

① Stellar Evolution (cont'd),
compact objects:
from White Dwarfs to Neutron Stars to Black Holes

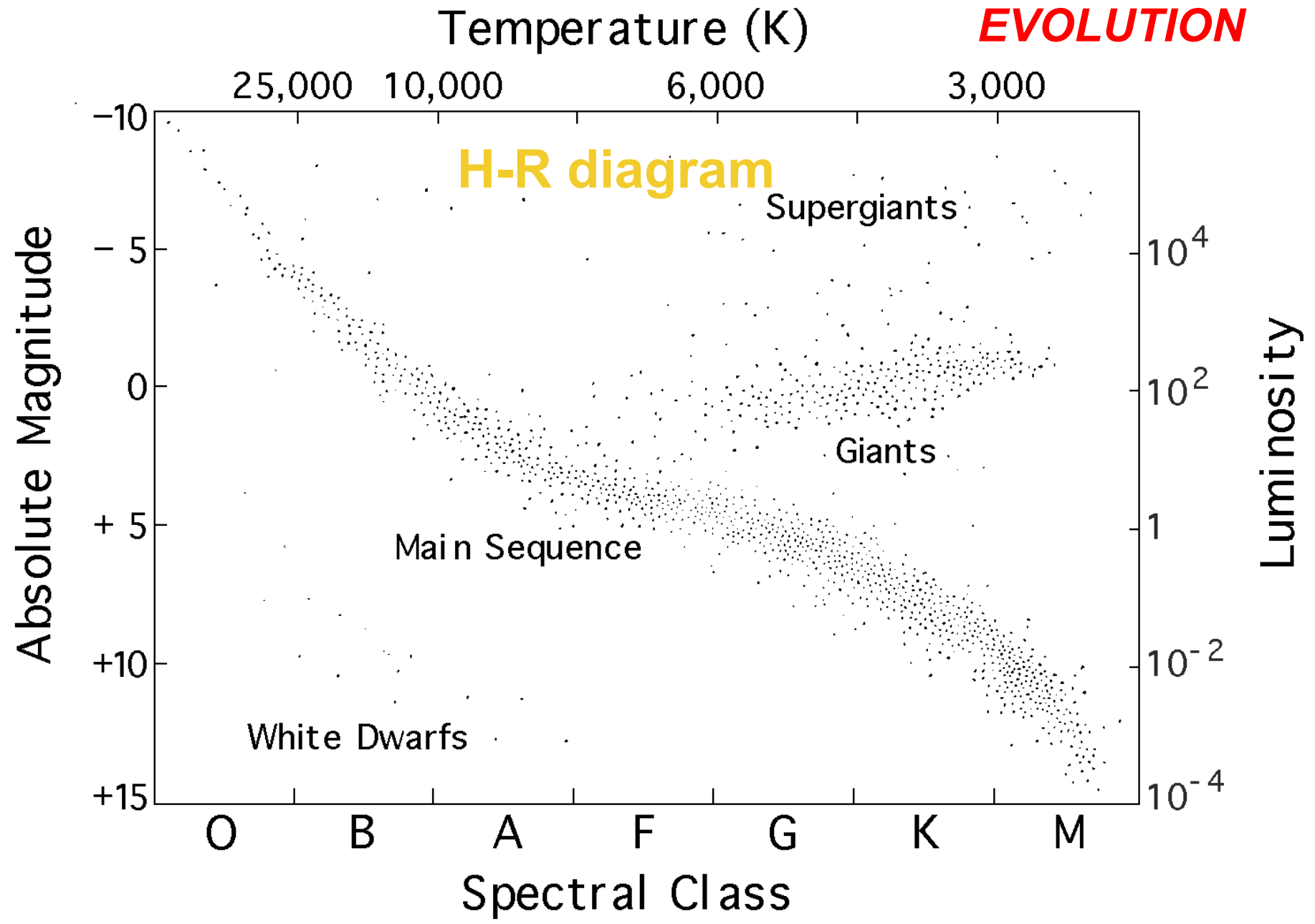
① *in the next lectures:*

① History of the Milky Way: Herschel, Kapteyn et al.

① The idea of spiral nebulae as island universes (Kant)

① The Great Debate of 1920: Curtis vs. Shapley

EVOLUTION



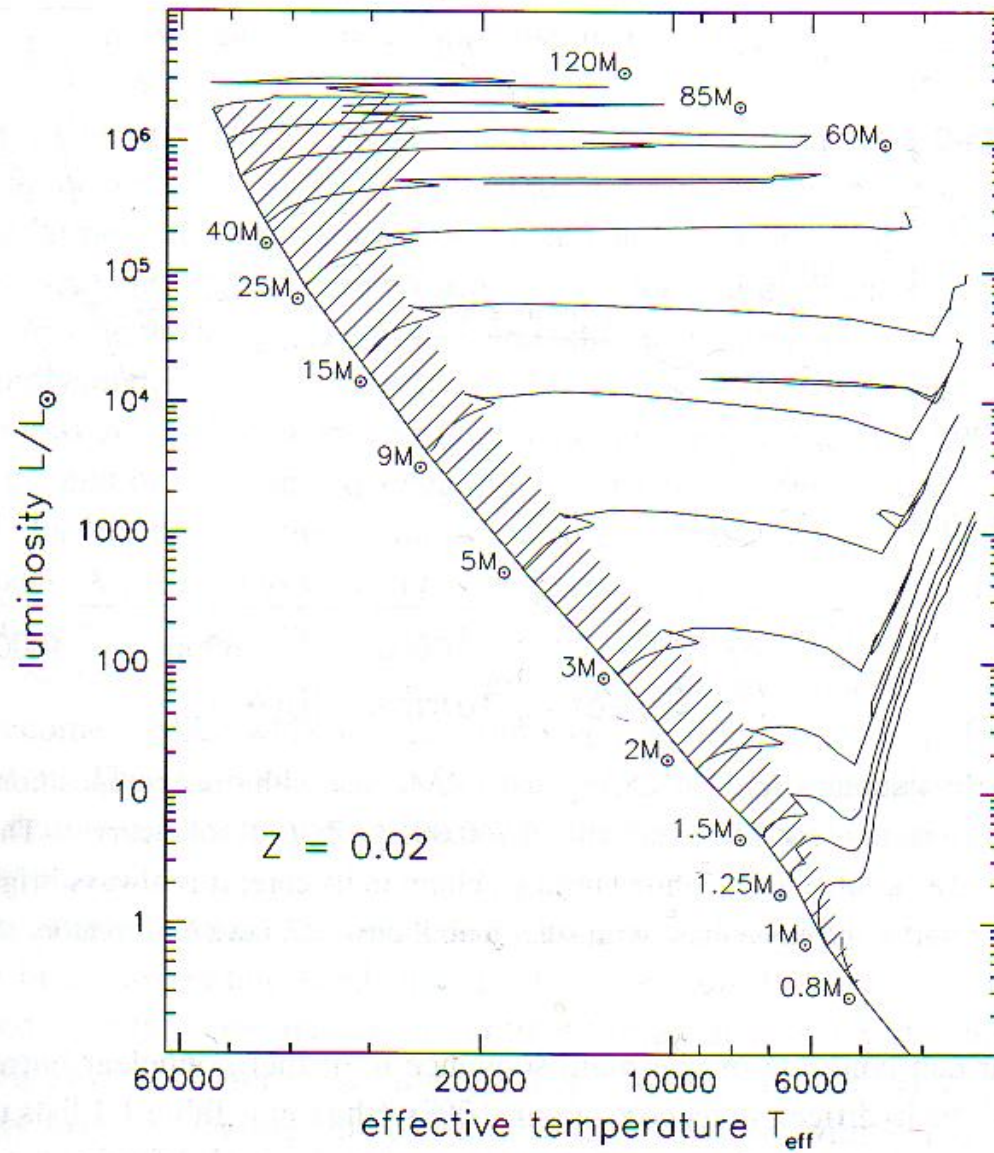
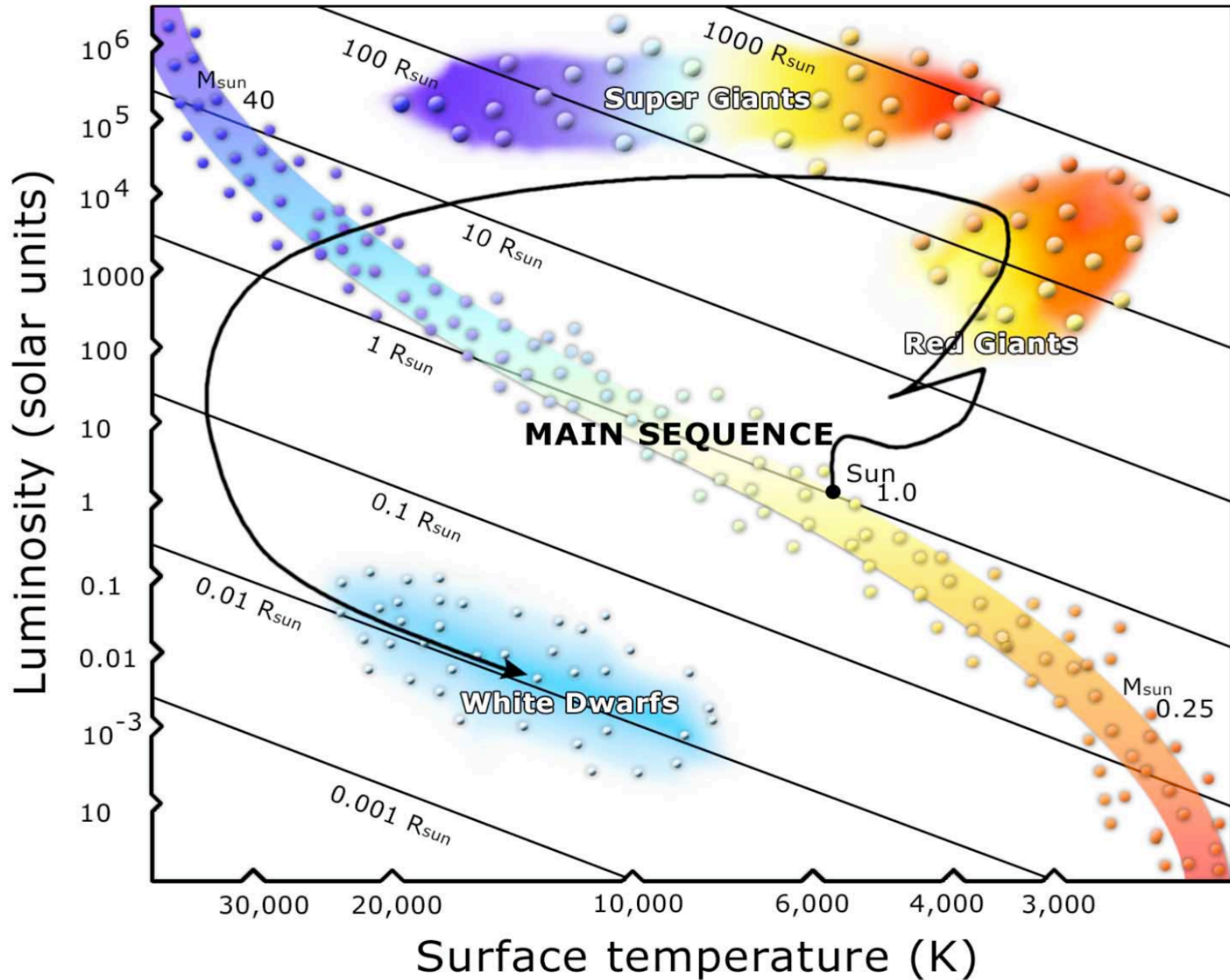
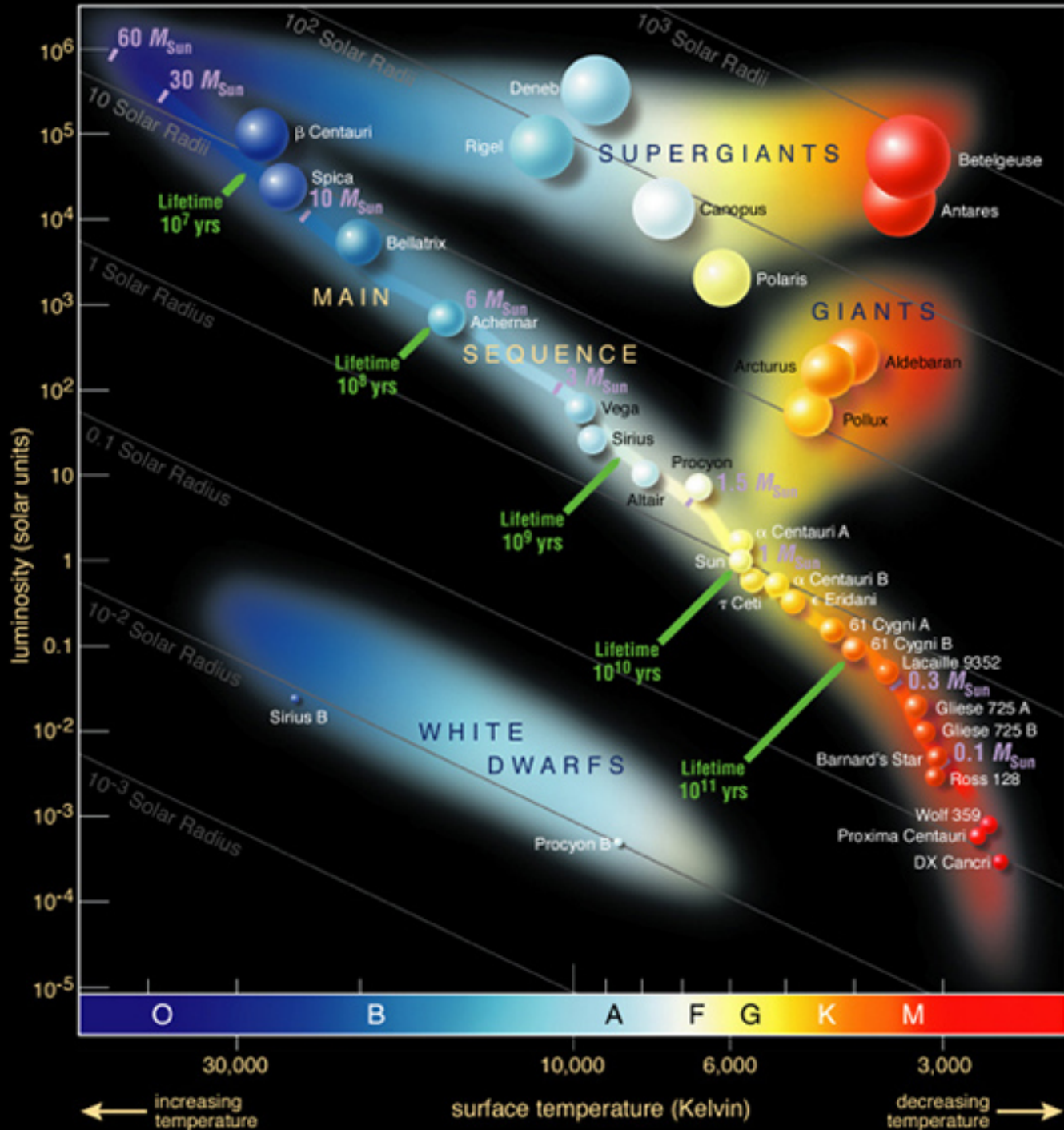


Figure 1.4 Luminosity and effective temperature during the main-sequence and later lives of stars with solar composition: hatched region shows where the star burns hydrogen in its core. Only the main-sequence track is shown for the $0.8M_{\odot}$ star – Geneva Observatory tracks.

Evol. of the sun on the Hertzsprung-Russell diagram





The Main Sequence Phase

Stars spend most of their lifetime on the Main Sequence, producing energy by hydrogen fusion.

The MS is characterized by hydrostatic equilibrium, and thermal equilibrium.

Location on the MS is determined by the star's mass.

Fusion takes place in the core. Energy is transported outward by radiation and convection.

All stars lose mass throughout their lifetimes, by stellar winds. More massive stars lose mass at higher rates.

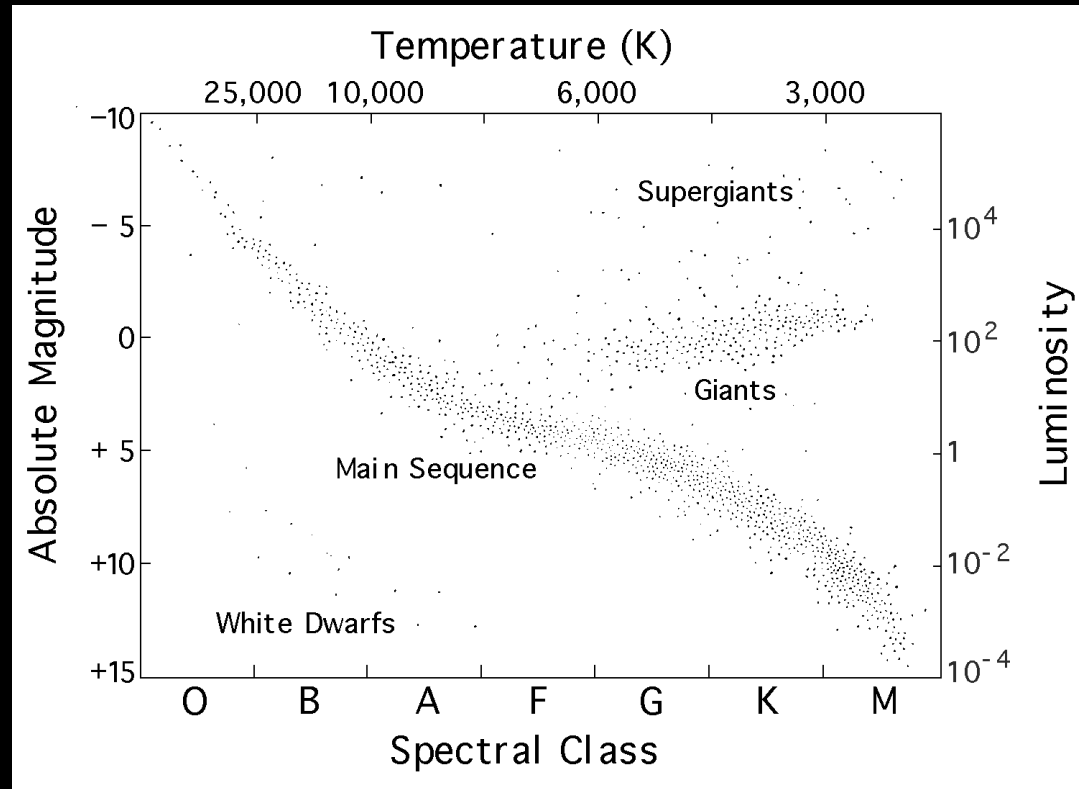


Table 1.1 Stellar models with solar abundance, from Figure 1.4

<i>Mass</i> (\mathcal{M}_{\odot})	L_{ZAMS} (L_{\odot})	T_{eff} (K)	<i>Spectral</i> <i>type</i>	τ_{MS} (Myr)	τ_{red} (Myr)	$\int(Ld\tau)_{\text{MS}}$ (Gyr $\times L_{\odot}$)	$\int(Ld\tau)_{\text{pMS}}$ (Gyr $\times L_{\odot}$)
0.8	0.24	4860	K2	25 000	—	10	—
1.0	0.69	5640	G5	9800	3200	10.8	24
1.25	2.1	6430	—	3900	1650	11.7	38
1.5	4.7	7110	F3	2700	900	16.2	13
2	16	9080	A2	1100	320	22.0	18
3	81	12 250	B7	350	86	38.5	19
5	550	17 180	B4	94	14	75.2	23
9	4100	25 150	—	26	1.7	169	40
15	20 000	31 050	—	12	1.1	360	67
25	79 000	37 930	—	6.4	0.64	768	145
40	240 000	43 650	O5	4.3	0.47	1500	112
60	530 000	48 190	—	3.4	0.43	2550	9
85	1 000 000	50 700	—	2.8	—	3900	—
120	1 800 000	53 330	—	2.6	—	5200	—

Note: L and T_{eff} are for the zero age main sequence; spectral types are from Table 1.3; τ_{MS} is main-sequence life; τ_{red} is time spent later as a red star ($T_{\text{eff}} \lesssim 6000$ K); integrals give energy output on the main sequence (MS), and in later stages (pMS).

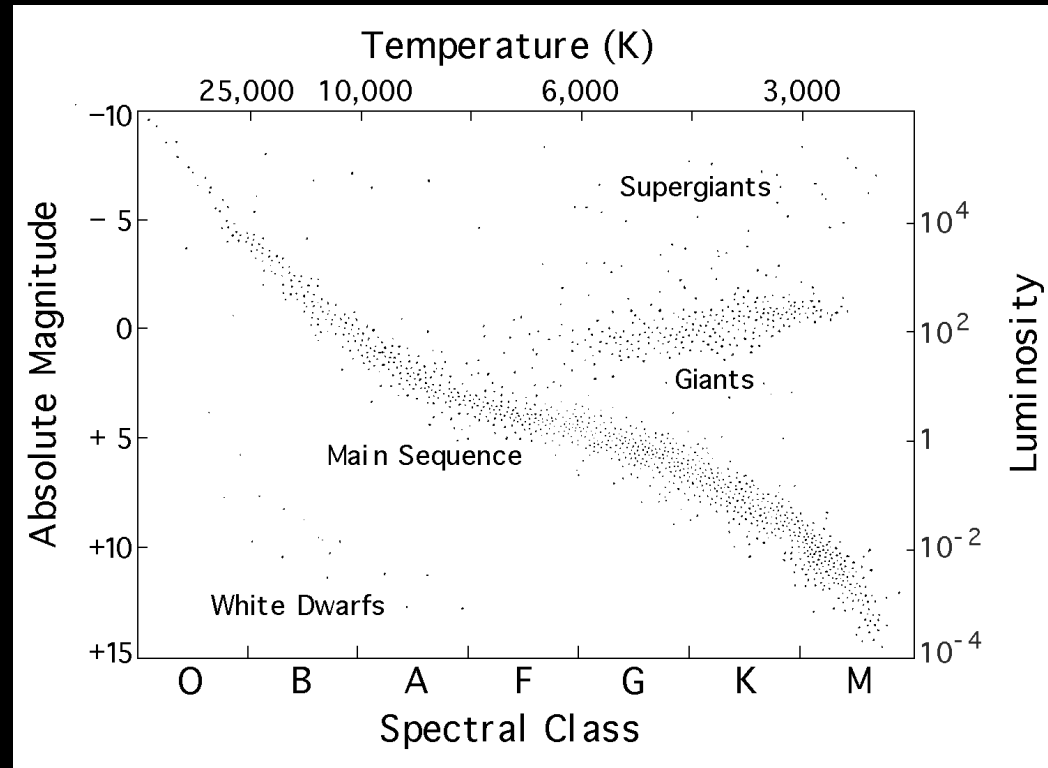
The Main Sequence Phase – low mass stars

Very small stars (< 0.3 solar masses) are fully convective

Small and intermediate mass stars have radiative cores and convective envelopes – the higher the mass of the star, the smaller the convective zone

The location of the convective layer may change as the star evolves. This leads to mixing of material and “dredge up” of nuclear burning products to the surface.

The products of this “dredge up” can be observed – heavy elements are detected in the spectra of evolved stars. This constitutes crucial evidence for nuclear energy generation in the interior.



Which regions of a star are convective and which radiative?

That depends on its mass...

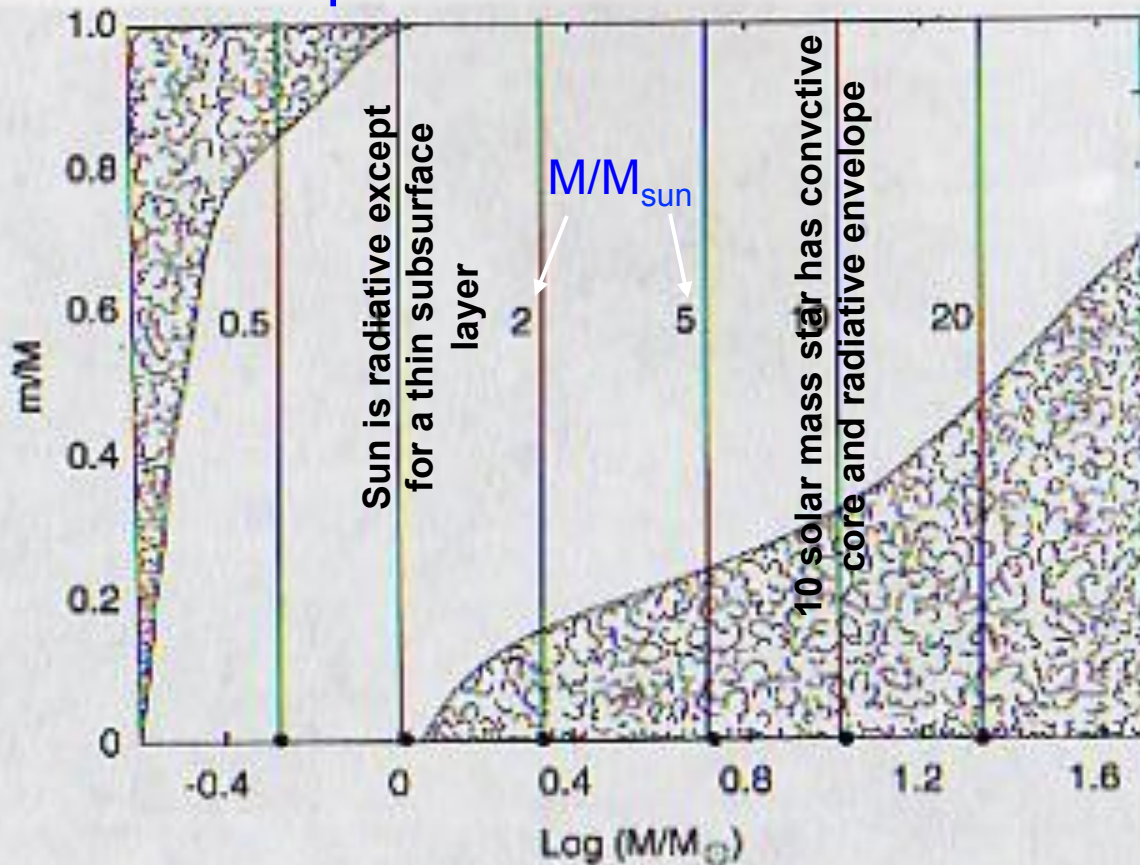
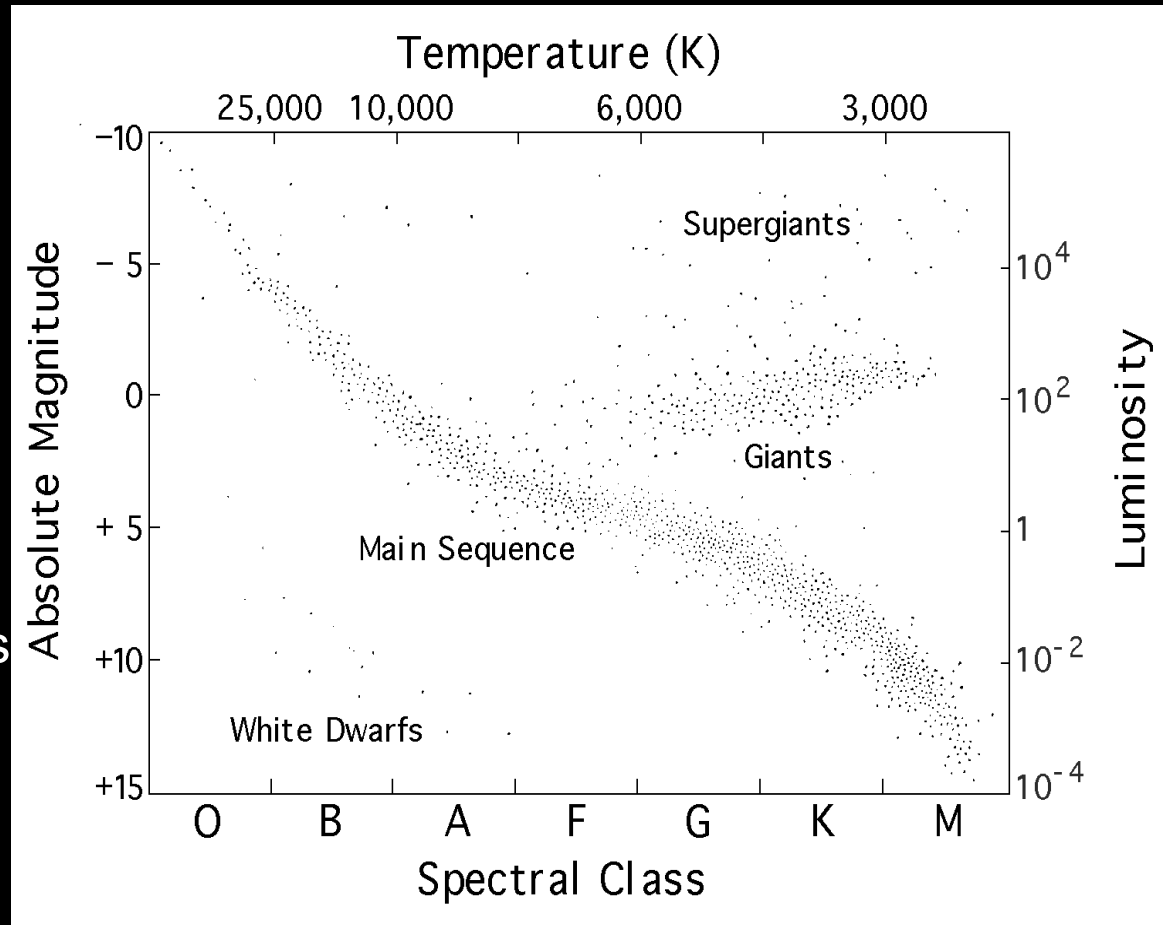


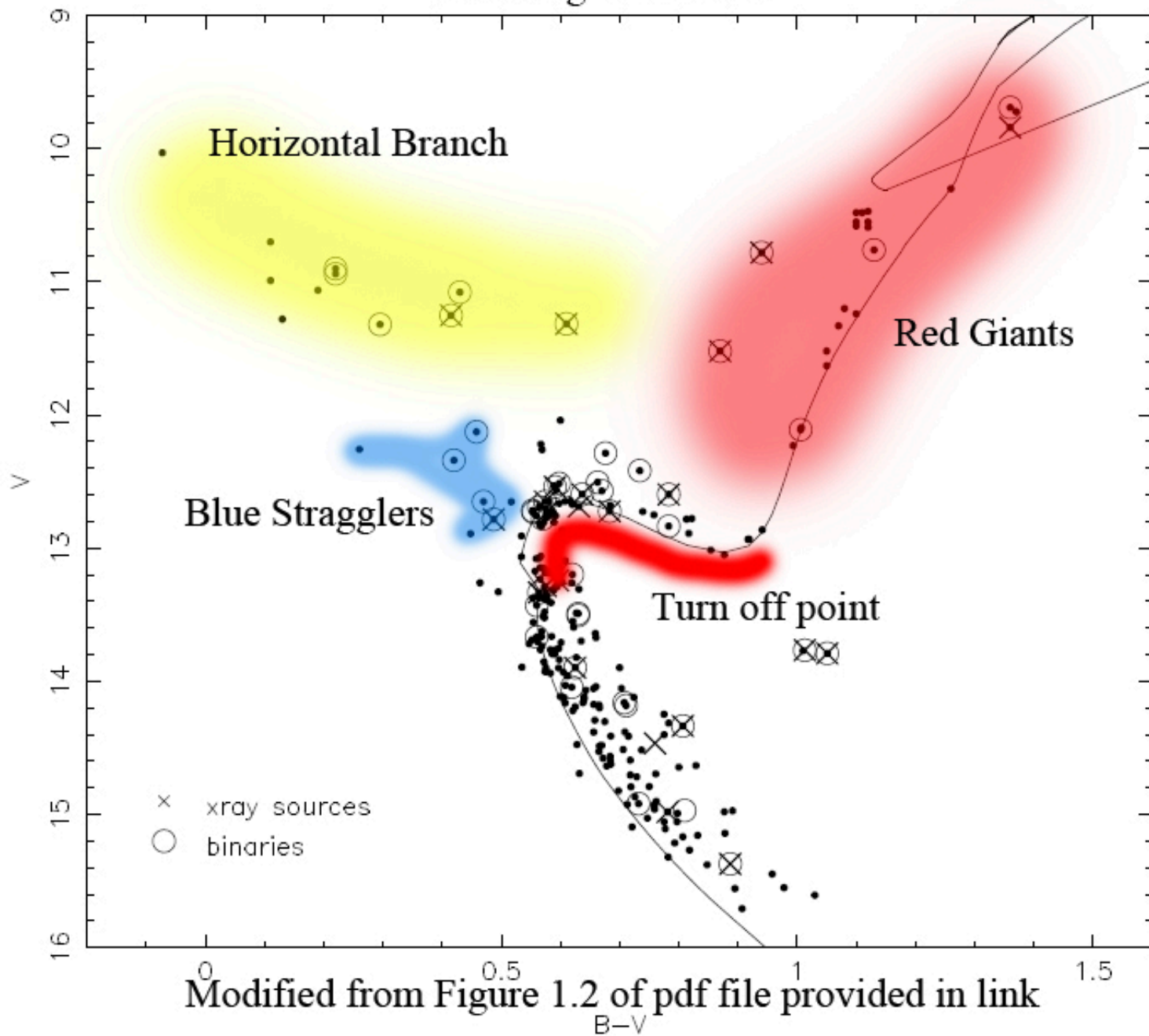
Figure 8.4 The extent of convective zones (shaded areas) in main-sequence star models as a function of the stellar mass [adapted from R. Kippenhahn & A. Weigert (1990), *Stellar Structure and Evolution*, Springer-Verlag].

The Main Sequence Phase – high mass stars

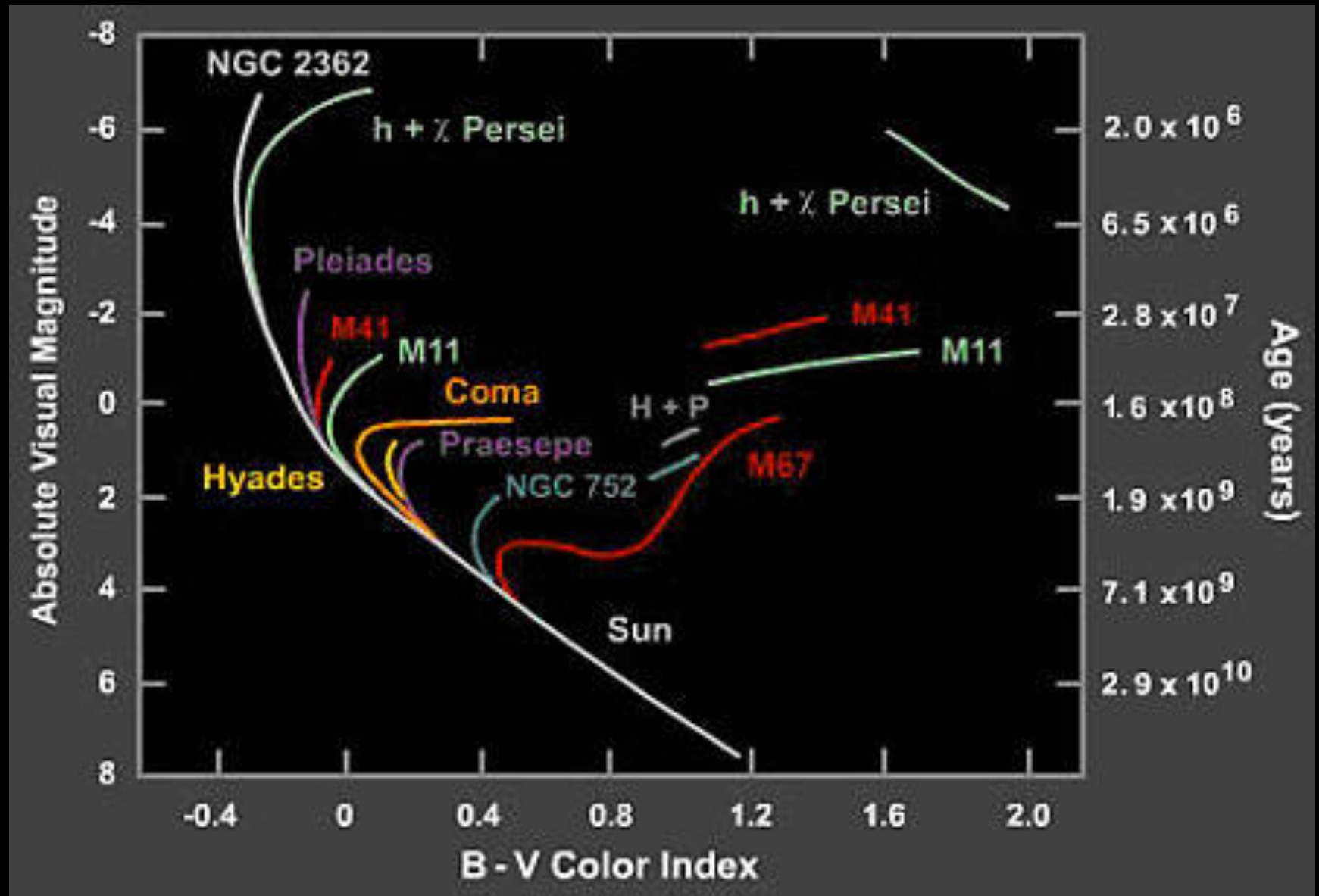
- High mass stars have convective cores and radiative envelopes.
- High mass stars also have strong stellar winds.
- High mass stars evolve more quickly than low mass ones.
- Very massive stars can lose enough material due to stellar winds that the mass loss slows down the rate of evolution of the star.
- Some stars ($M > 30$ solar masses) can lose almost their entire envelopes while still in the main sequence phase.



HR Diagram of M67



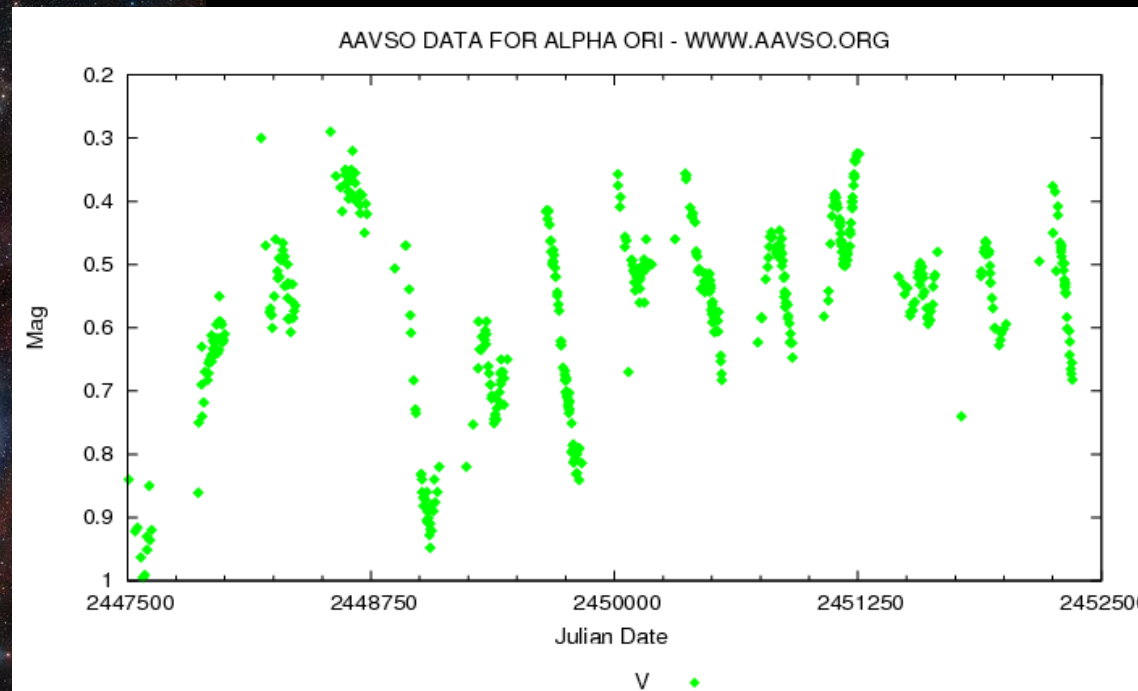
Stellar clusters of various ages & turnoff points



Orion constellation in a deep exposure, showing gas nebulae

Star α Ori = α Orionis = Betelgeuse has variable brightness

α Ori



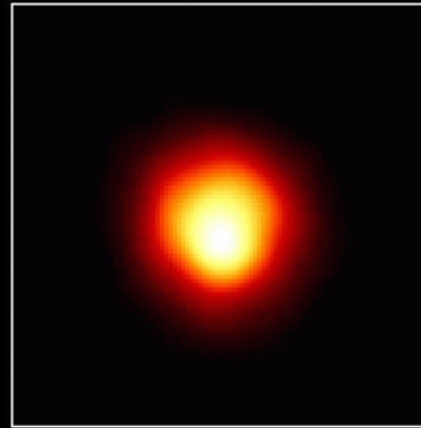
The Red Giant phase

When the MS star has exhausted its core hydrogen, nuclear burning in the core ceases, and the core becomes isothermal. An isothermal core cannot remain stable if its mass is above the so-called Schönberg-Chandrasekhar limit. The core then contracts rapidly, and heats up, while the envelope expands and cools.

Hydrogen burning continues in a shell surrounding the core.

Since the core releases gravitational energy during contraction, the difference is to an extent gained by the envelope, which expands (there is in fact no strict conservation of energy; **opacity** plays a role in energy transport as known from stellar structure equations).

Results of numerical modeling show that the envelope expands during the core contraction phase.



Size of Star

Size of Earth's Orbit

Size of Jupiter's Orbit



Atmosphere of Betelgeuse

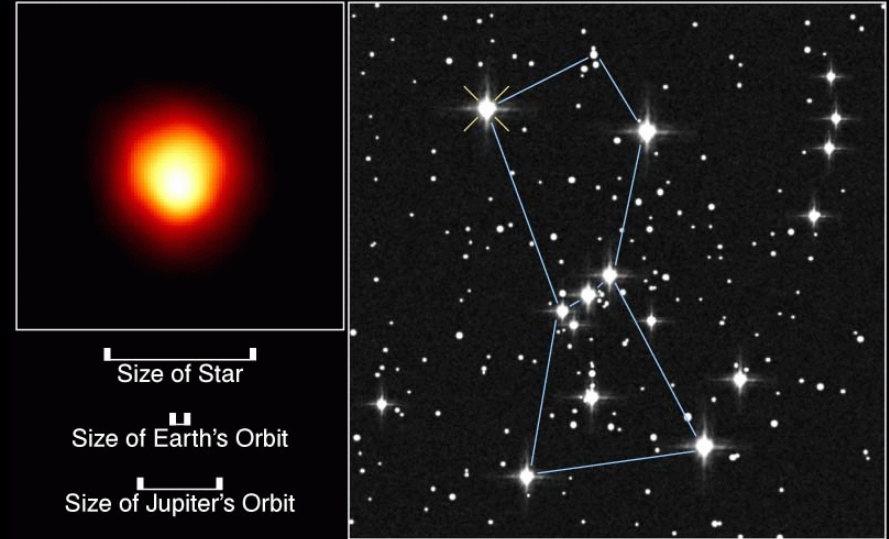
PRC96-04 · ST Sci OPO · January 15, 1995 · A. Dupree (CfA), NASA

HST · FOC

The Red Giant phase

The contraction of the core is a very rapid process relative to the MS lifetime of the star. Hence, it is difficult to observe – if we look at some sample of stars, most are on the MS, and we do observe some Red Giants, but it is very unlikely that we will “catch” the star right at the moment when it is undergoing core contraction. Instead, we will more likely observe the RG after the envelope has already expanded.

In relatively small stars ($M < 2$ solar masses) the hydrogen-depleted cores develop the right conditions for electron degeneracy. In this case, the core pressure is given by electron degeneracy pressure, and the core contraction and transition to the Red giant phase take place more gradually.

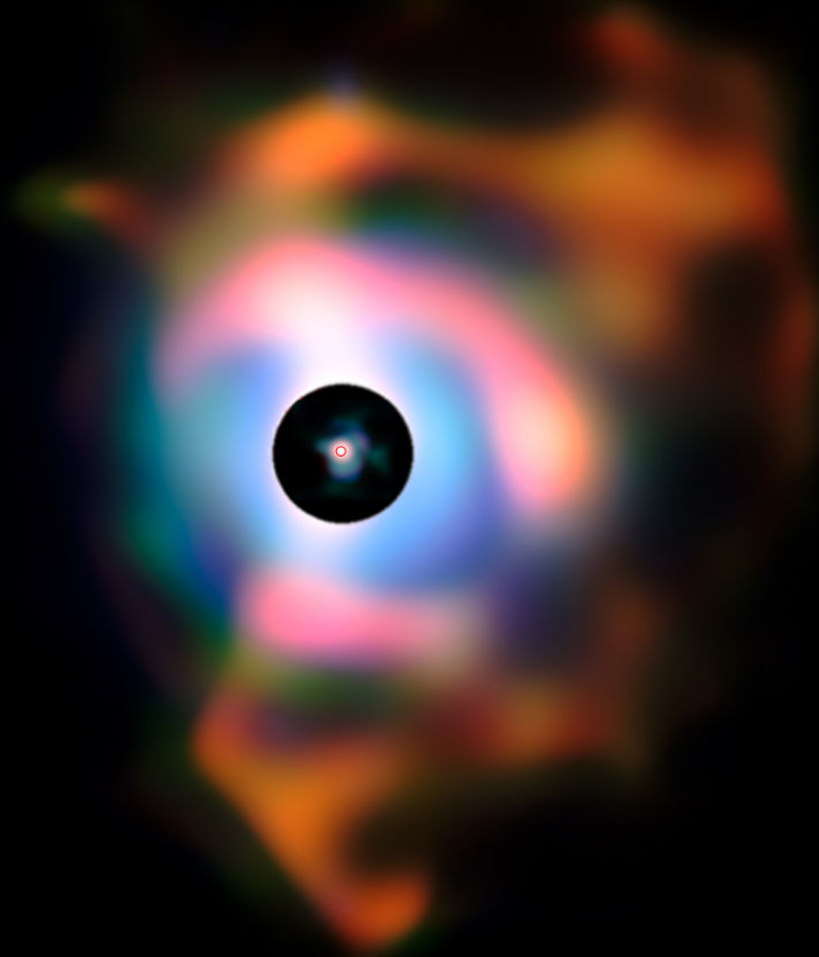


Atmosphere of Betelgeuse
PRC96-04 · ST ScI OPO · January 15, 1995 · A. Dupree (CfA), NASA

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Betelgeuse – a red supergiant ending its life soon,
now shedding mass

8-20 M_{sun} , 10 Myr age, $\sim 10^5 L_{\text{sun}}$, distance 150-250 pc



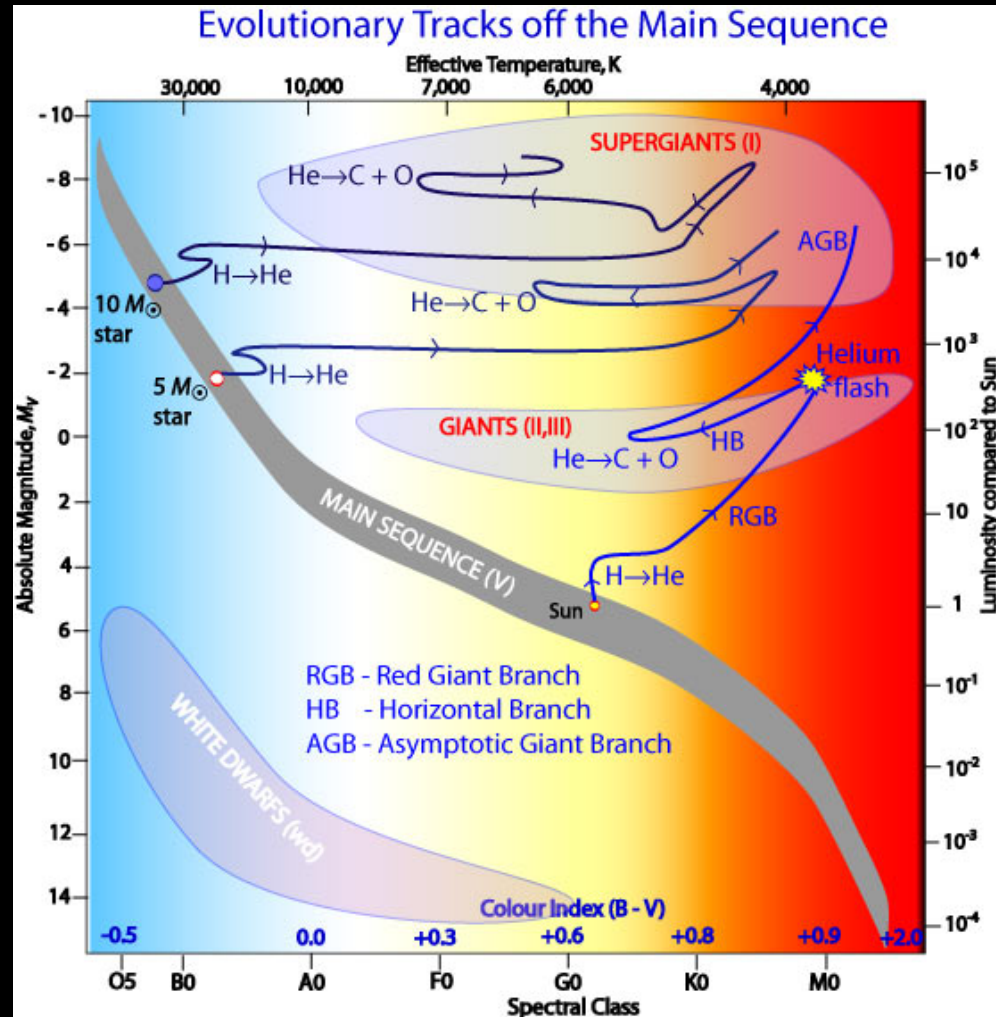
The Helium burning phase

The helium burning phase is much shorter than the hydrogen burning phase.

Helium burning produces about 1/10 the energy per unit mass compared to Hydrogen burning. But the star's luminosity is higher by about an order of magnitude than on the MS

Low mass stars ($0.7-2.0 M_{\text{sun}}$) have degenerate cores. In this case, He burning is unstable, leading to a runaway nuclear reaction called the **Helium flash**. In this process, the temperature rises steeply, the core expands, and the degeneracy is lifted; then regular, stable helium burning sets in.

When the core expands, the envelope contracts, luminosity drops, and the temperature in the envelope rises. The star is now on the horizontal branch. Location on the horizontal branch depends on the thickness of the hydrogen shell.



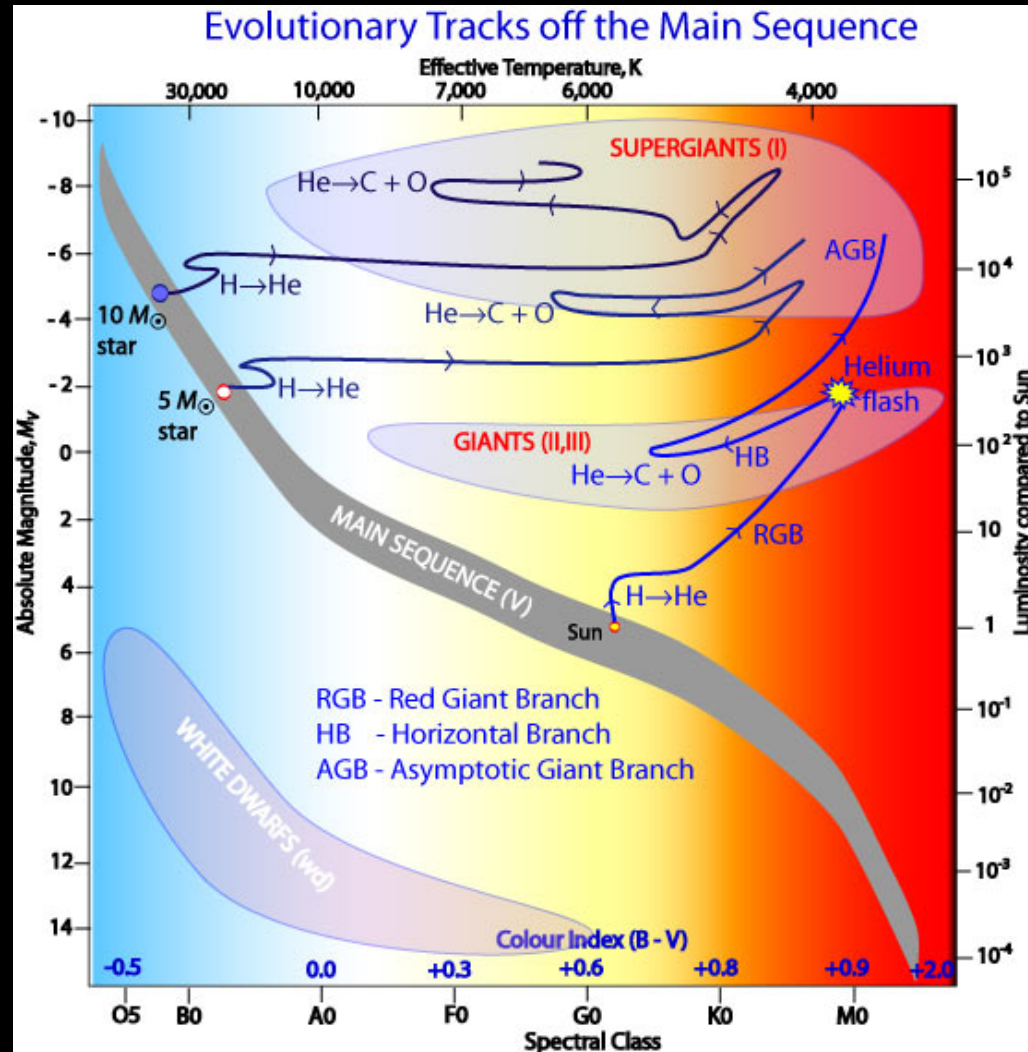
The Helium burning phase – variable stars

Horizontal branch stars toward the blue end have relatively thin hydrogen shells. The envelopes are radiative. These stars undergo a dynamical instability causing pulsations with periods of a few hours. They are known as **RR Lyrae** variables.

Intermediate mass helium burning stars may also undergo pulsations, on periods from a few to about 30 days. These stars are known as **Cepheid variables**.

Cepheid variables exhibit a Period vs Luminosity Relation. This makes them very important as standard candles.

Cepheids were first discovered by Henrietta Leavitt, who observed stars of variable luminosity in the Small Magellanic Cloud, and found a linear relation between the log of the peak luminosity and the log of the period of the star.



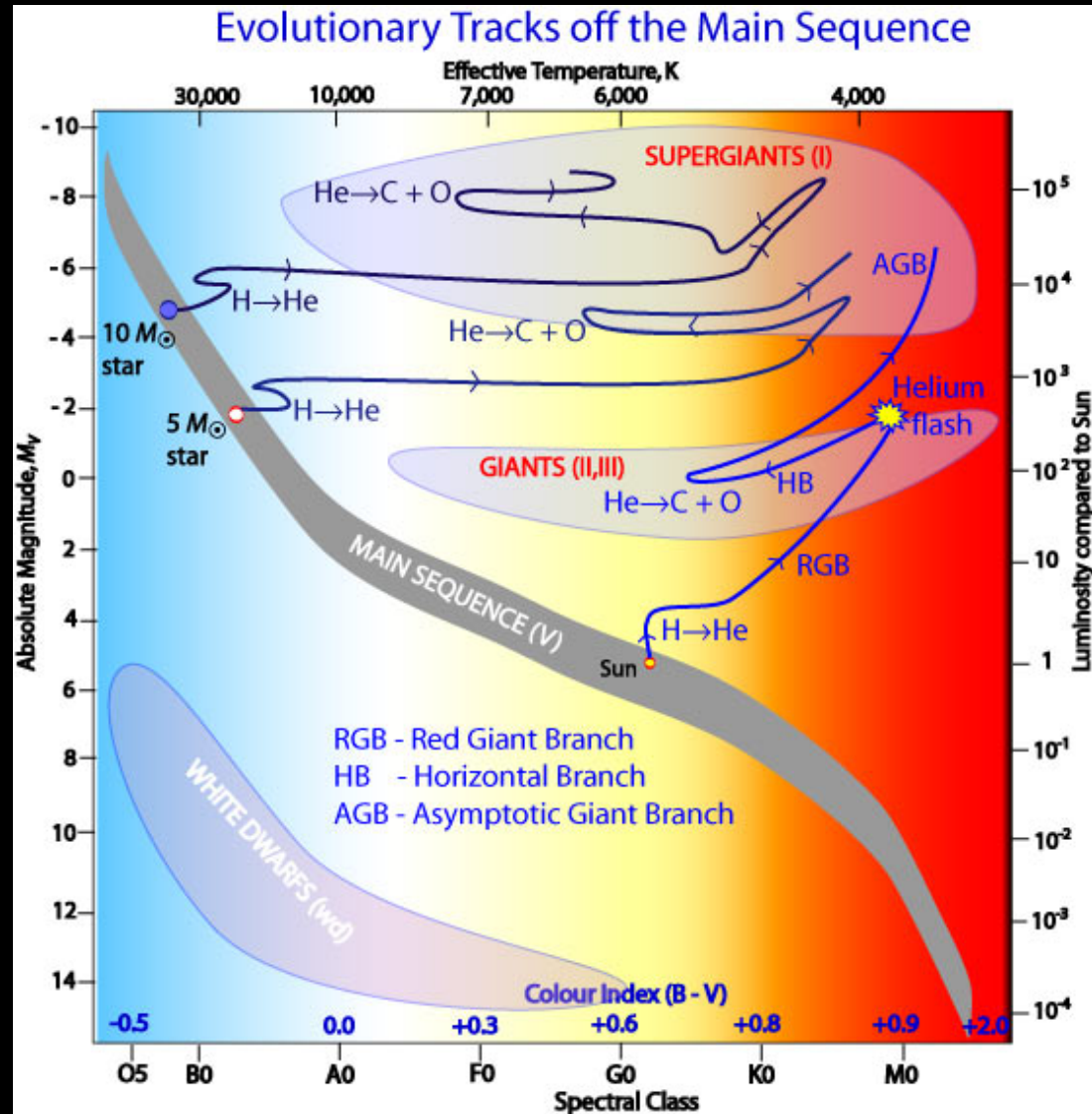
The Asymptotic Giant branch

Helium burning produces a carbon-oxygen core. When the helium gets depleted, the core again contracts and heats up, and the envelope expands even further. The star is now on the **asymptotic giant branch (AGB)**.

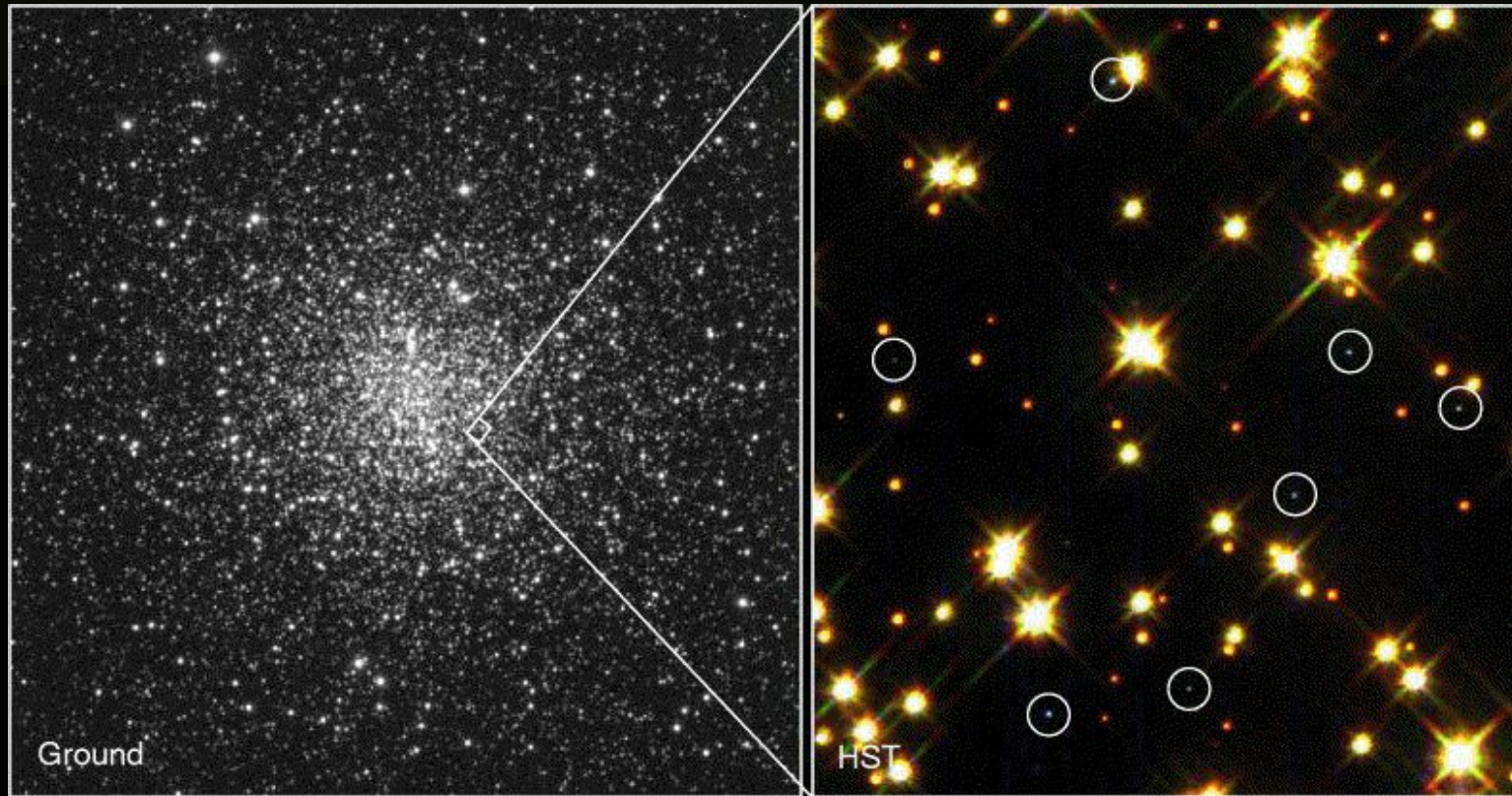
Both helium and hydrogen burning continue, in 2 shells around the CO core. This configuration is thermally unstable, leading to a series of thermal pulses.

The luminosity of the evolved star is determined by the core mass, independent of the total mass.

A strong stellar wind develops, leading to rapid mass loss. The rate of mass loss is well described by an empirical relation.



Observations of White Dwarfs from the ground and HST

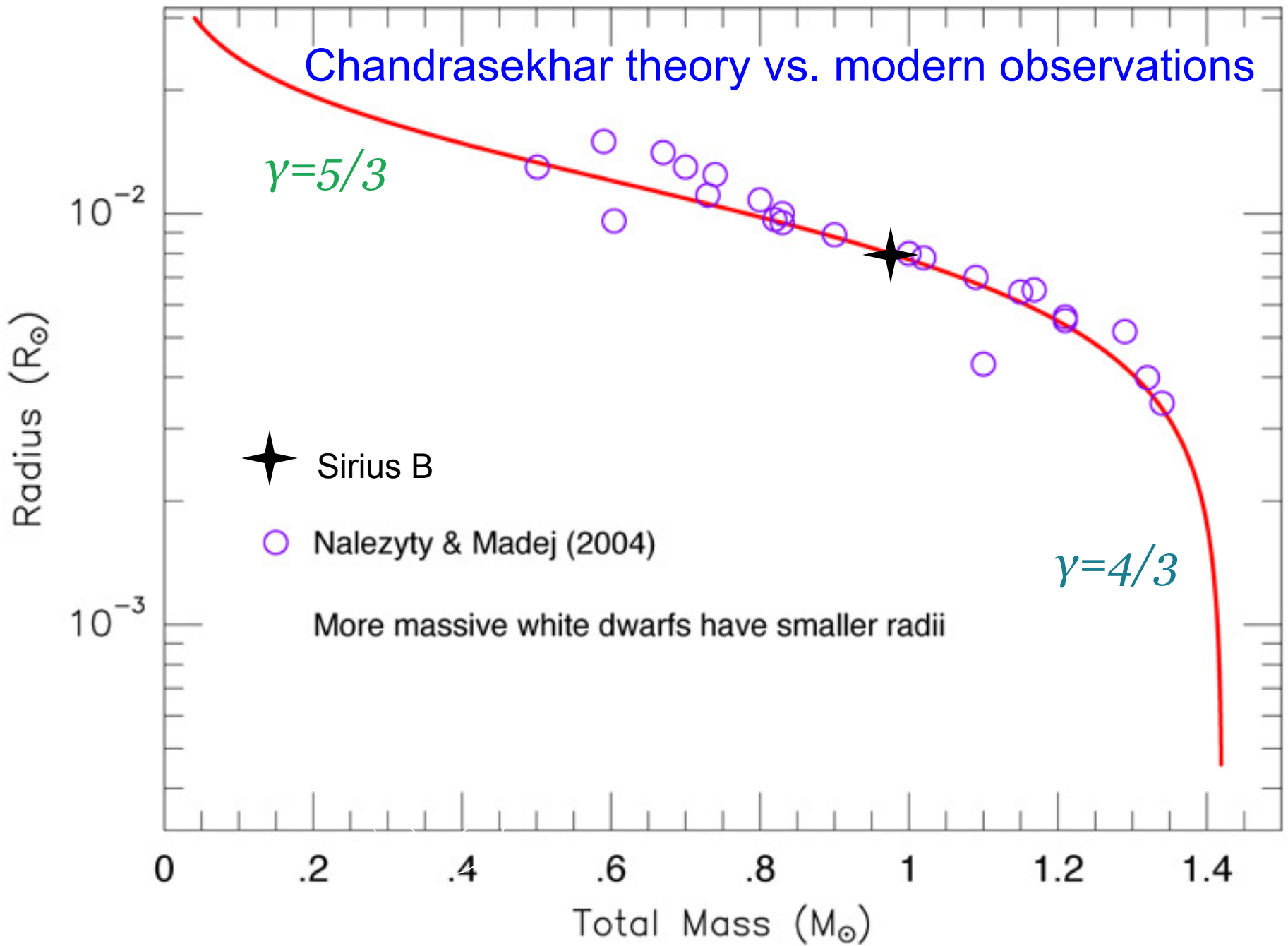


When a low or intermediate mass star exhausts its fuel supply, the core contracts, and the outer layers are blown off in what is known as a planetary nebula.

The bare core remains behind, initially hot enough to ionize the surrounding nebula, then slowly cooling. These stars are blue and very faint, and are known as white dwarfs.

The white dwarf is held up against gravitational collapse by electron degeneracy pressure. In 1931, S. Chandrasekhar showed that in order for electron degeneracy pressure to match the force of gravity, the white dwarf mass cannot exceed a maximum known as the Chandrasekhar limit, $M \sim 1.47$ solar masses.

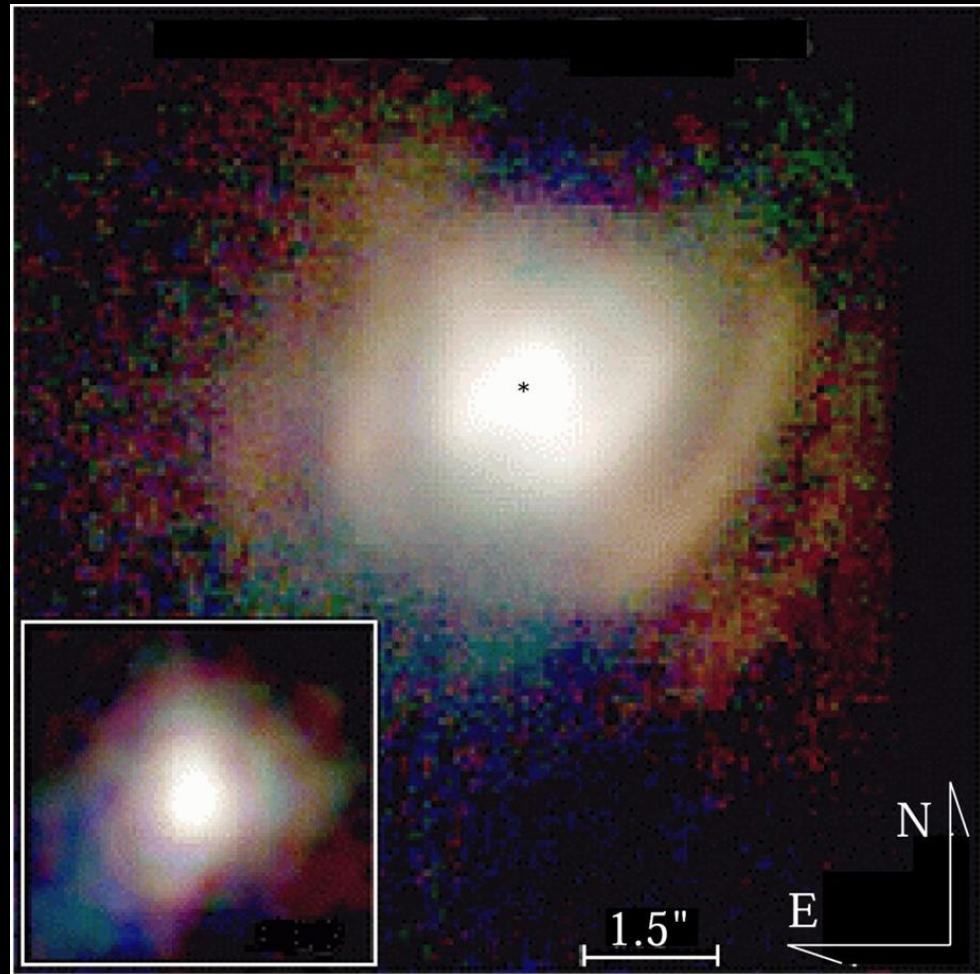
Chandrasekhar theory vs. modern observations



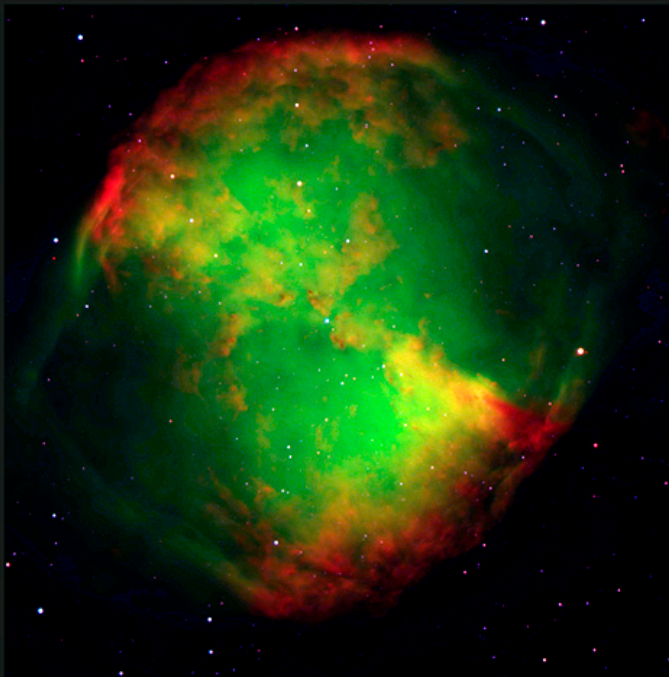
The planetary nebula phase

At the end of the AGB phase, low and intermediate mass stars shed their outer envelopes, which were already very diffuse and weakly gravitationally bound in the AGB phase.

The ejection of the outer layers is associated with a very strong stellar wind, known as a superwind. The mechanisms behind superwinds are not very well understood. However, we know that they exist from observational evidence (the rate of mass loss can be estimated from observations).



Some planetary nebulae



Planetary Nebula NGC 6853 (M 27) - VLT ANTU+FORIS1

ESO PR Photo 38a/98 (7 October 1998)

© European Southern Observatory



0.9532 μm [S III filter]



After the ejection of the outer shell, the remaining inner part of the envelope contracts and heats up to about 30,000 K. This produces highly energetic photons, which ionize material in the ejected shell, causing it to glow.

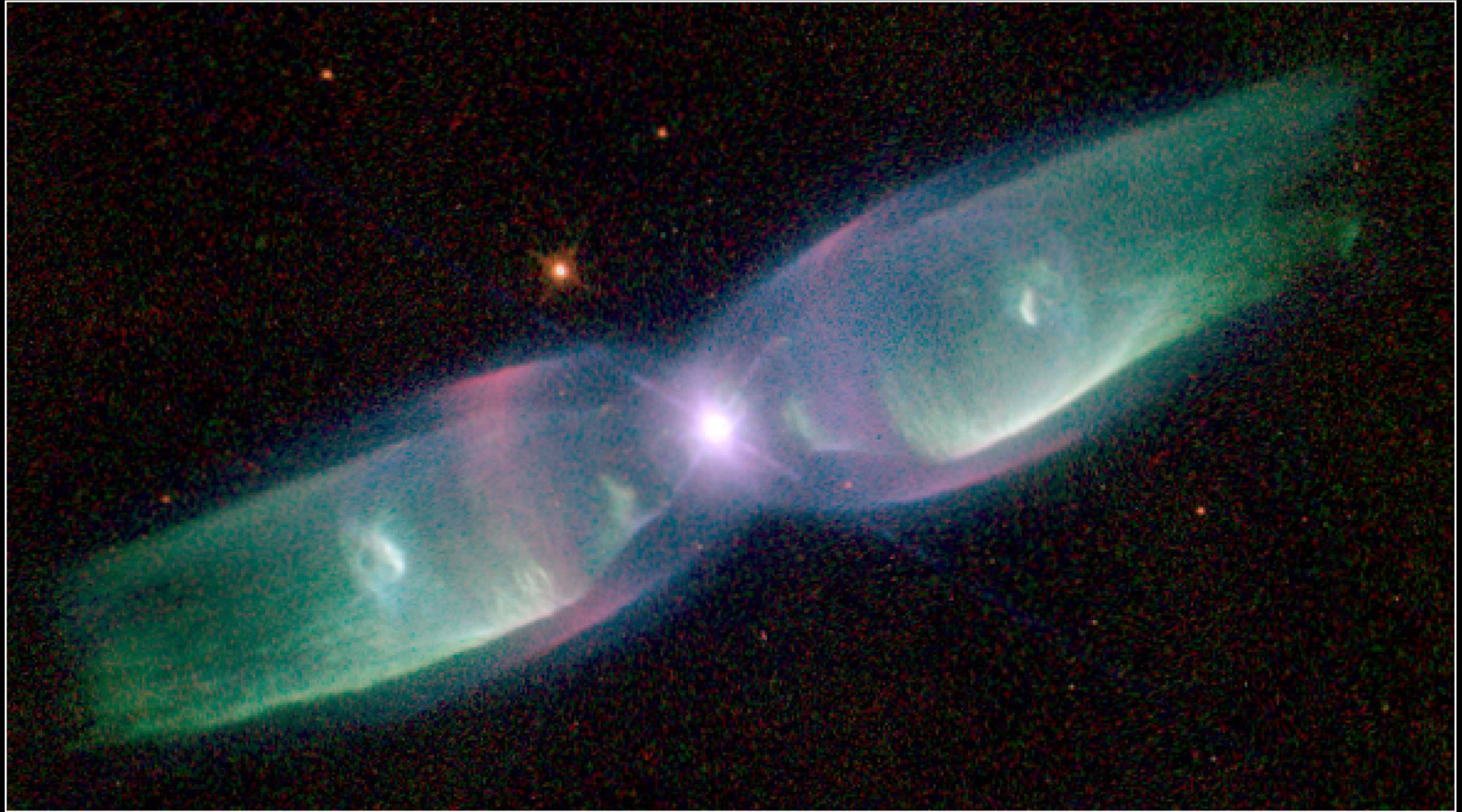
This is known as a **planetary nebula**.

The core of the star remains behind, and is seen as a hot central source. This is the planetary nebula nucleus, which will slowly cool to form a white dwarf.

More planetary nebulae



The Hourglass Nebula

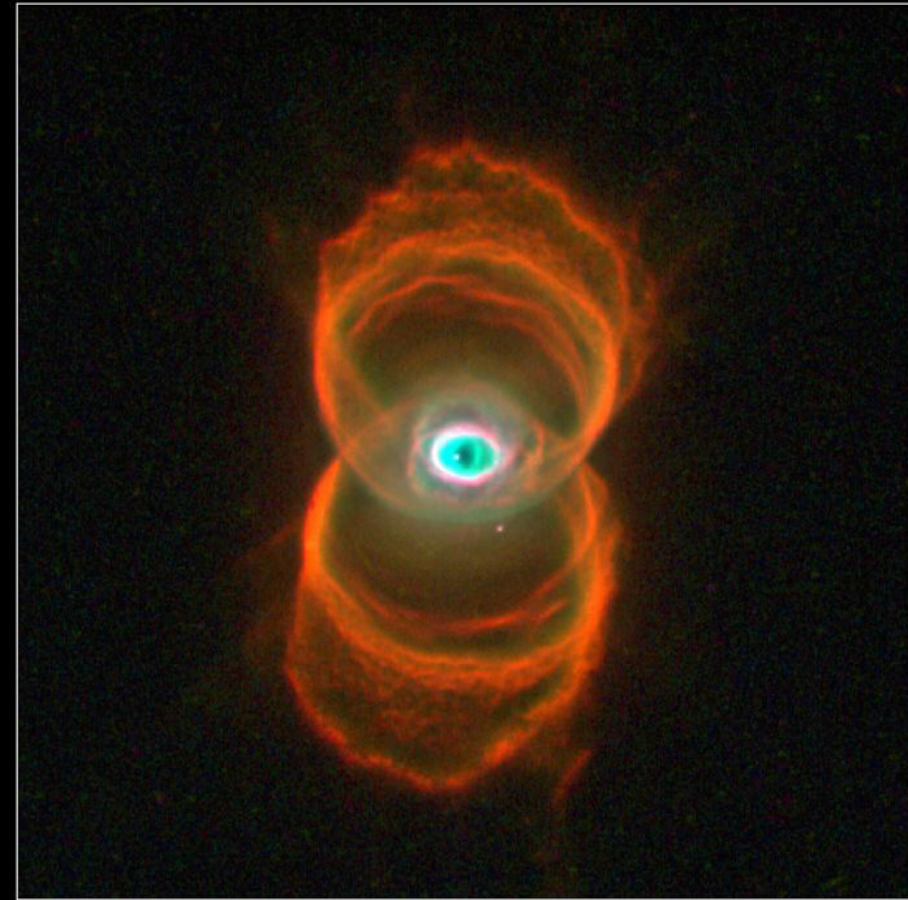


Planetary Nebula M2-9
Hubble Space Telescope • WFPC2

more planetary nebulae



WFPC2 F673N [S II]
ACS/WFC F658N H α
ACS/WFC F502N [O III]



Eta Carinae = η Car A + η Car B

The most massive star known. $120 M_{\text{sun}}$, was $150 M_{\text{sun}}$.
 $5,000,000 L_{\text{sun}}$, $60-800 R_{\text{sun}}$

Pulsating: 1-7 mag (1837 - Great Eruption for 18 yr, observed by John Herschel).

A 5yr period binary with another massive star η Car B: $30 M_{\text{sun}}$,

1,000,000 L_{sun}



John Herschel
SIR JOHN FREDERICK WILLIAM HERSCHEL, BART. F.R.S.

John Herschel



Eta Carinae

X-ray picture



The evolution of massive stars: Wolf-Rayet stars

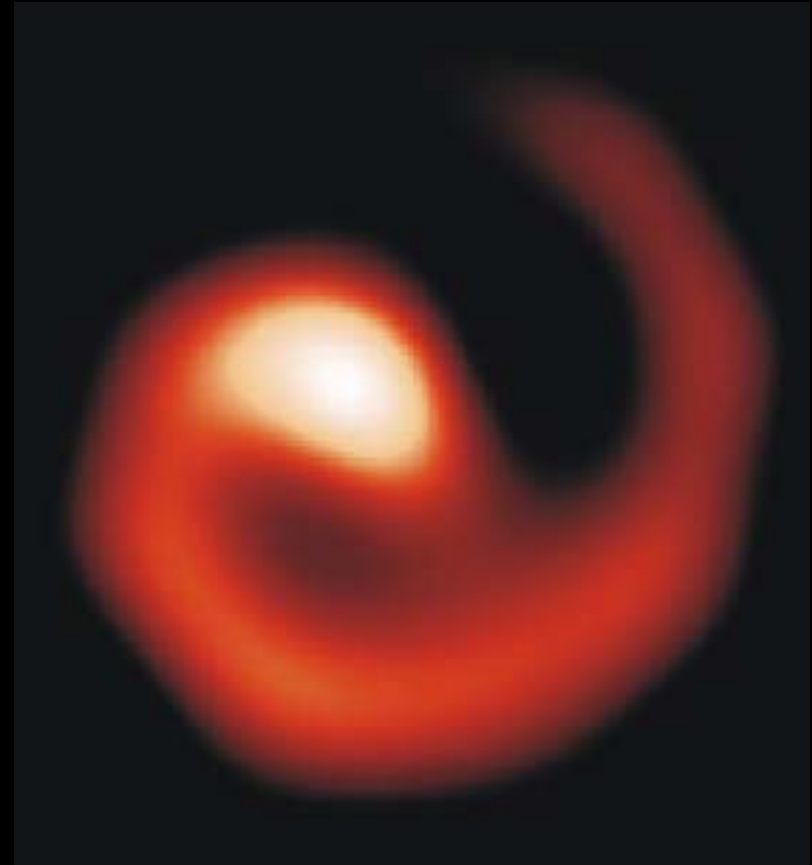
(5-10 M_{sun})

Very massive stars ($M > 10$ solar masses) have strong stellar winds and lose mass rapidly at all stages of evolution, including the main sequence.

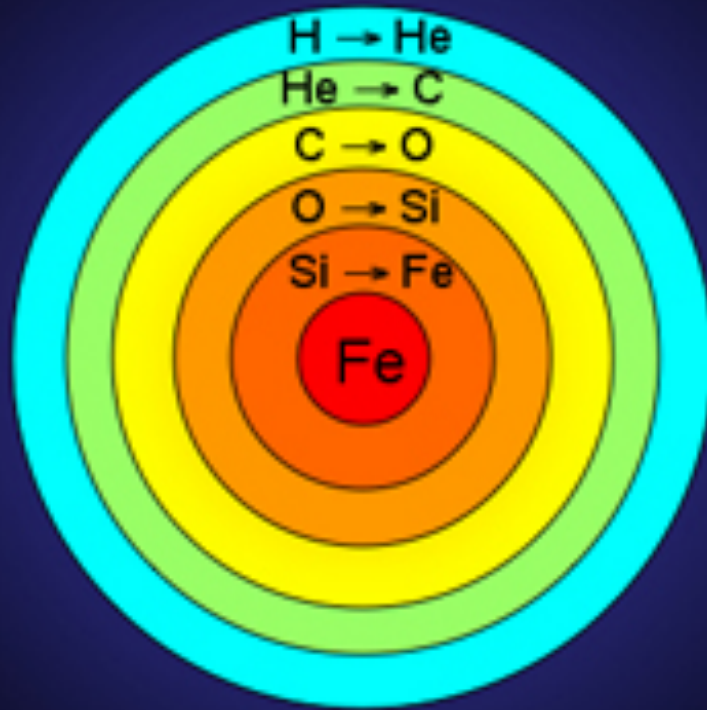
The electrons in their core do not become degenerate until the final burning stages. The core at that point consists of iron (Fe). Other elements – hydrogen, helium, carbon, oxygen, and silicon, burn in successive layers (moving inward).

The luminosity is almost constant, at all stages of the evolution. These stars move horizontally across the HR diagram.

Stars with $M > 30$ solar masses may lose all, or almost all, of their hydrogen envelopes while still on the MS. An example of this is what are known as **Wolf-Rayet stars** (M about 5-10 solar masses). They are highly luminous, hydrogen-depleted cores of the most massive stars.



- timescales and layering in massive stars



For a 25 solar mass star:

Stage	Duration
$H \rightarrow He$	7×10^6 years
$He \rightarrow C$	7×10^5 years
$C \rightarrow O$	600 years
$O \rightarrow Si$	6 months
$Si \rightarrow Fe$	1 day
Core Collapse	1/4 second

supernovae: core collapse of massive star

<http://astronomy.swin.edu.au/cosmos/c/core-collapse>

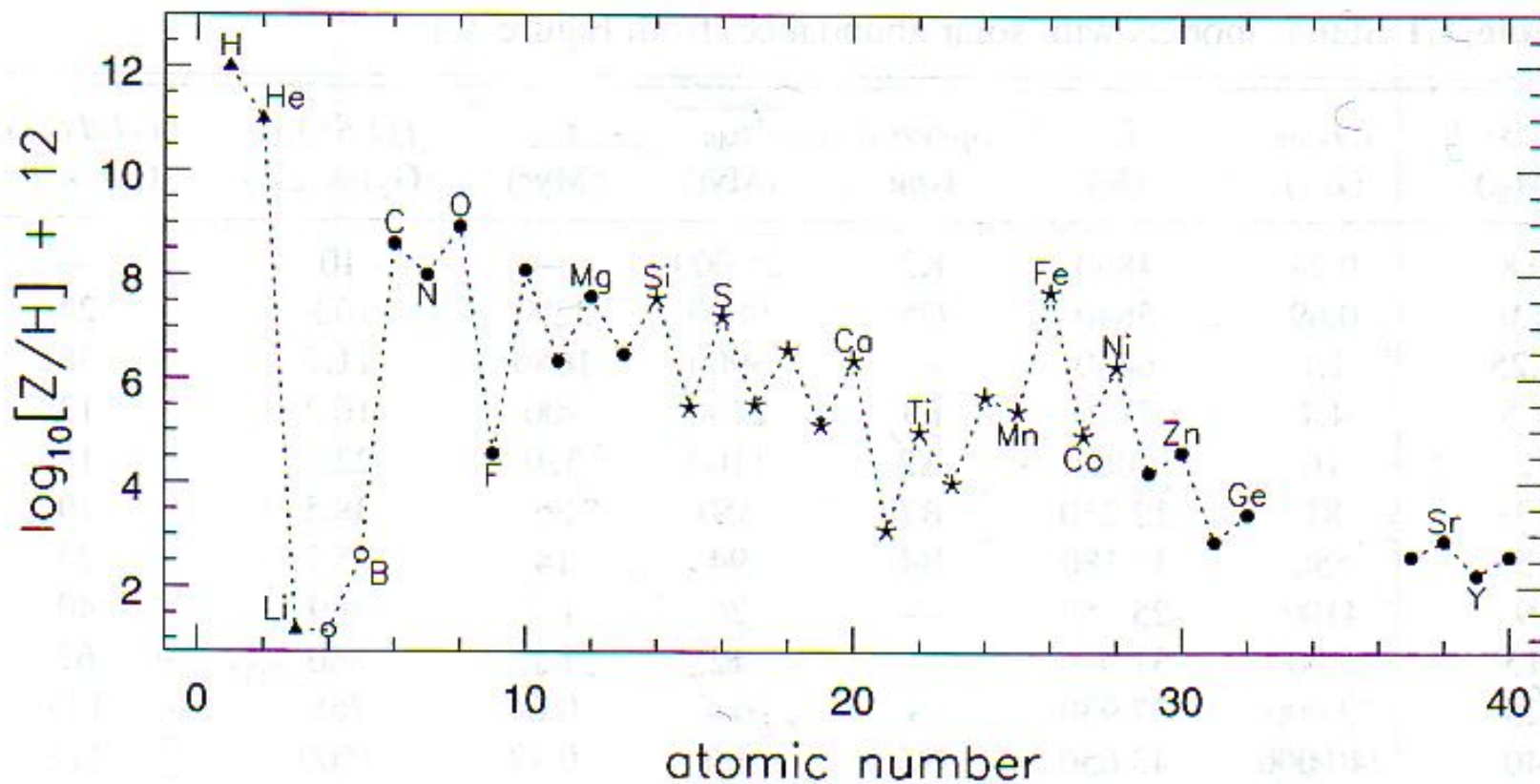


Figure 1.3 Logarithm of the number of atoms of each element found in the Sun, for every 10^{12} hydrogen atoms. Hydrogen, helium, and lithium originated mainly in the Big Bang; the next two elements result from the breaking apart of larger atoms, and the remainder are ‘cooked’ in stars. Filled dots show elements produced mainly in quiescent burning; asterisks indicate those made largely during explosive burning in a supernova – from Anders & Grevasse.

Tiny but almost as bright as galaxies:
supernovae, Active Galactic Nuclei, Quasars (extreme AGNs)



← a bright supernova

Supernovae – Type II

Massive stars at the end of their lifetimes develop iron cores. The core is degenerate and grows as shell burning continues. When the mass of the core exceeds the Chandrasekhar limit, the core starts to contract rapidly.



© Royal Observatory, Edinburgh / Anglo-Australian Observatory

Two types of instabilities develop. First, electrons are captured by heavy nuclei, which reduces the pressure. Second, the degenerate matter is less sensitive to temperature changes, and the temperature rises quickly. Eventually, this leads to photodisintegration of iron nuclei. This reaction is endothermic, and absorbs about 2 MeV per nucleon. Both of these processes reduce the energy and the pressure in the core, which now collapses in almost free fall.

As the temperature rises even further, highly energetic photons break the helium nuclei into protons and neutrons, a reaction which absorbs about 6 MeV per nucleon.

Supernovae – Type II

Finally, the density rises enough for the free protons to merge with free electrons to form neutrons. This reduces the number of particles, which reduces the pressure even further. The neutron gas becomes degenerate at a density of about 10^{18} kg/m³.

There is a big bounce when matter gets so dense that neutrinos get trapped and form gas that bounces.

The total time it takes for the core to collapse is a few hundred milliseconds!

The outer layers of the star are violently ejected, in an explosion that imparts high velocities to the material, and that briefly (for a few days or weeks) outshines an entire galaxy.

Elements heavier than magnesium are formed in the shockwave of the supernova explosion, in which the temperature and pressure are high enough to permit their formation.

The ejected material can be seen as a shell-like structure, that expands at first quickly but then at a decelerating rate, around the collapsed core, and eventually mixes with the interstellar medium.

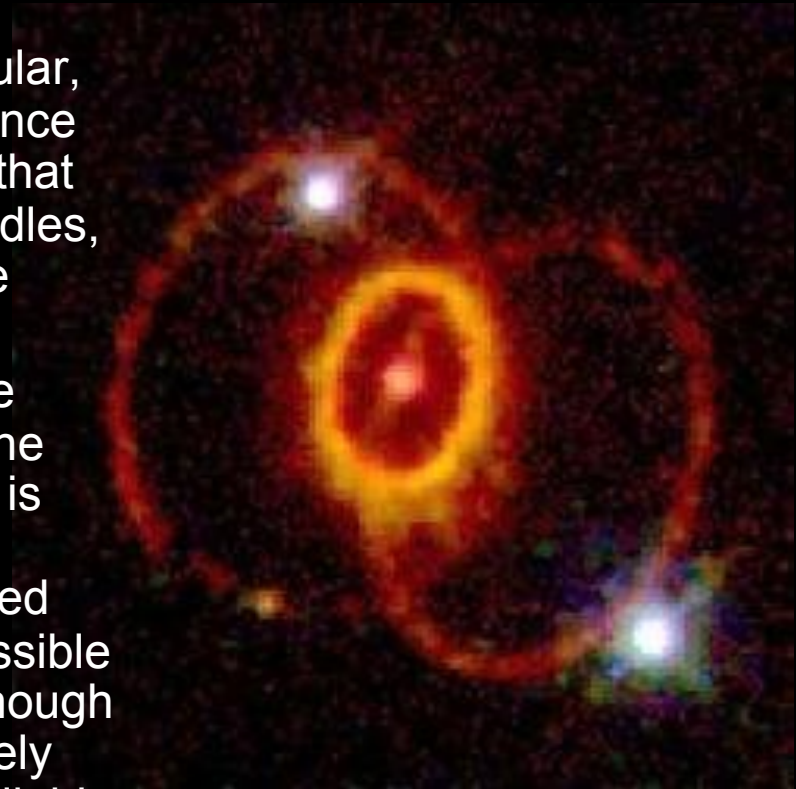


A Type Ia supernova is formed when a white dwarf surpasses the Chandrasekhar limit and undergoes core collapse. Since the white dwarf existed before the explosion, this can only happen in a binary system. When the companion star reaches the red giant stage, material from its outer layers accretes onto the white dwarf, increasing the WD's mass.

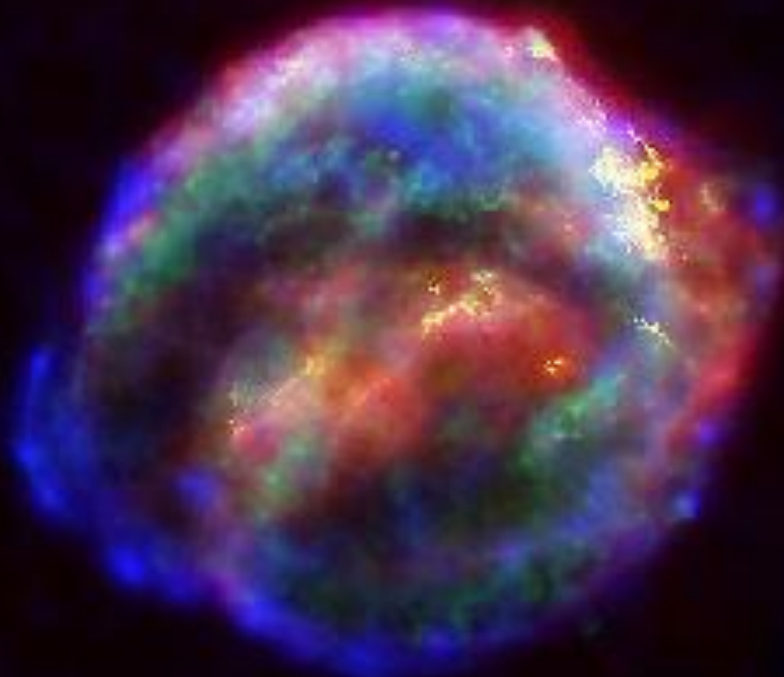
The lightcurves of type Ia supernovae show remarkable similarity to each other – in particular, the luminosity as a function of time elapsed since the explosion is the same for all. This means that Type Ia supernovae make good standard candles, which makes them very important for distance determination to far away galaxies.

A Comment: Observations of Type Ia supernovae were a crucial part of recent work that led to the conclusion that the expansion of the universe is accelerating. This in turn leads to the current discussion about, and theories of, what is called the “dark energy”. These conclusions are possible because Type Ia supernovae are luminous enough to be seen at very large distances, and they rely heavily on the assumption that SN Ia's are reliable standard candles.

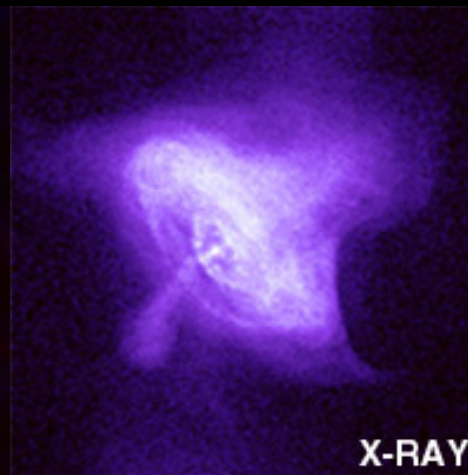
Supernovae – Type Ia



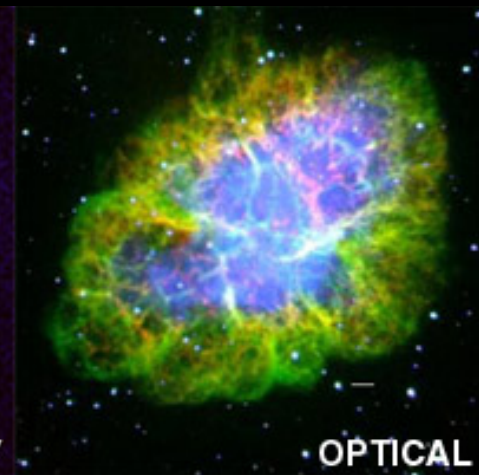
Observations: remnants carry away only 10^{42} J energy (kinetic), while they radiate in all wavelength ranges only 10^{44} J, total. Compare this with the gravitational energy change between the initial white dwarf-like object and a neutron star: $\sim GM^2/R_{\text{final}} \sim 10^{46}$ J. Where's $>90\%$ of energy? It is needed to create and propel **neutrinos**, as in $p \rightarrow n + (e^+) + (\nu_e)$, a.k.a. **neutronization**



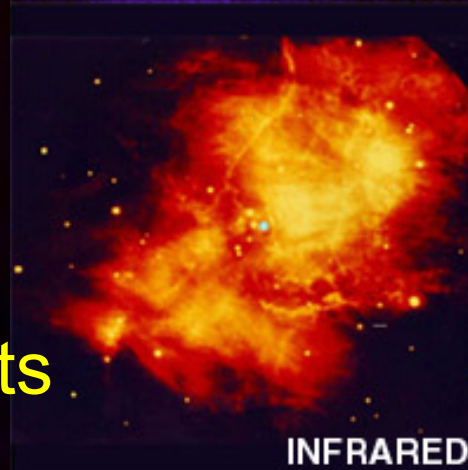
Supernova remnants



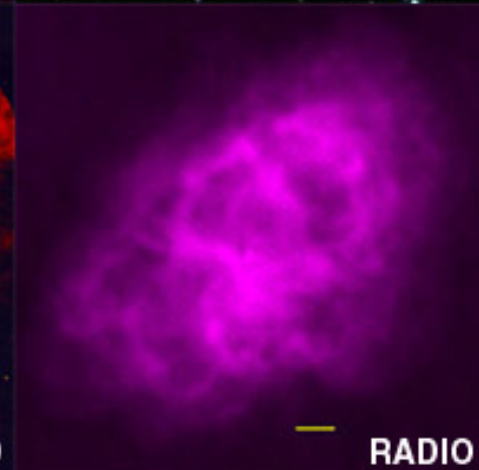
X-RAY



OPTICAL



INFRARED

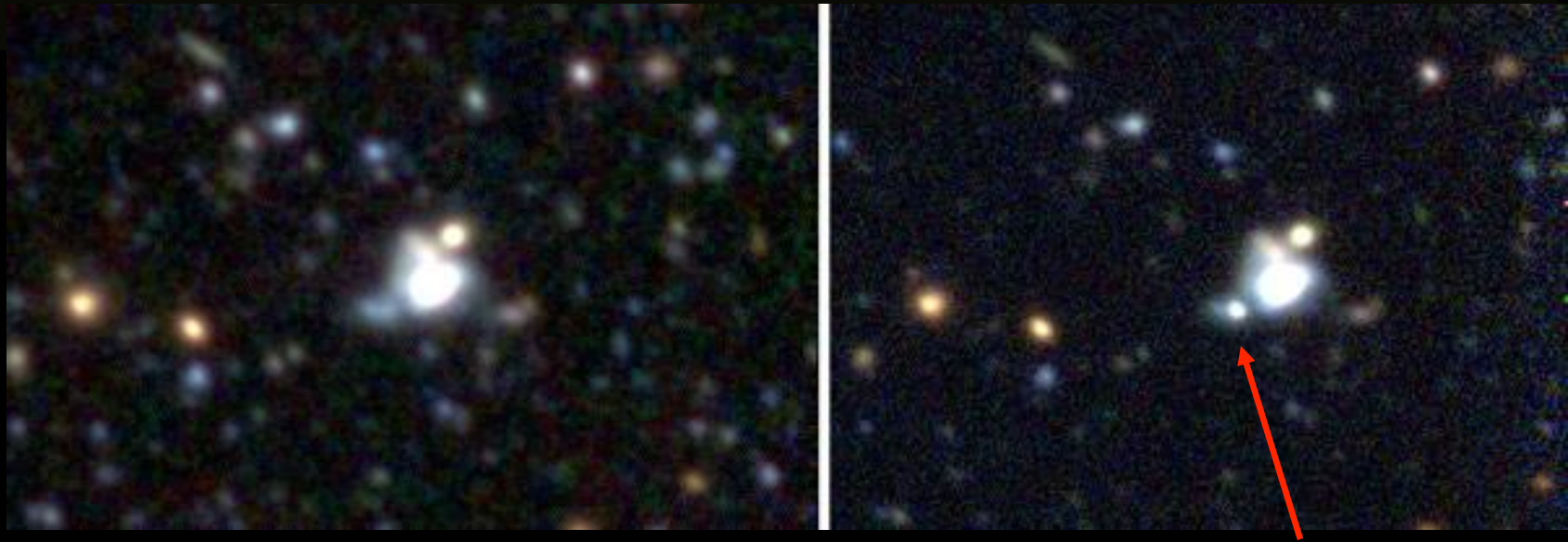


RADIO

SNLS-03D3bb

Before & after

SN explosion



A recent bizarre supernova (2003)

The Canada-France-Hawaii Telescope observed the host galaxy before the supernova (left) and afterward (right).

(Image: Andy Howell (UofT)/Supernova Legacy Survey/CFHT)

Bizarre supernova breaks all the rules

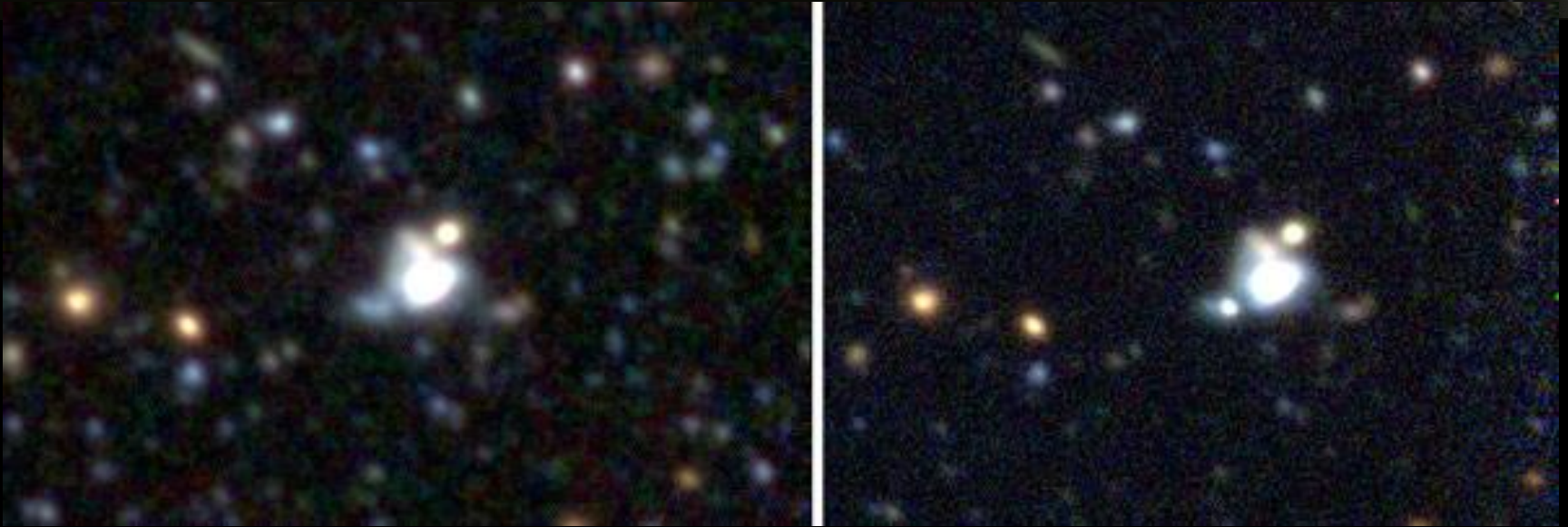
- September 2006; *NewScientist.com* news service; by Maggie McKee; see the papers in *Nature* (vol 443, p 283 and p 308)

A Type Ia supernova more than twice as bright as others of its type has been observed, suggesting it arose from a star that managed to grow more massive than the Chandrasekhar limit.

This mass cut-off was thought to make all such supernovae explode with about the same intrinsic brightness, allowing astronomers to calculate their distance based on how bright they appeared to be through telescopes. In fact, it was observations of type Ia supernovae that led to the surprising discovery in 1998 that some mysterious entity was causing the universe's expansion to speed up.

Now, astronomers led by Andrew Howell of the University of Toronto in Canada have found what appears to be a type Ia supernova that is **2.2 times as bright** as others of its class. Called SNLS-03D3bb, it lies about 3 billion light years away, a distance obtained from the redshift in the spectrum of its host galaxy. *It's the first such case in about 400 well-studied SN. High brightness, along with other clues from the supernova's spectrum, suggests that the white dwarf exploded with **2.1 solar masses** of material – significantly above the Chandrasekhar mass.*

SNLS-03D3bb



Possible explanations of the “Champagne SN” (*from an Oasis song*)

<https://www.youtube.com/watch?v=R04sLCmtz3E>

1. White dwarf may have been spinning so fast that centrifugal force allowed it to exceed the mass limit.
2. Alternatively, the supernova may have resulted from the merger of two white dwarfs, so that the limit was only violated momentarily.

Neutron stars, Pulsars

the remnants of some supernovae

predicted in 1930s by:

Swiss astrophysicist F. Zwicky,

Soviet physicist L.D. Landau.

Structure theory by J. Oppenheimer (American) &
G. Volkoff (Russian-Canadian)

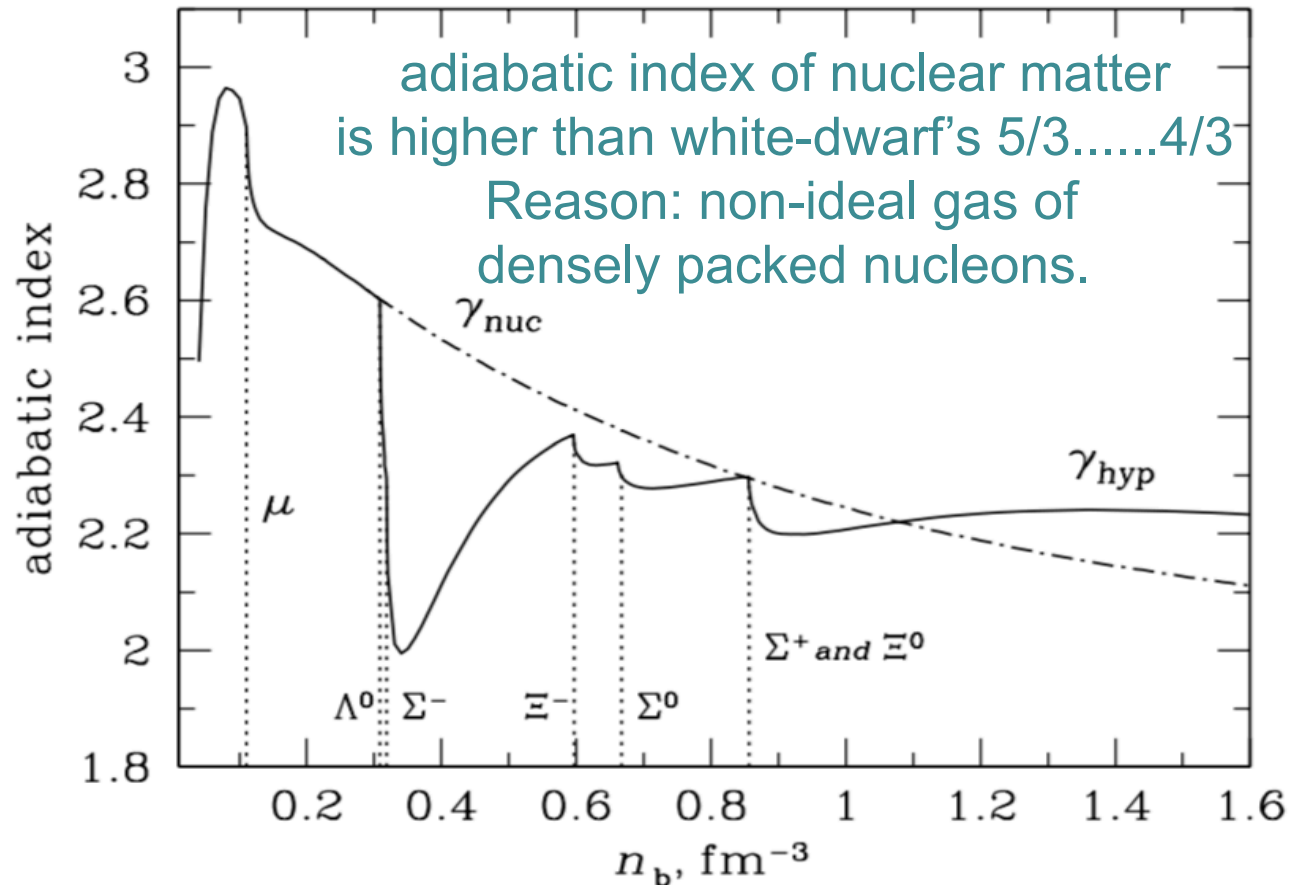


Figure 5.15. The adiabatic index γ versus n_b in a neutron-star core. Calculations are performed for the model EOS of Glendenning (1985). The solid line (γ_{hyp}) is for the hyperonic matter (vertical dotted lines indicate thresholds for the appearance of muons and hyperons); the dot-and-dashed line (γ_{nuc}) corresponds to the case in which the appearance of hyperons is artificially forbidden.

From the structure eqs.

(hydrostatics + mass eq.+polytropic EOS)

$$Gm/R^2 = (1/\rho) dP/dr$$

$$dm/dr = 4\pi r^2 \rho$$

$$P = \text{const. } \rho^\gamma$$

you *should* be able to show that:

$$M/R^2 \sim M^2 R^{-5} \rho^{-1}, \quad \rho \sim M/R^3, \quad P \sim \rho^\gamma \rightarrow$$

$$R^{3\gamma-4} \sim M^{\gamma-2}$$

$\gamma \rightarrow \infty$ $R \sim M^{+1/3}$ incompressible solid bodies

$\gamma = 2$ $R \sim M^0$ brown dwarfs, neutron stars

$\gamma = 5/3$ $R \sim M^{-1/3}$ nonrelativistic white dwarf

$\gamma = 4/3$ $R^0 \sim M$ relativistic WDs, NSs

(Chandrasekhar mass M_{Ch} , Oppenheimer-Volkoff $M_{\text{O-V}}$)

compare: $R \sim M^{0.5...1}$ normal stars (non-adiabatic gas,
energy production equation, radiative transport)

Structure equations strongly affected by General Relativity

Calculating \mathcal{R}_{ik} for the metric (6.1) and using the Einstein equations (6.6), one comes to the three relativistic equations of hydrostatic equilibrium for a static spherically symmetric neutron star,

$$\frac{dP}{dr} = -\frac{G\rho m}{r^2} \left(1 + \frac{P}{\rho c^2}\right) \left(1 + \frac{4\pi P r^3}{m c^2}\right) \left(1 - \frac{2Gm}{c^2 r}\right)^{-1}, \quad (6.7)$$

$$\frac{dm}{dr} = 4\pi r^2 \rho, \quad \text{this is just to show differences with the structure equations that you know,} \quad (6.8)$$

$$\frac{d\Phi}{dr} = -\frac{1}{\rho c^2} \frac{dP}{dr} \left(1 + \frac{P}{\rho c^2}\right)^{-1}. \quad \text{do not memorize!} \quad (6.9)$$

Equation (6.7) is called the Tolman-Oppenheimer-Volkoff equation of hydrostatic equilibrium (Tolman 1939; Oppenheimer & Volkoff 1939; also see § 1.2). Equation (6.8) describes mass balance; its apparently Newtonian form is illusive because the proper volume of a spherical shell, given by Eq. (6.4), is not simply $4\pi r^2 dr$. Finally, Eq. (6.9) is a relativistic equation for the metric function $\Phi(r)$. These equations should be supplemented by an EOS, $P = P(\rho)$.

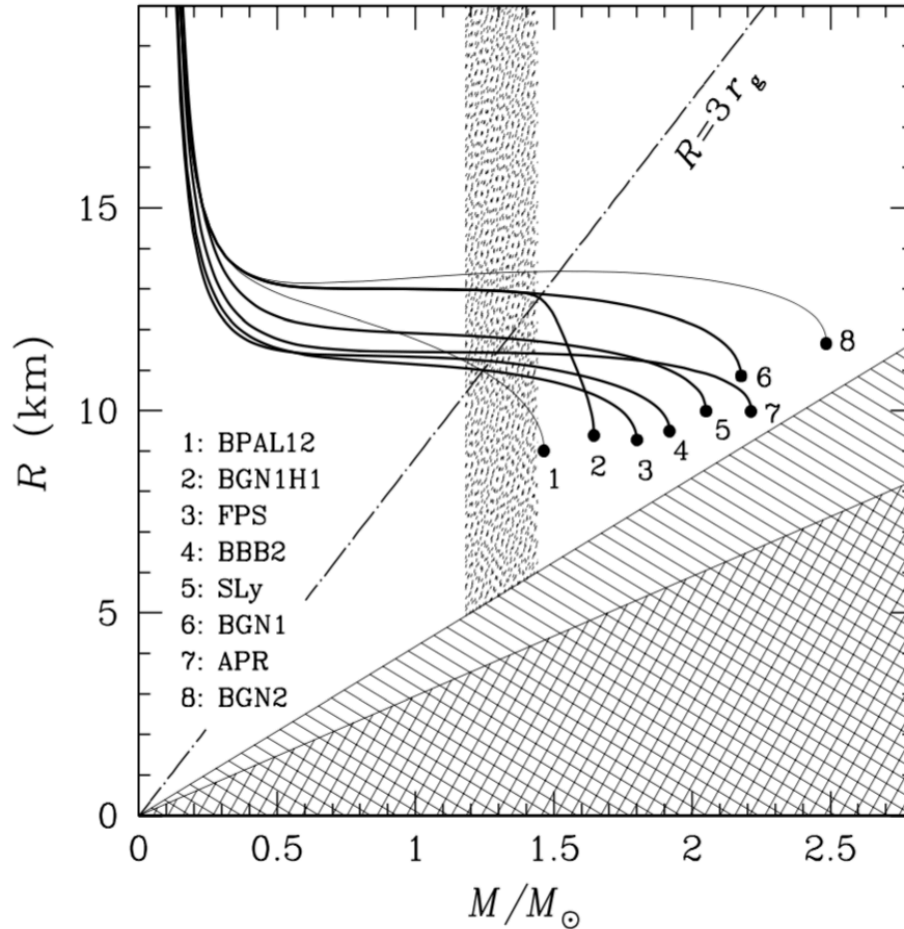


Figure 6.5. Circumferential neutron star radius R versus gravitational mass M for the selected EOSs of dense matter. The doubly hatched area ($R < r_g$) is prohibited by General Relativity. The entire hatched triangle is prohibited by General Relativity combined with the condition $v_s \leq c$. The shaded vertical band corresponds to the measured range of masses of double neutron star binaries (§9.1.2c). The dot-and-dashed line $R = 3r_g$ shows the radius of the innermost stable orbit of a test particle rotating around a compact object with a given mass.

NS radii predicted by different theoretical models 1-8 are almost independent of mass, which is predicted if the polytropic index ~ 2

Haensel et al (2016)
“Neutron Stars – I”

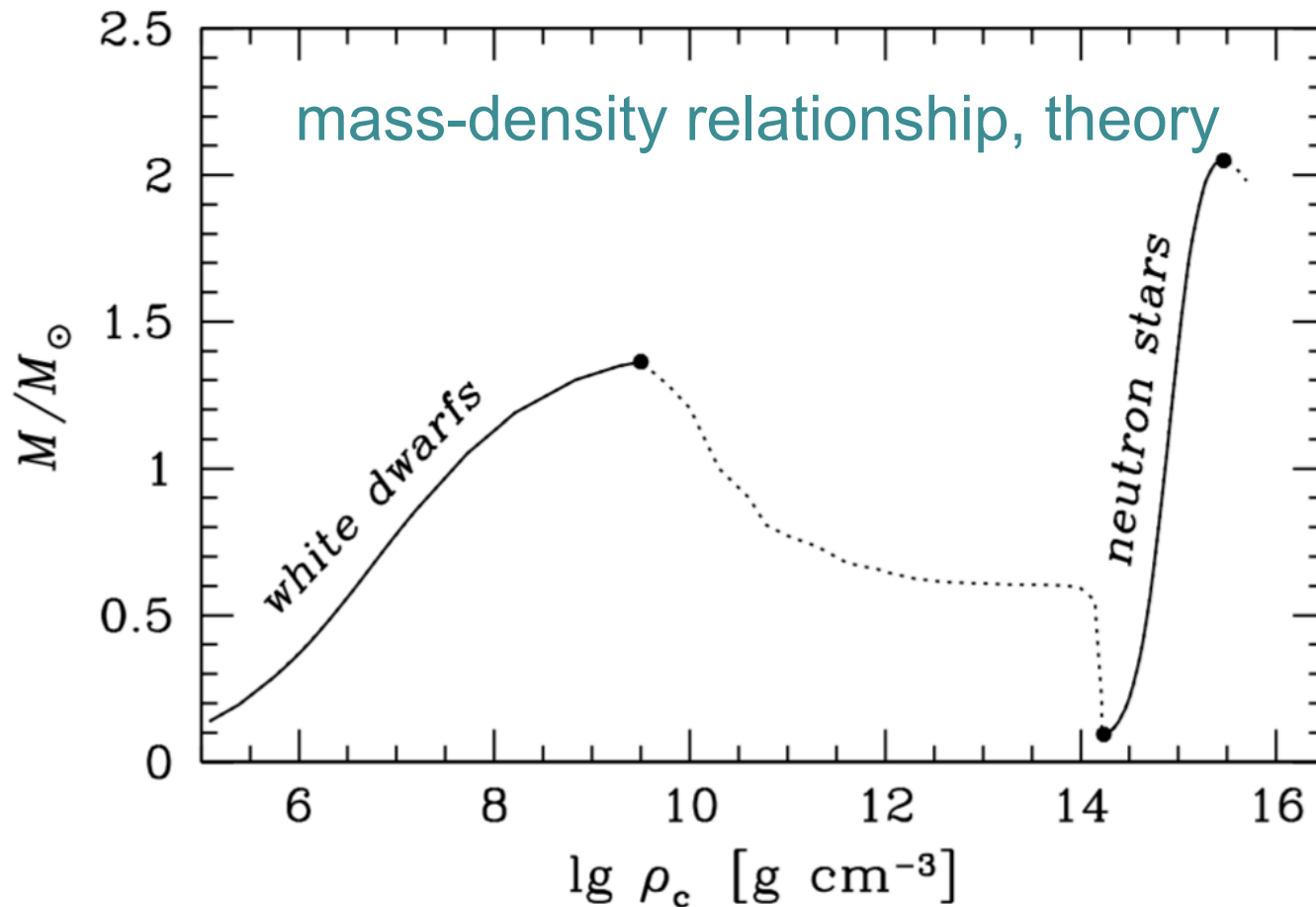


Figure 6.2. Schematic dependence of masses of equilibrium stellar models (built of cold dense matter) versus central density. Solid fragments show stable stars (either white dwarfs or neutron stars) while dotted fragments show unstable stars. Filled dots indicate maximum-mass and minimum-mass stars.

NEUTRON STARS

were discovered as PULSARS by Jocelyn Bell



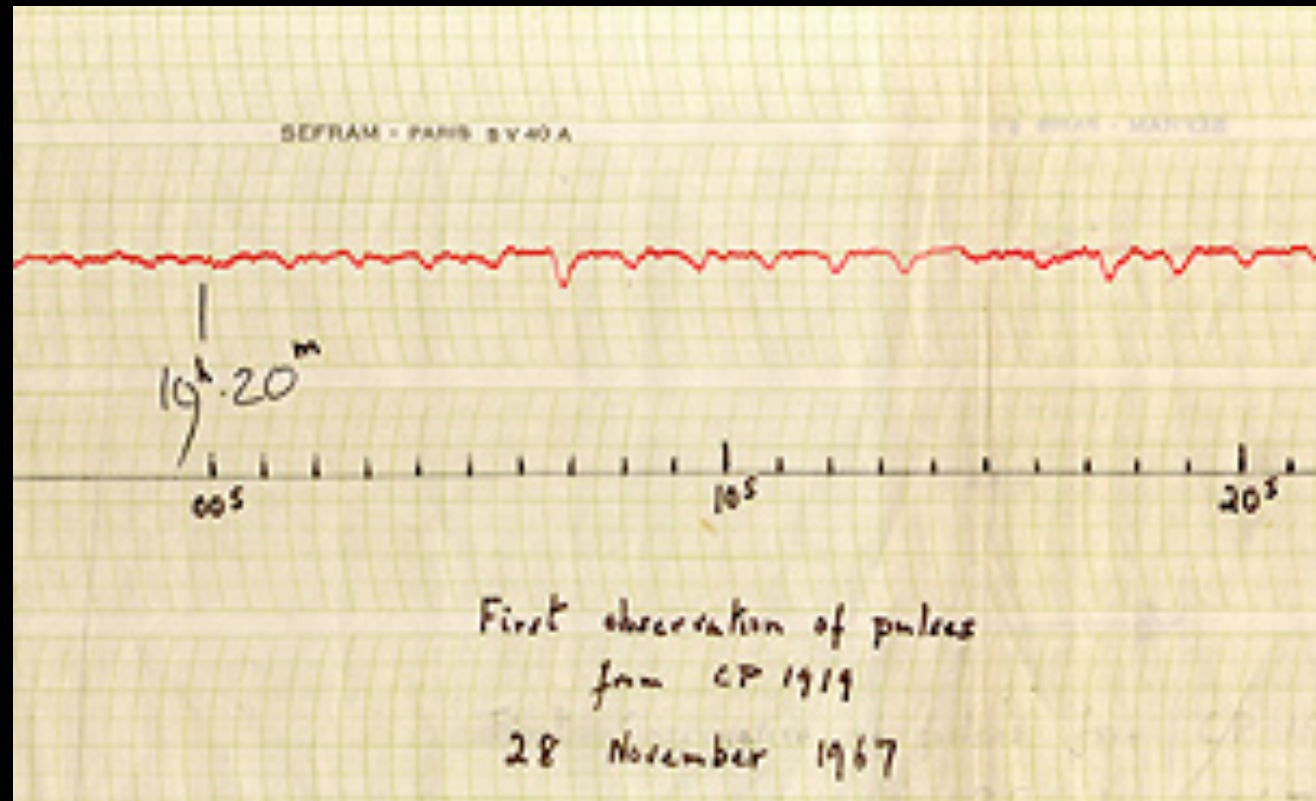
Jocelyn Bell Burnell was born in northern Ireland.

After receiving a B.S. degree in physics from Glasgow University, Scotland, she went to Cambridge University, England, where she earned her doctorate in radio astronomy in 1969.

She made the discovery as a graduate student in 1967 and rather scandalously was not honored with Nobel prize, although her supervisor was!

NEUTRON STARS discovered as **PULSARS** by Jocelyn Bell in 1967 using a radio telescope Her thesis was about quasars (quasi-stellar objects, now known to be powered by supermassive black holes.)

Her advisor was Tony Hewish.



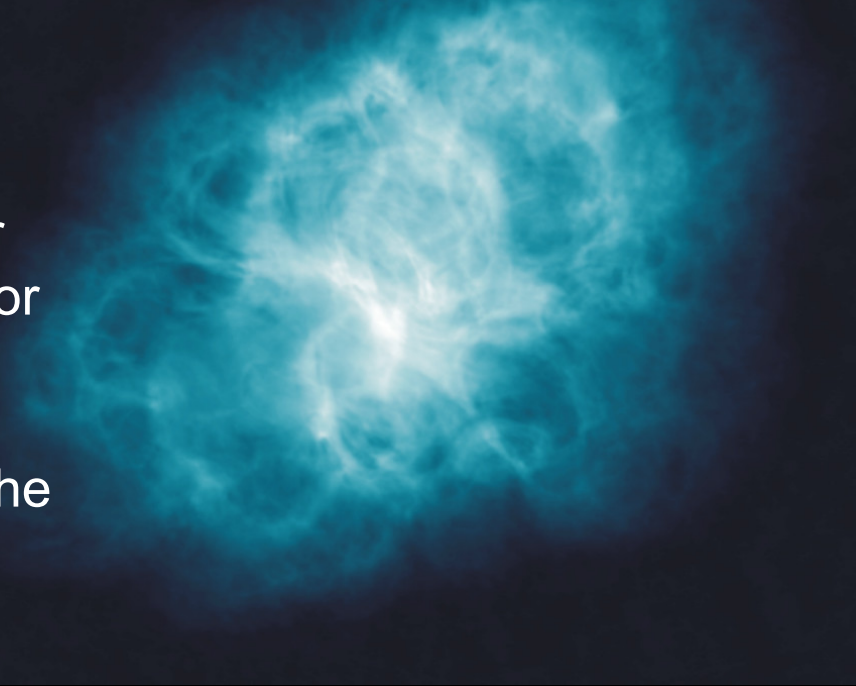
Neutron stars

Neutron stars are the collapsed remnants of the iron cores of massive stars that have undergone supernova explosions. The core is held up by neutron degeneracy pressure.

The masses of neutron stars also have an upper limit (Oppenheimer-Volkoff limit), which is higher than the Chandrasekhar limit for white dwarfs. The calculations for neutron degeneracy involve also the strong nuclear force, about which not enough is yet known to uniquely know the equation of state of the degenerate neutron matter. It is estimated that the **upper limit for neutron stars lies between 2 and 3 solar masses.**

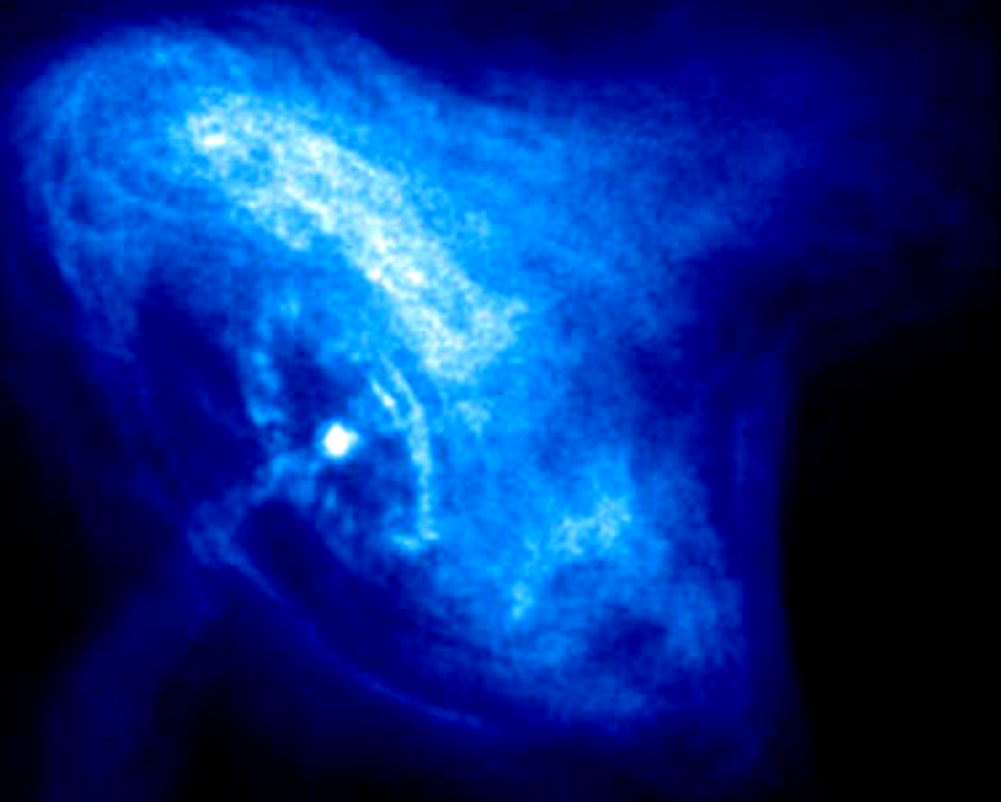
Example: a neutron star of 1.5 solar masses would have a radius of 12-15 km.

Neutronization of matter
 $p^+ + e^- \rightarrow n^0 + \nu_e$
during supernova explosion

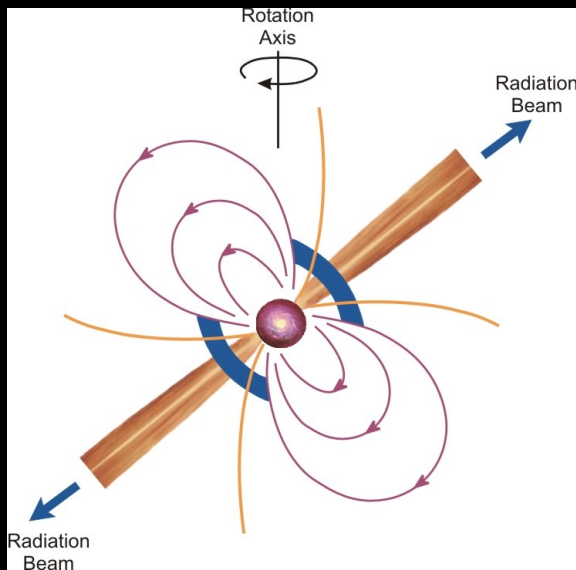


The Crab Nebula, a SN 1054 remnant w/NS containing P=33.5 ms pulsar

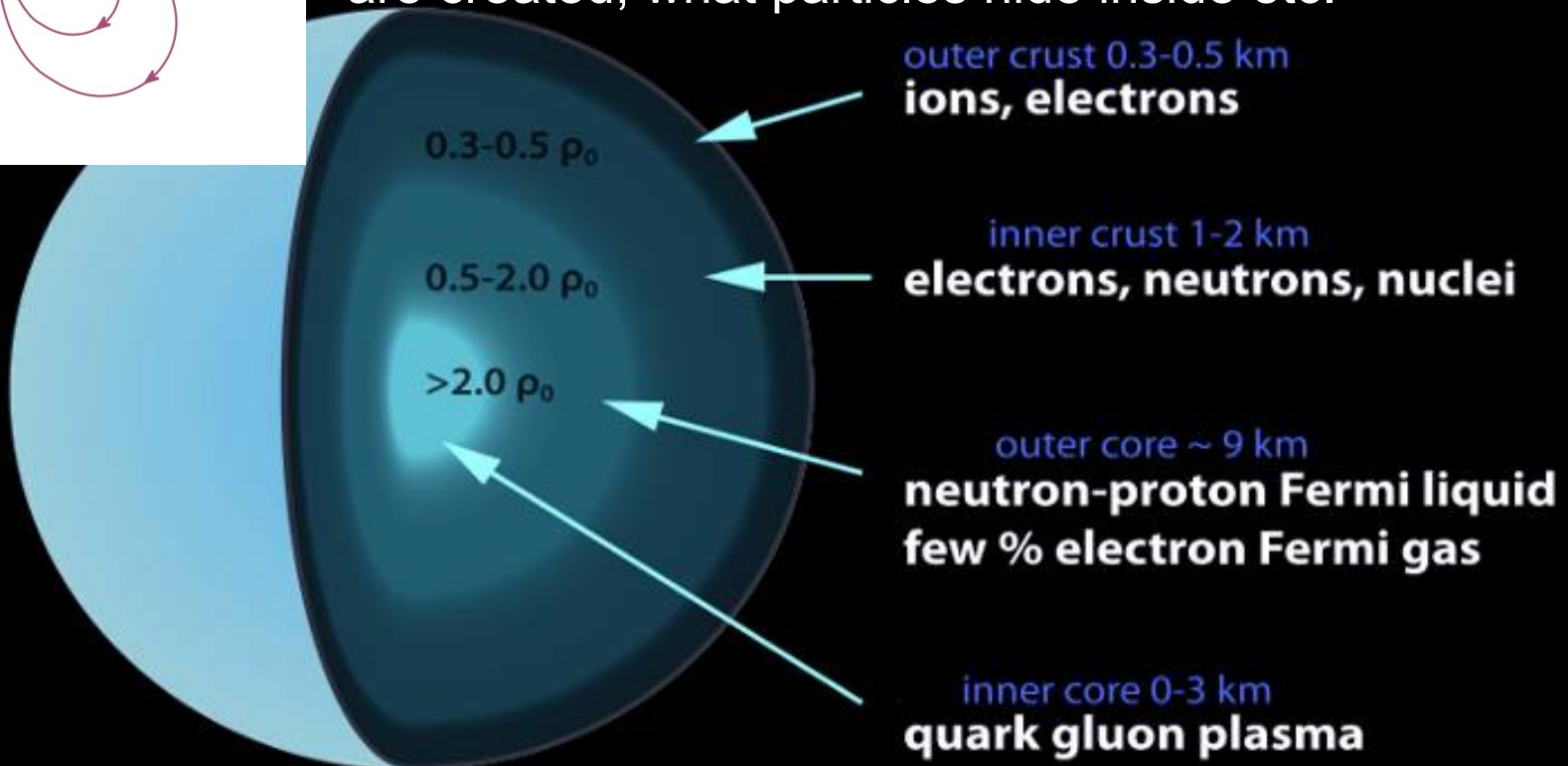
Effect of a neutron star on the inner regions of the Crab Nebula over a period of several months. Animation.



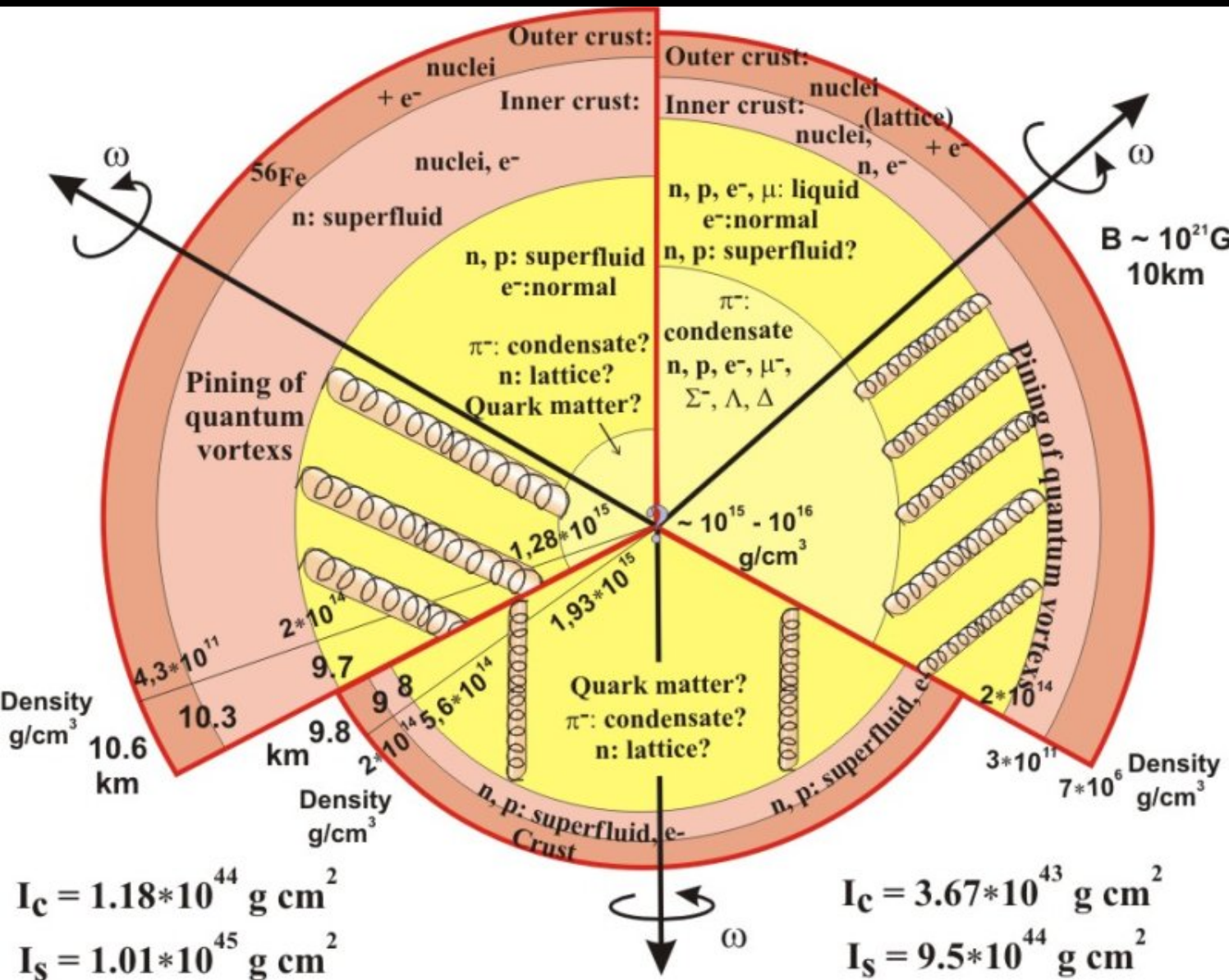
NS = a new type of star, made of neutron-enriched superdense n-crystal matter; superfluid & superconducting



Starquakes occur as the rotation of a magnetar (highly magnetized neutron star) slows down. A very energetic quake on **SGR 1806–20** was detected in 2004. In the first 0.2 s, the flare released as much energy as the Sun radiates in $\frac{1}{4}$ Myr. Pulsars still keep many secrets, e.g. how exactly their beams are created, what particles hide inside etc.



PS. We still don't know everything about pulsars, in particular how their beams are created, or even precisely what's inside them



see all those different models and the question marks?

Density 10¹⁵g/cm³ (a backpack ⇒ mass of a 10-km comet)

Messier 1 (M1) is an expanding nebula discovered in 1731 by John Bevis. It is a remnant of a supernova from year 1054, seen by the Chinese astronomers. M1 harbors a pulsar. We know $> 10^3$ such objects, and 10^5 may be present in our Galaxy.

2008



Black holes & their history

John Michell (1724 – 1793)

clergyman and natural philosopher, member of Royal Academy

He was the first person known to propose the existence of what was later named derisively Black Holes.

<https://www.youtube.com/watch?v=3pAnRKD4raY>

In the above video, there is a GIF animation, the foreground black hole is lensing the image of the background galaxy (it's just an animation, not real observations).

BH acts as gravitational lens, bending light much more strongly than the sun does it, in the classical solar eclipse experiment that we have discussed (the closer the light passes near a given mass, the larger the deflection; it's a simple inverse proportionality – and black hole is very small!)

In a paper for the *Philosophical Transactions of the Royal Society of London*, read on 27 November 1783, Michell first proposed that there were “**dark stars**”:

“If there should really exist in nature any bodies, whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us; or if there should exist any other bodies of a somewhat smaller size, which are not naturally luminous; of the existence of bodies under either of these circumstances, **we could have no information from sight**; yet, if any other luminous bodies should happen to revolve about them we might still perhaps from the motions of these revolving bodies infer the existence of the central ones with some degree of probability, as this might afford a clue to some of the apparent irregularities of the revolving bodies, which would not be easily explicable on any other hypothesis.”

This was correct, yet much ahead of his time...

"He died in quiet obscurity", states the American Physical Society, "and his notion of a 'dark star' was forgotten until his writings re-surfaced in the 1970s".

Pierre-Simon Laplace (1749 –1827)

French mathematician, physicist, astronomer, atheist and... Napoleon's Minister of the Interior!

(Napoleon later wrote critically of his brief appointment:

“Laplace did not consider any question from the right angle: he sought subtleties everywhere, conceived only problems, and finally carried the spirit of ‘infinitesimals’ into the administration.”)

Laplace had a wide knowledge of all sciences and dominated all discussions in the French *Académie*. He wrote *Mécanique céleste* – textbook on celestial dynamics published in five volumes from 1799 to 1805.

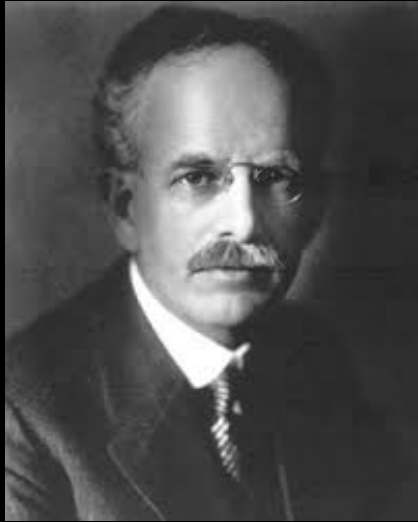
(He loved the phrase: "It is easy to see that...")

In 1796 Laplace had essentially the same idea as Michell, but developed it mathematically: $g = GM/R^2 \sim G(\rho R^3)/R^2 \sim R$ where g is the acceleration of gravity on the surface of a body of the assumingly constant density ρ , while M & R are its mass & radius.



German astrophysicist **Karl Schwarzschild (1873-1916)** – [pron.: shvARTS-shild or ~shwartz-shield] was known for his great contribution to radiation transfer inside the sun, dynamics of stellar systems, and introduction of gratings in spectrometry. In the trenches of WWI, being drafted as a soldier of the German army, in 1915 he first solved Einstein's field equations (published in 1915!) in case of a spherical, non-rotating compact object. Important expression below is known as Schwarzschild metric, but you don't need to understand it.

Much later, a metric for a rotating black hole was also found by Roy Kerr.



The Schwarzschild Solution, 1

$$d\tau^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\phi^2 - r^2 \sin^2 \phi d\theta^2$$

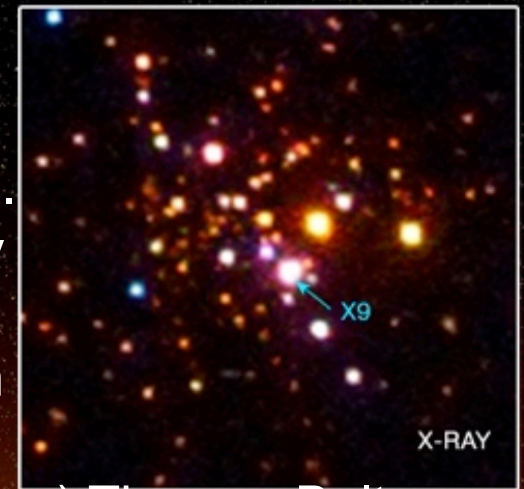


Schwarzschild radius

= horizon radius of a non-rotating black hole

$R_{\text{Sch}} = 2GM/c^2$ (G,c = const.). Notice that $R_{\text{Sch}} \sim M$ ⁵⁷

After the theoretical discovery, a long time had to pass until astronomers first detected a believable example of a black hole. This happened in 1971, in a binary star emitting X-rays, named Cygnus X-1 (X-ray source no. 1 in constellation Cygnus = Swan).



UofT student (later professor) Thomas Bolton was a co-discoverer!

A newer example, X9, is an oxygen-rich white dwarf from 47 Tucannae globular cluster (on the left in this drawing). **WD** is in the closest known pair with a black hole (less likely, a neutron star). Period is only $P=28^m$. It has been discovered in 2017 by NASA Chandra X-ray observatory.⁵⁸

Low-mass (stellar-mass) Black Holes



form in supernova explosions
(illustration: type II supernova remnant)

LOW-MASS BLACK HOLES

Small, i.e. stellar-mass black holes (BH) are a natural end-product of stellar evolution, just like neutron stars. Whether one or the other forms, depends on the initial mass of the star. Stars $>10 M_{\text{sun}}$ are thought to produce black holes directly \rightarrow supernova type II (core-collapse supernova)

Another important route to low-mass and therefore small BHs (since their effective radius is proportional to their mass) is when a white dwarf or neutron star is in a binary system with a star, which sends a stream of gas onto the compact object.



Eventually, the Chandrasekhar mass is exceeded, the star must collapse \rightarrow supernova type Ia *with no compact remnants*

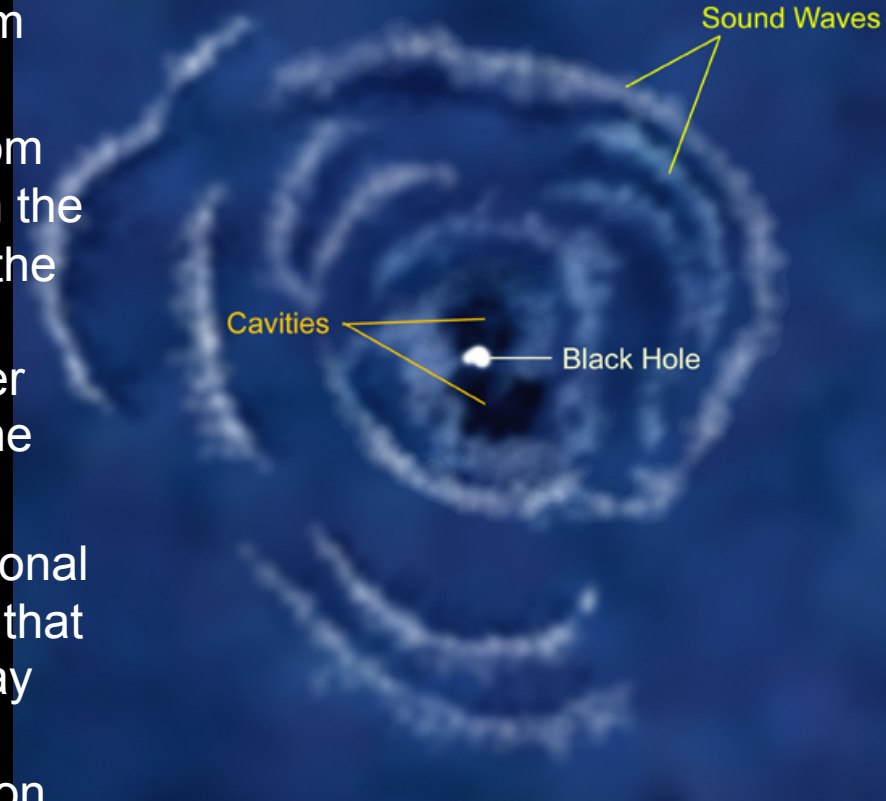
Black holes

If the mass of the collapsed stellar core exceed the limit for what can be held up by neutron degeneracy pressure, the material collapses further. There is not any other thing known in physics at the present time to prevent it from forming a black hole.

Black holes are characterized by a distance from the center called an event horizon, at which the gravitational pull is just strong enough that the escape velocity equals the speed of light. Hence, we in the outside universe can never receive information about anything inside the event horizon of the black hole.

Black holes are detected through their gravitational influence on other objects, such as hot gas that accretes onto the black hole, and emits x-ray radiation as the atoms travel at very high speeds just before crossing the event horizon.

The picture at the right is based on data taken by the Chandra X-ray observatory. It shows sound waves that show up as differences in the temperature of the radiation emitted by material that surrounds what is thought to be the black hole in Perseus.



THE NOBEL PRIZE IN PHYSICS 2020



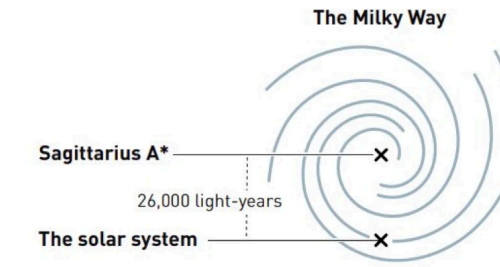
Roger Penrose

“for the discovery that black hole formation is a robust prediction of the general theory of relativity”

Reinhard Genzel

“for the discovery of a supermassive compact object at the centre of our galaxy”

Andrea Ghez



Awarded were discoveries about the supermassive BHs.

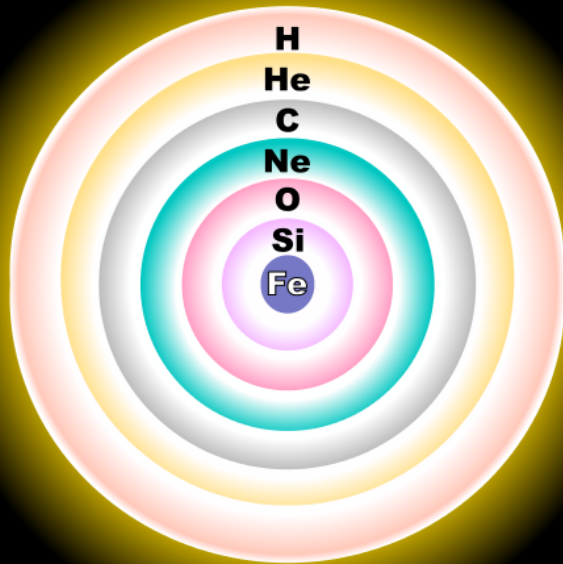
We cover them in the next lectures on galaxies.

Here let's finish the story of stellar-mass black holes.

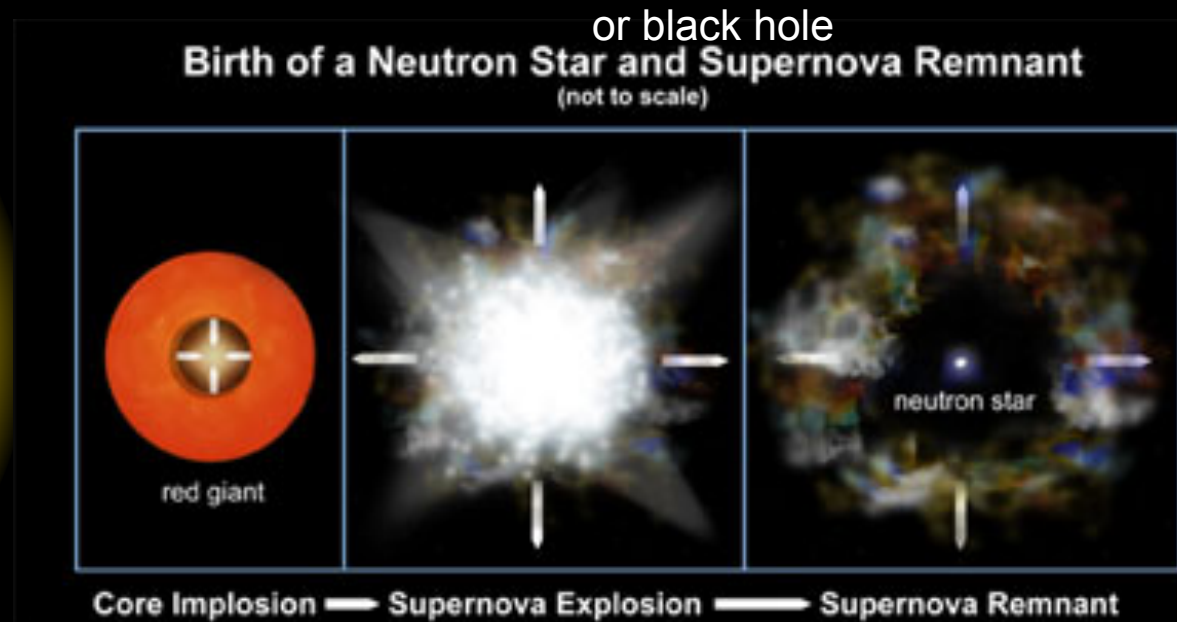
Supernovae are of 2 different types.. only one yields black holes.

1. Small, i.e. stellar-mass black holes (BH) are a natural end-product of stellar evolution, just like neutron stars.

Whether NS or BH forms, depends on the initial mass of the star:
Stars $>10 M_{\text{sun}}$ are thought to produce black holes directly via **supernova type II** (or core-collapse supernova).



a massive star's core



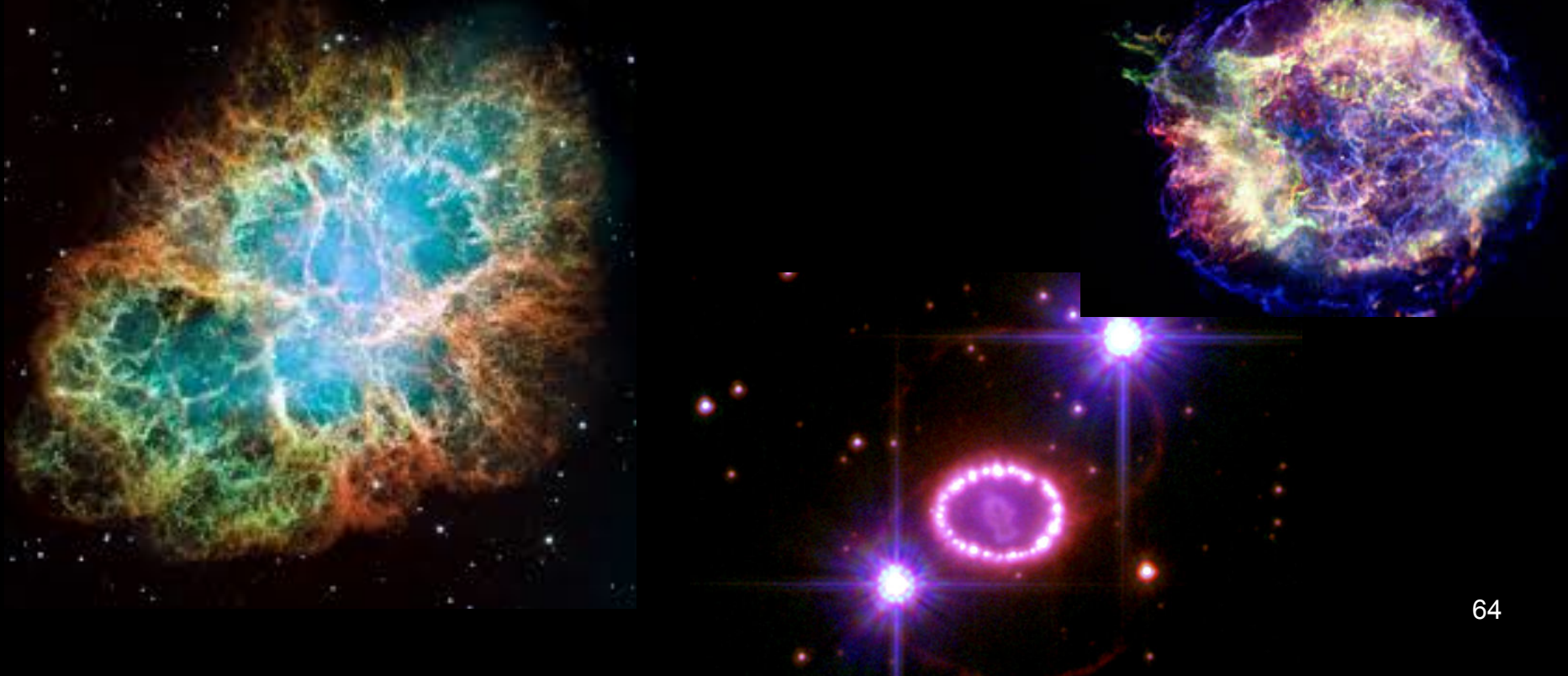
Supernova remnants: Why an explosion and not just implosion?

When a massive star's collapse is slowed down by quantum forces of degeneracy, gas splashes outward with a fraction of the speed of light. We call it **supernova type II**.

Three examples of core-collapse remnants are shown below:

M1

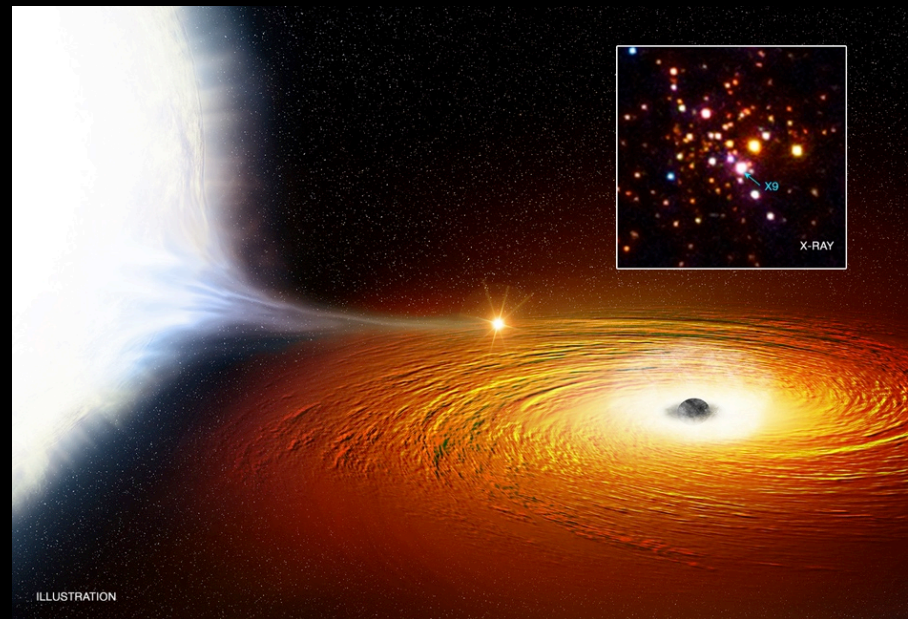
two other SN II remnants:



Stellar-mass black holes can be born in other ways

2. Another important route to stellar explosion is when a white dwarf or neutron star is in a binary system with a normal star, which sends a stream of gas onto that compact object.

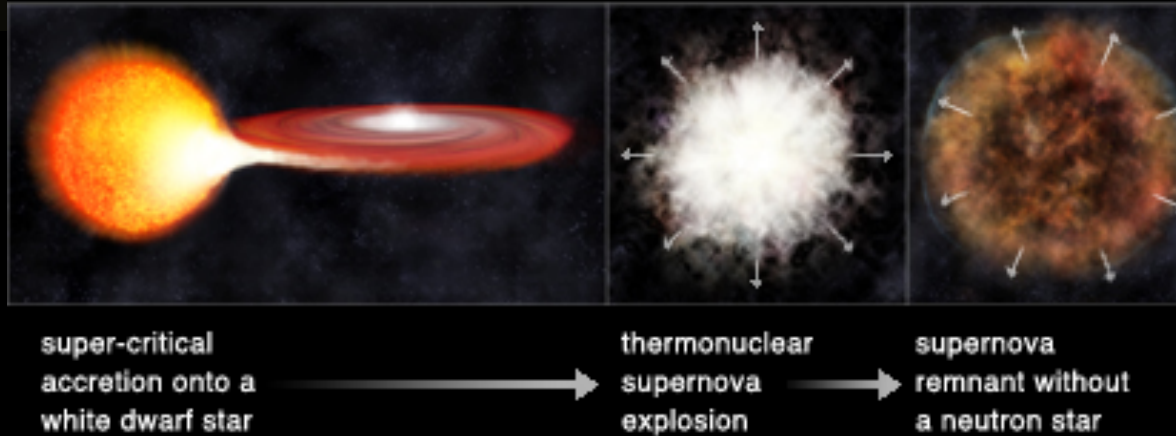
Illustration is re-used 😊
now the black object
is WD, and white star is
a normal star:



If the compact object is a White Dwarf, eventually the Chandrasekhar mass limit is exceeded, and it must collapse becoming **supernova type Ia**.

TYPE Ia (THERMONUCLEAR) SUPERNOVA

(NOT TO SCALE)



Since type Ia supernova starts from the well studied, known conditions ($M = M_{Ch}$), it almost always has a similar, standard brightness curve and peak luminosity. This makes it a great 'standard candle'.

However, remember that type Ia supernovae do not produce pulsars or black holes!

3. If instead a *neutron star* is stealing gas from a normal star, sometimes we get a BH, when NS collapses after exceeding a limit mass (Oppenheimer-Volkoff mass limit $>$ Chandra mass, but). That's a rare event, though.

