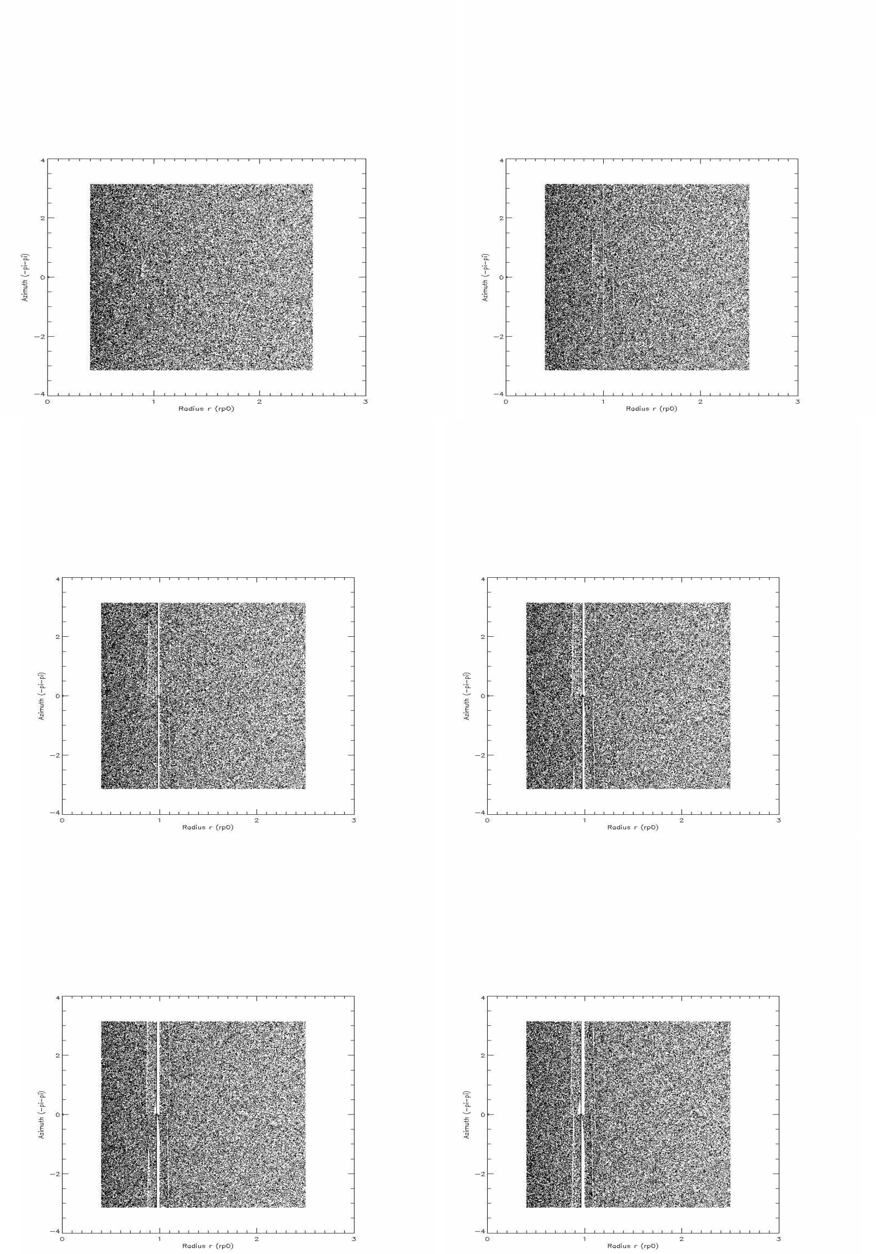

Figure 14: The temporal surface density evolution of a two-sided disk with β=-1.5 and q=0.0050 at the times t= 4 (top left), 8 (top right), 16, 24, 32 (bottom left) and 40 (bottom right) initial periods $P(r_{p0})$. The surface density plots are for 10^5 particles and show very little migration and the "slow" formation of a gap.

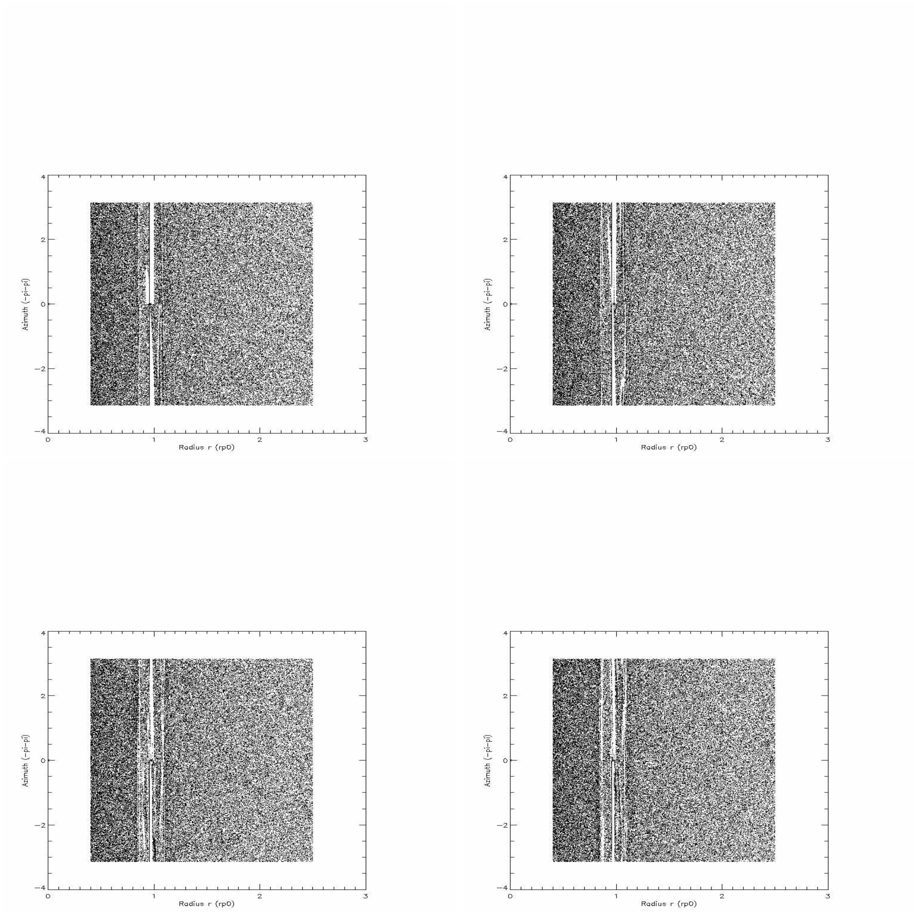

Figure 15: The temporal surface density evolution of a two-sided disk with β=-1.5 and q=0.0050 at the times t= 60 (top left), 80, 150 and 180 (bottom right) initial periods $P(r_{p0})$. The surface density plots are for 10^5 particles and show very little migration and the "slow" formation of a gap.

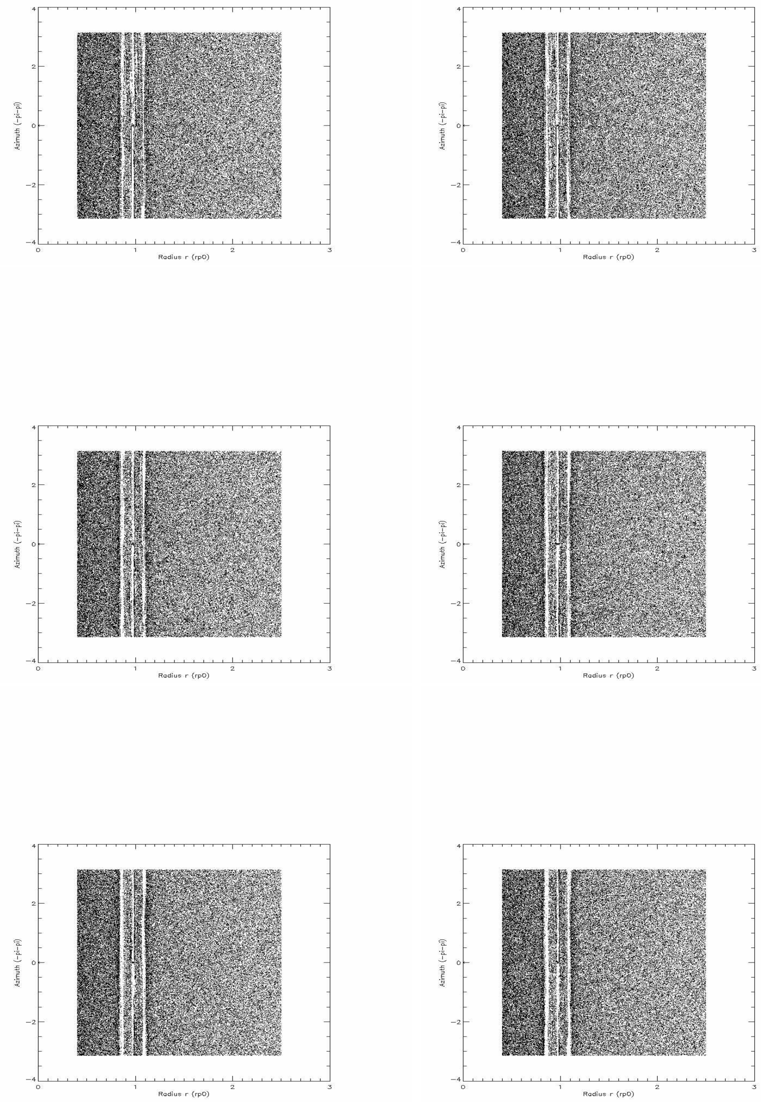

Figure 16: The temporal surface density evolution of a two-sided disk with β=-1.5 and q=0.0050 at the times t= 215 (top left), 250 (top right), 285, 320, 355 (bottom left) and 390 (bottom right) initial periods $P(r_{p0})$. The surface density plots are for 10^5 particles and show very little migration and the "slow" formation of a gap.

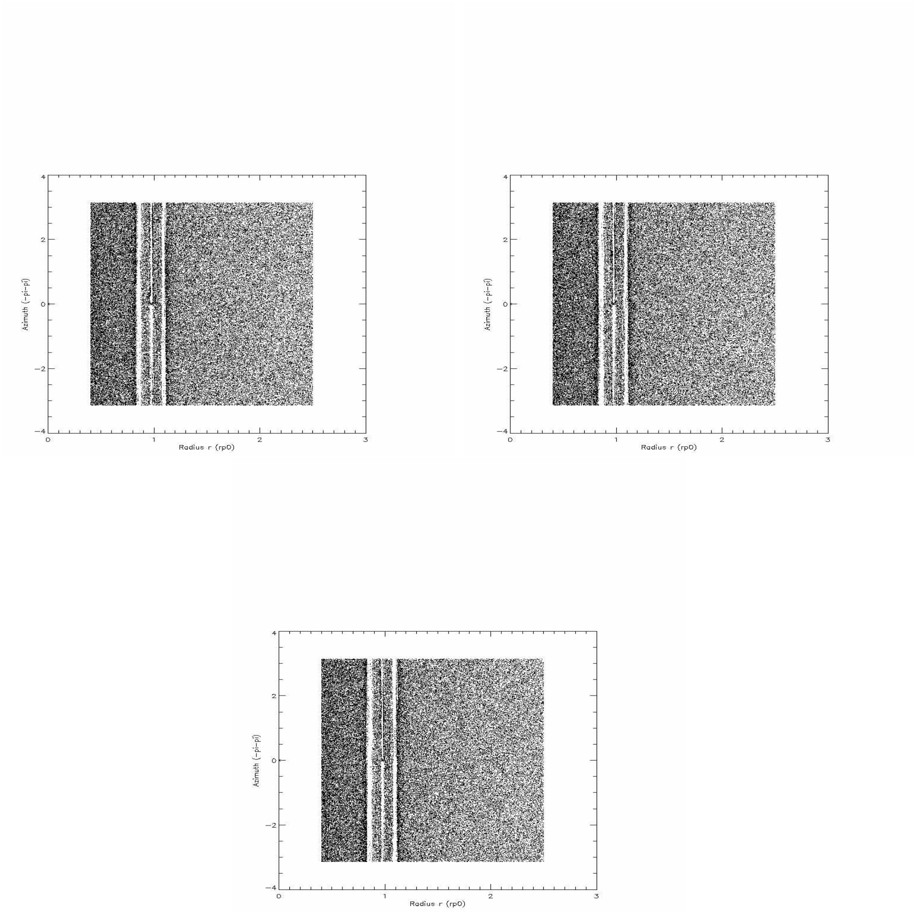

Figure 17: The temporal surface density evolution of a two-sided disk with β=-1.5 and q=0.0050 at the times t= 425 (top left), 460 and 495 (bottom right) initial periods $P(r_{p0})$. The surface density plots are for 10^5 particles and show very little migration and the "slow" formation of a gap.

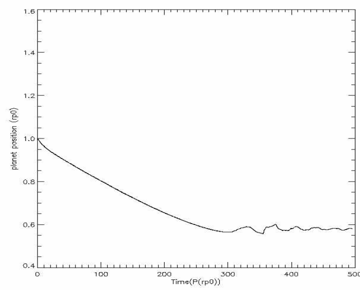

Figure 18: Planet migration of a planet outside an inner one-sided disk with β=-1.5 and q=0.0050. On the x-axis we have time in initial periods $P(r_{p0})$ and on the y-axis we have planet radius in initial planet radius. We see a fast runaway migration to roughly 300 $P(r_{p0})$ when the planet feels a surface density gradient in the opposite direction which changes the migration to outward migration. At the same time both the inner and the outer CR regions are filled with particles and the close encounters on each side of the planet balances each other and the planet migration stops at roughly 0.6 r_{p0}.

the gradient is abrupt and the new outward leaning gradient is large enough? Does this mean that the planet can migrate between the both, inner and outer, edges of the disk? This has not been examined here but would be intresting to know.

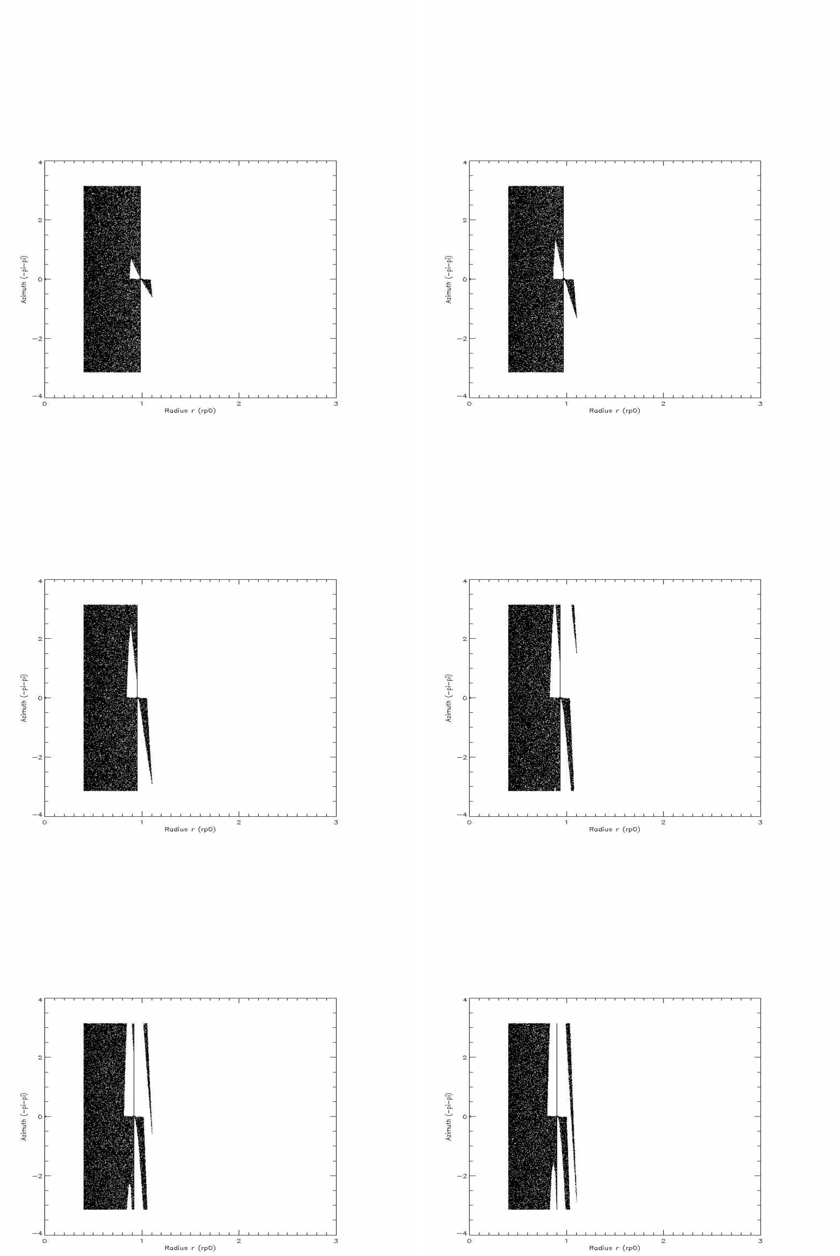

Figure 19: The temporal surface density evolution of an inner one-sided disk with β=-1.5 and q=0.0050 at the times t= 4 (top left), 8 (top right), 16, 24, 32 (bottom left) and 40 (bottom) initial periods $P(r_{p0})$. The surface density plots are for 10^5 particles and show a clear runaway situation with a surface density gradient that promotes inwards migration. Notice the particles that gather as a line at the planets radius because they co-rotate with the planet and never get to conjunction.

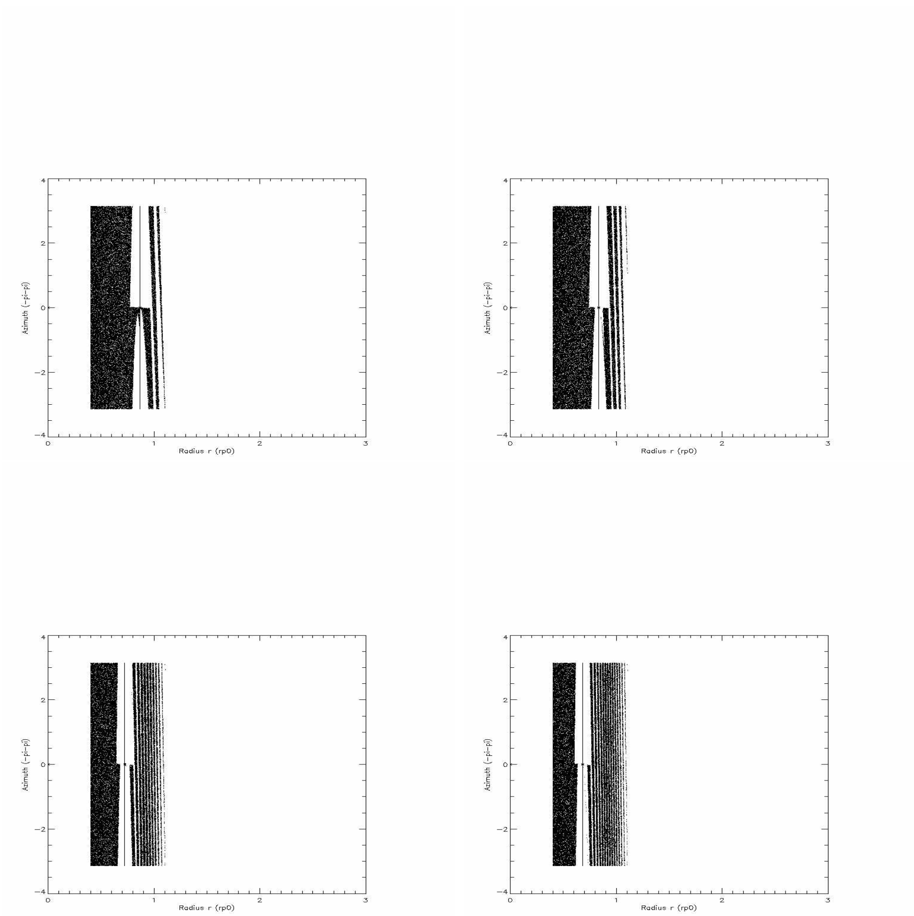

Figure 20: The temporal surface density evolution of an inner one-sided disk with β=-1.5 and q=0.0050 at the times t= 60 (top left), 80, 150 and 180 (bottom right) initial periods $P(r_{p0})$. The surface density plots are for 10^5 particles and show a clear runaway migration situation with a clear surface density gradient that promotes inward migration.

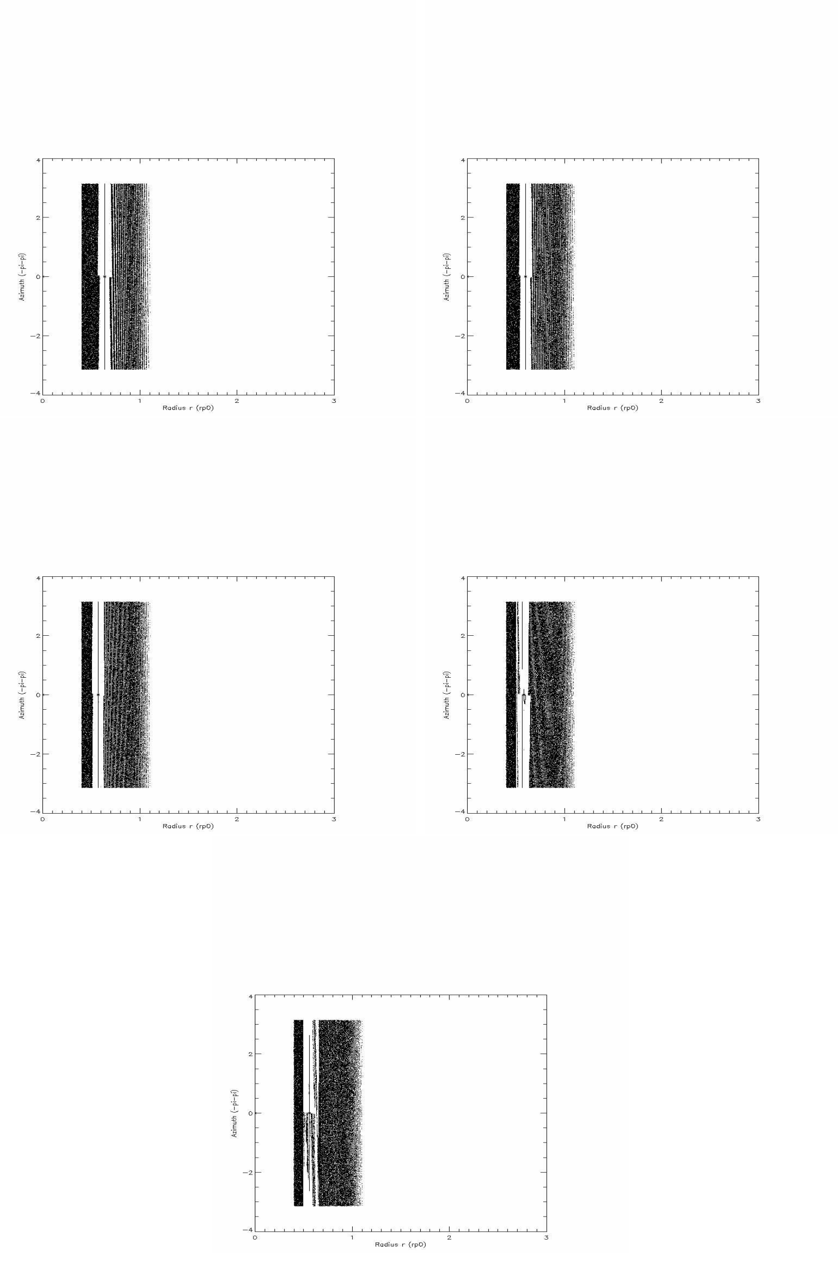

Figure 21: The temporal surface density evolution of an inner one-sided disk with β=-1.5 and q=0.0050 at the times t= 215 (top left), 250 (top right), 285, 320 and 355 (bottom) initial periods $P(r_{p0})$. The surface density plots are for 10^5 particles and show a clear runaway migration situation first, which follows by a bounce (and also outward migration) when the surface density gradient changes direction and after that the CR region fills on both sides which gives a situation similar to a two-sided disk.

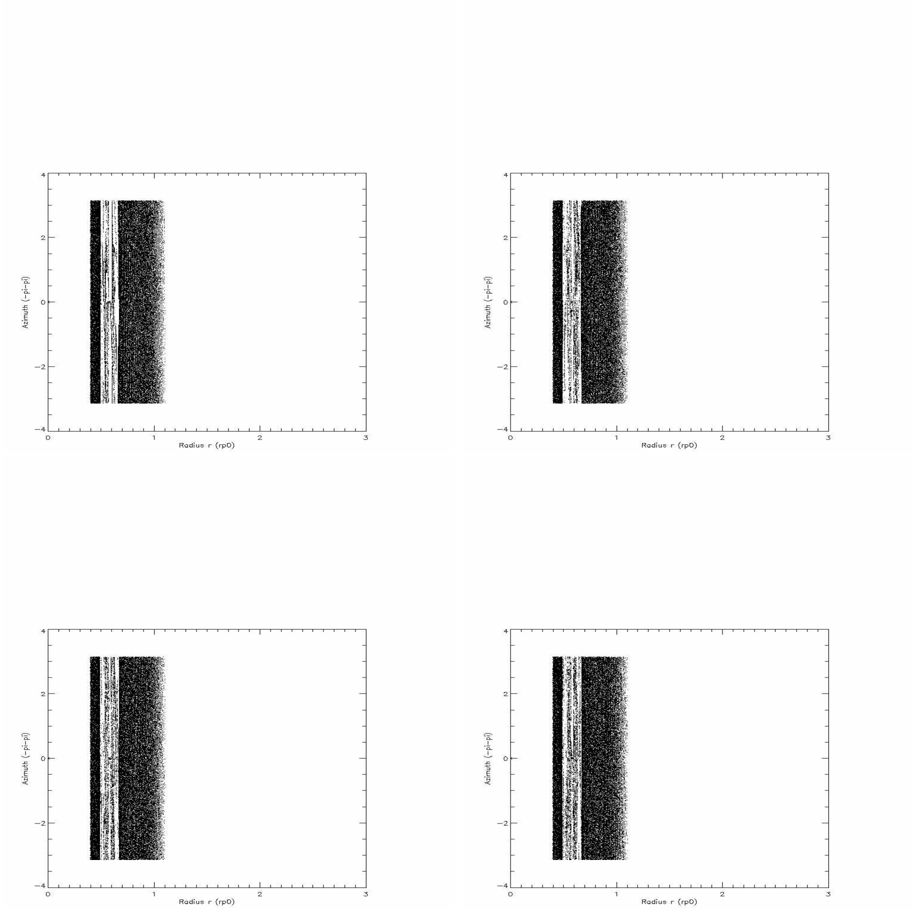

Figure 22: The temporal surface density evolution of an inner one-sided disk with β=-1.5 and q=0.0050 at the times t= 390 (top left), 425, 460 and 495 (bottom right) initial periods $P(r_{p0})$. The surface density plots are for 10^5 particles and show the "slow" formation of a gap.

5 Discussion

The solar nebula theory explains the formation of planets via the coagulation of molecules to dust and accumulation of dust to planetesimals and the accretion of these planetesimals to protoplanets. This forms protoplanets with cores of a mass up to 10 M_E. The solar nebula theory also explains the formation of the more massive planets, the gaseous giant planets of mass larger than 10 M_E, through a model of additional runaway accretion of icy gas onto these protoplanets. This icy material can only be found outside the so called ice condensation line (since else the temperature are too high and thus the ice melts), which lies roughly 2-5 AU from the central star. The solar nebula theory means that no gaseous giant planets can be formed inside the ice condensation lines. The observations of extrasolar planets have found a large number of gaseous giant planets in orbits very close to their central star. This is in contrast with the solar nebula theory and the theories are now trying to explain these observations by assuming that the giant planets have been formed outside the ice condensation line but have interacted with the disk the planets have been formed from and have moved radially, also called migrated, inwards to their present positions.

The theories of migration, the radial motion of a planet that interacts with a gaseous disk, have mostly been made for Lindblad resonance torques. These theories have assumed that the torques from two areas (one slightly inside the planets radius and one slightly outside the planets radius), very close to the planets orbit, called together the CR region, almost give equally large but opposite directed effects on the planet and thus can be neglected. The theories then suggest that the migration is mostly inwards and one of two types. These two types are called type I and type II migration. For type I migration we have a case where the planet is not massive enough to give the disks surface density enough disturbance to open a significant gap (or dip) and a little difference in Lindblad resonance torques between the outer and inner disk give an inward migration. This type of migration can be examined by linearizing the equations of continuity and conservation of angular momentum. The other type of migration is for very massive planets in which case the planet opens a significant gap and the evolution of the planet is dominated by the viscous inward flow and drifts along with the viscous flow. Both types of migration are derived assuming that the initial surface density profile are such that the torques from both sides of the CR region balance each other and these torques do not give any effect on the planets migration.

The effects of the co-rotation torques are at least one order of magnitude larger than the effects of the Lindblad resonance torques[43] so if there is a difference in the torques from either side of the CR region then the co-rotation torques dominate the evolution. This co-rotation torque is dependant on a surface density gradient, which means a difference in the surface density profile both in the radial and azimuthal direction. First, there might be a difference in surface density in the radial direction between the area very close to the separatrices and the CR region as a hole, e.g. like a gap or a dip. Second, there might also be a difference in the surface density in the radial direction between the inner (inside the planets orbit) and outer (outside the planets orbit) CR regions. The most extreme gradient of this type is the one-sided disk, with particles only on one side of the CR region. Third, in a particulate model, like the model in this work, the number of particles in the inner CR region that interacts during a timestep may not be equal to the number of particles that interacts in the outer CR region. This also give a difference in surface density during one timestep in the azimuthal direction. This should not be a problem for the long term behaviour of the disk and the planet. This is normally not a problem in the hydro-dynamical codes since one assumes a smooth flux of particles. In fact, the effect of the azimuthal gradient is the difference in torques between the particles outside and in front of the planet (in the outer CR region) and the particles inside and behind the planet (in the inner CR region). In the particulate model and in the case of the one-sided disks this gradient and the second radial gradient type manifests itself in the same way.

Examining a situation similar to the first case, Masset and Papaloizou 2003 [46] gets a new type of migration: type III migration or runaway migration which is completely dominated by the co-rotational torques. It seems though, that these simulations do not have, from the very outset, two counteracting CR regions (second case) but rather, in order to derive the formulae, the theory assumes that the planet has an initial migration inwards, i.e. they assume a radial pattern close to the planet that is a gap or dip. If this initial migration then is small enough the particles that performs close encounters with the planet ends up in the outer CR region also after a half a libration period, i.e. half a horse shoe orbit. But when the migration is high enough the particles performing a close encounter does not enter the outer CR region but rather the outer LRT region after a libration period. This behaviour is similar to the situation of the interaction between a planet and a one-sided disk (see section 4.2.4), which is examined in this work.

Here in this work we examine a situation that may have all three types of gradients: Both types of radial gradients through an initial radial power law distribution and the azimuthal gradient through the temporal evolution of the particles as they rotate in a circular orbit. Even though the temporal evolution of the interactions between the particles and the planet are somewhat crude they should at least qualitatively give the correct behaviour of the system.

From assumption 1 in the code description (section 4.1.1) follows that we assume that all interactions occur at the conjunction only. This is assumed for simplicity in the code. Due to this assumption we may perhaps miss the correct behaviour of the particles actually in the Lagrangian points. These particles are represented by the line at the planets radius in the surface density plots. This could be, and probabaly is, a big problem giving larger, or less, effects of the corotating particles. This has not been considered nor examined here, since we only want a qualitative picture of the importance of the initial disk distribution on migration.

In Ida et al. [43] they have separated the separatrix region into three different regions depending on the distance to the planet. This could be necessary in order to have a region with a possible overlap of LRT and CRT and regions of only LRT or CRT in order to get the exact separatrix distance. Again we only want a qualitative picture and in this work we have, for simplicity in the program, assumed very sharp lines equally distant to the planet. This means that either there is a CRT or LRT. All this follows from the assumption 2 in the description.

An interesting thing to examine further could be how a relaxation of assumption 3 affects the results. It should be possible to relax the assumption of instant interaction, assumption 3, if one use a sufficiently big timestep so that the timestep is much bigger than the time it takes for the particles to travel the entire CR region, 2 b_i, with the relative velocity $v_{rel}(r)$. This gives a constraint to the timestep which then is given by

$$\Delta t = \frac{\Delta\tau}{2\pi} P(r_{p0}) >> \frac{2b}{v_{rel}(r)} \approx \frac{4}{3} \frac{1}{\Omega_K(r_{p0})} \tag{83}$$

or $\Delta\tau >> \frac{4}{3}$. Though if one use timesteps this big the change in θ over one timestep could be much larger than 2π and one has to consider this too. In this work we use timesteps smaller than this value, in order to be sure that every particles interaction is accounted for and to simplify the computer program, why we have to add assumption 3 and consider instant interactions. The results should be the same.

In this work we derived the impulse approximation expressions for the LRTs and CRTs in somewhat different ways than earlier. The biggest difference in the derivations are that the expressions in this work is for the interaction between the planet and one point mass particle. This forces us to derive the change in angular momentum ΔL per interaction instead of the time derivative, the torque $\frac{dL}{dt}$, of the angular momentum. Another important thing in these derivations are that since we have point mass particles the change in angular momentum is for a particle with mass M_i instead of the differential torque integrated over the radial surface density gradient of the entire (or half: outer or inner, which are then compared to each other) disk for a massflux $dm = \Sigma\, v_{rel}\, dr$.

The assumption 7 is that there are no accretion onto the planet. This may not necessarily be the case. In fact for planets more massive than 15 M_E there are a runaway accretion phase, which might give quite big difference in the results of the migration in the long run. We examine the situation for a Saturn mass planet and thus we actually should include this accretion phase but we examines the evolution over periods of 200-500 initial planet periods $P(r_{p0})$ and this is over such a short timescale that we might assume that there are very little accretion.

The fact that the effect on the planet migration from the co-rotational torques are more than one order of magnitude larger than the effect of the Lindblad resonance torques on the planet migration should, in order to solve the riddle of the Hot Jupiters, turn the theoreticians focus onto the corotatational torques and the different density gradients and how they affect the planet migration. Here it would be intresting to

In order to compare to the theory of Masset and Papaloizou 2003 [46] we have the same components of the CR region that they have even though we have not assumed these components from the beginning. They mean that, as the planet migrates, there are particles, entering the CR region from the LR region, which performs a jump. This component give a positive contribution, in the same direction, to the migration. They mean that there also are co-orbiting particles that are drifting in the opposite direction relative to the planets migration and thus giving a negative torque, which counteracts the migration, on the planet. In order to get a runaway migration they mean that there need to be a large difference in the mass between these two components. This difference in mass is called the co-orbital mass deficit and need to be of the planet mass size. In figure 4 we have such a situation. For the case β=1.5 the effect of the close encounters in the initial outer CR region are so much larger than that of the initial inner CR region that the migration causes the particle that enters the CR region to only perform one close encounter with the planet. We then have a situation with a small distribution, or number, of the particles that drift relative the planets migration while the particles that enters the CR region due to the migration and only performs one close encounter are large. This situation is similar to the runaway migration mode. The particles that initially are set into the negative component drifting relative to the planet should be exactly those particles in the inner (outer) CR region during the first libration period for outward (inward) migration. This has not been examined here but would be intresting to see.

Also for all one-sided disks we have such a situation since there are no particles inside the CR region drifting along with the planets migration but we have a lot of particles, since the planet migrates fast toward the higher number (surface) density, entering the CR region as the planet migrates. In fact this should be the reason why the migration is larger for a disk distribution that "promotes" migration, i.e. disk distributions where the number of particles increases in the same direction as the migration, than for a disk distribution that do not "promotes" migration.

From these simulations it seems that in order to get a runaway migration to be maintained over a long period of time (longer than roughly 200 periods) it seems necessary to have a distribution that promotes a runaway migration, i.e. the number (surface) density of the disk increases in the same direction as the planet migrates, else it migrates to the "equilibrium point" and settles to this radius and enters one of the two other migration regimes. These simulations also tells us that the dependance of the planet migration on the surface density gradient and thus the behaviour of the planet migration due to CRT is determining the short time behaviour of the planet migration before the effects due to CRT (or close encounters) is more or less balancing even for the most extreme distributions. Thus the long time behaviour (over more than 1000 initial periods) should be determined by viscosity and LRTs as long as these interactions dont produce large surface density gradients.

The intresting deviance of the two-sided disk with β=1.5 and q$\geq$0.004 compared to the other two-sided disks and the similarity of this deviating simulation compared with the outer one-sided disks seems to favour the idea that there are some value of the diskmass parameter that set off the runaway migration. One also have to notice that for disk distributions which promotes the runaway migration, (β=-1.5 for inward migration and β=1.5 for outward migration) there seem to be a value of q that couples-decouples runaway migration. But for disk distibutions that do not promote runaway migration it does not seem to be important what the value of q is. These distributions still do not promote much migration and give results similar to two-sided disks. The deviating simulation is for β=1.5 which do promote outward migration.

The implications for planet formation are that the interactions and the initial distribution of the CR particles are more important than earlier theories have asssumed and predicted. It is now necessary to include these types of torques in the complete early evolution of planetary system formation. Another implication for planetary formation is that the migration, earlier approximated to occur on timescales of 10^3 initial periods are in fact possibly even faster, on timescales of 100 initial periods. A third implication on the planetary system formation is the importance of the initial conditions, most importantly in the

CR region, in the disk. If initially there is a large imbalance between the torques of the corotating particles "inside and behind" and "outside and in front of" the planet there will be large migration. This phase of migration will occur as long as the imbalance is maintained. If instead the torques of the co-rotating particles "inside and behind" and "outside and in front of" the planet more or less balances, equals out, earlier types of migration due to Lindblad resonances take over.

6 Conclusion

- In the simulations of a planet in a two-sided disk we get little migration, ($< 0.05\ r_{p0}$) for all runs except for the runs with β=1.5 and q$\geq$ 0.004 which get a large outward migration.

- For two-sided disks with the same disk distribution, same β, but with different diskmass parameters q there was very little difference in the planets migration except for the most extreme distributions with β=-1.5 and β=1.5. It seems like the value of q have very little effect on the migration for the distributions -1$\geq$ β $\geq$1. For the more extreme distributions β=-1.5 and β=1.5 there were somewhat more difference in the migration for different q values, larger migration for larger q values, but the difference was still very small compared to the migration due to a one-sided disk.

- For two-sided disks with the same diskmass parameter, q, but with different disk distributions, β, there were quite big difference, a factor of several, in the planets migration. Actually there were more or less no migration, less than one percent of the initial planet position r_{p0}, for a planet in a disk with a distribution of -0.5$\leq$ β $\leq$0.5 while there were an inward migration of up to five percent of r_{p0} for a planet in a disk with β=-1.5 and an outward migration of up to five percent of r_{p0} for a planet in disk with β=1.5. For the steeper disk distributions there were more migration in both directions.

- The temporal evolution of the migration of a planet in a two-sided disk looks like a damped sinusoidal curve, as it adjusts itself to the initial disk distribution and strife for the "equilibrium point" where the co-rotational torques on each side of the planet equals out.

- For planets in a one-sided disk we got large migration, more than ten percent of the initial planet position r_{p0}, for the planet in all runs, both for inner disks and for outer disks, except in the outer disk case for β=-1.5 which did not migrate much at all.

- When a planet is placed outside an inner disk the planet migrates inward and when the planet is placed inside an outer disk the planet migrates outward no matter the distribution. For the distributions that do not "promote" fast migration, i.e. distributions in which the number (surface) density increases in the opposite direction as the planets migration, the planet migrates less and reaches a state which is similar to a two-sided disk much faster than disk distributions which "promotes" migration, i.e.

distributions in which the number (surface) density increases in the same direction as the planet migration.

- When we examine inner one-sided disks with the same initial disk distribution but with different diskmass parameters q there are similar patterns of migration for all values of the q except for the runs with β=-1.5 and q$\geq$0.035 for which there are a runaway migration. The planets in the other runs migrate fast to an "equilibrium point" within roughly 0.8 to 0.9 r_{p0} and stops and behaves similar to a planet in a two-sided disk. We also have larger migration for larger mass even though the difference is relatively small. The migration is inward toward the star.

- When we examine outer one-sided disks with the same initial disk distribution but with different values of the diskmass parameter q there are similar patterns as for the inner one-sided disks but here it is reversed so that for the distribution β=-1.5 the planet do not migrate much but for the other distributions there are a runaway migration. There are slightly more migration for larger q but also here the difference is very little. The direction of the migration is outward and thus opposite to inner one-sided disks as suggested by theory.

- For inner disks with the same diskmass variable q the two disk distributions β=0 and β=1.5 gave a very large migration inward ($r_p < 0.9\, r_{p0}$) first but then gave a result similar to that of a two sided disk and did not migrate much more. For β=-1.5 instead we got a runaway migration all the way to $r_p < 0.6\, r_{p0}$ within 300 periods (see below) before the planet settles and also behaves like a two-sided disk.

- For outer one-sided disks with same diskmass q but different distribution β we have a situation similar to that outside an inner disk but with an outward migration instead. Again it is the evolution of the β= -1.5 distribution that deviates from the other two distributions and behaves like a planet in a two-sided disk. For both β= 0 and β= 1.5 there is a fast "runaway" migration (of between 20-40 percent of the initial planet radius) within 200 orbits.

- From these simulations it seems that in order to get a runaway migration to be maintained over a long period of time (longer than roughly 200 periods) it seems necessary to have a distribution that promotes a runaway migration, i.e. the number (surface) density of the disk increases in the same direction as the planet migrates, else it migrates to the "equilibrium point" and settles to this radius and enters one of the two other migration regimes. These simulations also tells us that the dependance of the planet migration on the surface density gradient and thus the behaviour of the planet migration due to CRT is determining the short time behaviour of the planet migration before the effects due to CRT (or close encounters) is more or less balancing even for the most extreme distributions. Thus the long time behaviour (over more than 1000 initial periods) should be determined by viscosity and LRTs as long as these interactions dont produce large surface density gradients.

- The exchange of specific angular momentum of co-rotational torques during a close encounter is more than one order of magnitude larger than the exchange of specific angular momentum of Lindblad resonance torques during a distant encounter. This result is in accordance to the theory of Ida et. al. 2000 [43] even though the derivations of the exchange of specific angular momentum of close and distant encounters are somewhat different.

- Most of the exchange of specific angular momentum come from very close to the separatrices. The exchange of specific angular momentum drops more than a factor of several within a distance of 2 r_L from the separatrices.

- The behaviour of the exchange of specific angular momentum is similar to the behaviour in figure 5 of Ida et. al. 2000 [43].

- For a two-sided disk one can see the slow formation of a gap and little migration.

- For the inner one-sided disk one can see a fast migration to roughly 0.6 r_{p0} where it enters a scenario similar to a two-sided disk with not much further migration and gap formation. This probably occurs because the runaway migration decreases and the migration speed gets to low to force the particles end up outside the CR region after a close encounter and the CR region gets filled. After one libration period the CR region are filled on both sides and the effects in either direction are balancing. This can be a way for the planet to stop migrating in orbits very close to the central star.

Acknowledgements

First of all I would like to thank Pawel Artymowicz for your support, our giving discussions and for your patience. Secondly, I would like to thank everyone at the Department of Astronomy and Space Physics at Uppsala University. Third, I would like to thank my family, without you it would not have worked. And finally I would like to thank my friends Daniel Arnberg and Lars Mattson for very good discussions about astronomy in general and for a good stydy atmosphere.

A Appendix 1 - Derivations

A.1 Vertical structure of the disk

From equation 10 we have

$$\eta = \frac{GM}{R}\left(1+\left(\frac{z}{R}\right)^2\right)^{-\frac{1}{2}} \tag{84}$$

and we have the Taylor expansion

$$(1+x)^{-\frac{1}{2}} \approx 1 + \frac{-1}{2}\,x + \mathcal{O}(x^2) \tag{85}$$

for small x. Now for a quantity x that is small enough the square is even smaller and can be approximated to 0 so to the first order in x we have

$$(1+x)^{-\frac{1}{2}} \approx 1 + \frac{-1}{2}\,x. \tag{86}$$

This means that

$$\frac{GM}{R}\left(1+\left(\frac{z}{R}\right)^2\right)^{-\frac{1}{2}} \approx \frac{GM}{R}\,(1+\frac{-1}{2}\,\left(\frac{z}{R}\right)^2) \tag{87}$$

and we get

$$c_s^2\, ln\frac{\rho}{\rho_0} \approx \frac{GM}{R}\,(1-\frac{1}{2}\,\left(\frac{z}{R}\right)^2). \tag{88}$$

A.2 LRT - impulse approximation

Here we have the definition of the relative velocity

$$v_{rel} = r\,(\Omega_K(r)-\Omega_K(r_p)) = r\,\Omega_K(r) - r\,\Omega_K(r_p) \tag{89}$$

and if we use $r = r_p + b$ for particles in the outer disk we get

$$\begin{aligned} v_{rel}(r_p+b) &= (r_p+b)\,\Omega_K(r_p+b) - (r_p+b)\,\Omega_K(r_p) = \\ &= \sqrt{GM}\,(r_p+b)^{-\frac{1}{2}} - (r_p+b)\,\Omega_K(r_p) = \\ &= \sqrt{GM}\,r_p^{-\frac{1}{2}}\,(1+\frac{b}{r_p})^{-\frac{1}{2}} - (r_p+b)\,\Omega_K(r_p) = \\ &= r_p\,\Omega_K(r_p)\,(1+\frac{b}{r_p})^{-\frac{1}{2}} - (r_p+b)\,\Omega_K(r_p) \end{aligned} \tag{90}$$

and if one use the Taylor expansion of $(1+\frac{b}{r_p})^{-\frac{1}{2}}$ to the first order of $\frac{b}{r_p}$, i.e. use eq. 86, we get

$$\begin{aligned} v_{rel}(r_p+b) &\approx r_p\,\Omega_K(r_p)\,(1-\frac{1}{2}\,\frac{b}{r_p}) - r_p\,\Omega_K(r_p) - b\,\Omega_K(r_p) = \\ &= -\frac{3}{2}\,b\,\Omega_K(r_p). \end{aligned} \tag{91}$$

If we instead use $r = r_p - b$ for particles in the inner disk we get

$$\begin{aligned}
v_{rel}(r_p - b) &= (r_p - b)\,\Omega_K(r_p - b) - (r_p - b)\,\Omega_K(r_p) = \\
&= \sqrt{GM}\,(r_p - b)^{-\frac{1}{2}} - (r_p - b)\,\Omega_K(r_p) = \\
&= \sqrt{GM}\,r_p^{-\frac{1}{2}}\,(1 - \frac{b}{r_p})^{-\frac{1}{2}} - (r_p - b)\,\Omega_K(r_p) = \\
&= r_p\,\Omega_K(r_p)\,(1 - \frac{b}{r_p})^{-\frac{1}{2}} - (r_p - b)\,\Omega_K(r_p) \qquad (92)
\end{aligned}$$

and if one use a Taylor expansion of $(1 + \frac{-b}{r_p})^{-\frac{1}{2}}$ to the first order of $\frac{-b}{r_p}$, i.e. use eq. 86, we get

$$\begin{aligned}
v_{rel}(r_p - b) &\approx r_p\,\Omega_K(r_p)\,(1 + \frac{-1}{2}\,\frac{-b}{r_p}) - r_p\,\Omega_K(r_p) + b\,\Omega_K(r_p) = \\
&= +\frac{3}{2}\,b\,\Omega_K(r_p). \qquad (93)
\end{aligned}$$

In equation 18 we have

$$\Delta v_{\parallel} = \sqrt{v_{rel}^2(r) - \Delta v_{\perp}^2} - v_{rel} = v_{rel}(r)\,\left[(1 + \frac{-\Delta v_{\perp}^2}{v_{rel}^2(r)})^{\frac{1}{2}} - 1\right] \qquad (94)$$

and if the parameter $x = \frac{-\Delta v_{\perp}^2}{v_{rel}^2(r)}$ is small enough we may use a Taylor expansion

$$(1 + x)^{\frac{1}{2}} \approx 1 + \frac{1}{2}\,x + \mathcal{O}(x^2) \qquad (95)$$

and if x is small enough the square of x is even smaller and may be neglected and to the first of order of x we have

$$(1 + x)^{\frac{1}{2}} \approx 1 + \frac{1}{2}\,x \qquad (96)$$

and we get

$$\Delta v_{\parallel} \approx v_{rel}(r)\,[(1 + \frac{1}{2}\,\frac{-\Delta v_{\perp}^2}{v_{rel}^2(r)}) - 1] = -\frac{\Delta v_{\perp}^2}{2\,v_{rel}(r)}. \qquad (97)$$

A.3 CRT - impulse approximation

From the definition of the exchange of specific angular momentum during a jump of an outer disk particle from radius $r_p + b$ to radius $r_p - b$, see equation 31, we have

$$\begin{aligned}
\Delta l &= \sqrt{GM\,(r_p - b)} - \sqrt{GM\,(r_p + b)} = \\
&= \sqrt{GMr_p}\,\left[(1 - \frac{b}{r_p})^{\frac{1}{2}} - (1 + \frac{b}{r_p})^{\frac{1}{2}}\right]. \qquad (98)
\end{aligned}$$

If the parameter $x = \frac{b}{r_p}$ is small enough we may use equation 95 and 96 so we get

$$(1 - \frac{b}{r_p})^{\frac{1}{2}} - (1 + \frac{b}{r_p})^{\frac{1}{2}} = 1 - \frac{1}{2}\,\frac{b}{r_p} - (1 + \frac{1}{2}\,\frac{b}{r_p}) = -\frac{b}{r_p} \qquad (99)$$

and

$$\Delta l \approx -r_p^2\,\Omega_K(r_p)\,\frac{b}{r_p}. \tag{100}$$

From the definition of the exchange of specific angular momentum during a jump of an inner disk particle from radius $r_p - b$ to radius $r_p + b$, see equation 32, we have instead

$$\begin{aligned} \Delta l &= \sqrt{GM\,(r_p+b)} - \sqrt{GM\,(r_p-b)} = \\ &= \sqrt{GMr_p}\left[(1+\frac{b}{r_p})^{\frac{1}{2}} - (1-\frac{b}{r_p})^{\frac{1}{2}}\right]. \end{aligned} \tag{101}$$

and

$$(1+\frac{b}{r_p})^{\frac{1}{2}} - (1-\frac{b}{r_p})^{\frac{1}{2}} = 1 + \frac{1}{2}\,\frac{b}{r_p} - (1 - \frac{1}{2}\,\frac{b}{r_p}) = +\frac{b}{r_p} \tag{102}$$

so

$$\Delta l \approx +r_p^2\,\Omega_K(r_p)\,\frac{b}{r_p}. \tag{103}$$

A.4 Radius shift of LRT

If r is the new radial coordinate and r_0 is the initial radial coordinate then for a particle that performs a shift that is caused by LRT we have

$$\Delta l^{LRT} = \sqrt{GMr} - \sqrt{GMr_0}. \tag{104}$$

This means that

$$\Delta l^{LRT} + \sqrt{GMr_0} = \sqrt{GMr}. \tag{105}$$

or

$$(\Delta l^{LRT} + \sqrt{GMr_0})^2 = (\sqrt{GMr})^2. \tag{106}$$

so

$$GMr = GMr_0 + 2\,\sqrt{GMr_0}\,\Delta l^{LRT} + (\Delta l^{LRT})^2 \tag{107}$$

and thus we have

$$\Delta r^{LRT} = r - r_0 = 2\,\sqrt{\frac{r_0}{GM}}\,\Delta l^{LRT} + \frac{1}{GM}\,(\Delta l^{LRT})^2. \tag{108}$$

If one use eq. 21 and 23 together with equation 72 then we get

$$\begin{aligned} \Delta r_i^{LRT} &= 2\,\sqrt{\frac{r_0}{GM}}\, \pm\, \frac{16}{27}\,\mu^2\,(r_p^2\,\Omega_K(r_p))\,\left(\frac{b_i}{r_p}\right)^{-5} + \\ &+ \frac{1}{GM}\,\left(\pm\,\frac{16}{27}\,\mu^2\,(r_p^2\,\Omega_K(r_p))\,\left(\frac{b_i}{r_p}\right)^{-5}\right)^2 \\ &= \pm\frac{32}{27}\,\mu^2\,\sqrt{r_p\,r_0}\,\left(\frac{b_i}{r_p}\right)^{-5} + \frac{16^2}{27^2}\,\mu^4\,r_p\,\left(\frac{b_i}{r_p}\right)^{-10} \end{aligned} \tag{109}$$

If μ^2 is small enough then μ^4 is even smaller so the second term is neglected and thus to the first order of specific angular momentum we have

$$\Delta r_i^{LRT} \approx \pm\frac{32}{27}\,\mu^2\,\sqrt{r_p\,r_0}\,\left(\frac{b_i}{r_p}\right)^{-5}. \tag{110}$$

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