# ASTB23. Lecture L23.
# Galaxies are mostly dark matter

---

**Mergers**

**Computational astrophysics**

**Active galactic Nuclei**

**Gravitational Lenses**

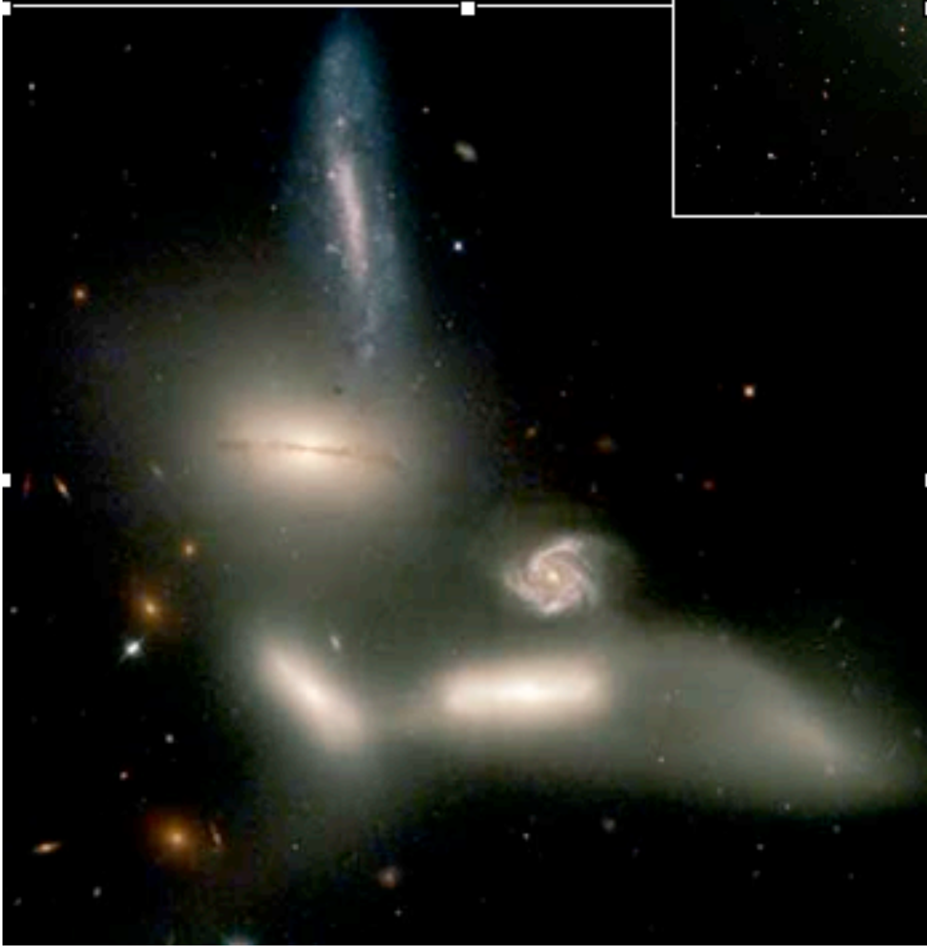
**Superluminal motions**

**Dark Matter – history, evidence, (bad) alternatives**

# Galaxy mergers – computational wizardry in the 21st century

Large Galaxies:
form from the successive merging of smaller galaxies, star clusters and gas clouds.

See next slide for simulation.



# GALAXY MERGERS

a movie about merging galaxies by John Dubinski (UofT)
The future of Galaxy and Andromeda galaxy
https://www.youtube.com/watch?v=DPvCdU6cepY

much more verbal & a simulation of how we'd see it
https://www.youtube.com/watch?v=r8YQsFZyGzw

https://www.youtube.com/watch?v=C0XNyTp5brM
Chris Mihos STScI simulation

https://www.youtube.com/watch?v=opP7ttCCC20
the simulation of dark matter and normal matter is performed in this merger calculation

HOW DO WE DO THE SIMULATIONS in 2023?

We solve dynamics equations dv/dt = F/m on supercomputers

https://www.top500.org/lists/top500/2023/11/

| Rank | System | Cores | Rmax (PFlop/s) | Rpeak (PFlop/s) | Power (kW) |
|---|---|---|---|---|---|
| 1 | **Frontier** - HPE Cray EX235a, AMD Optimized 3rd Generation EPYC 64C 2GHz, AMD Instinct MI250X, Slingshot-11, HPE<br>DOE/SC/Oak Ridge National Laboratory<br>United States | 8,699,904 | 1,194.00 | 1,679.82 | 22,703 |
| 2 | **Aurora** - HPE Cray EX - Intel Exascale Compute Blade, Xeon CPU Max 9470 52C 2.4GHz, Intel Data Center GPU Max, Slingshot-11, Intel<br>DOE/SC/Argonne National Laboratory<br>United States | 4,742,808 | 585.34 | 1,059.33 | 24,687 |
| 3 | **Eagle** - Microsoft NDv5, Xeon Platinum 8480C 48C 2GHz, NVIDIA H100, NVIDIA Infiniband NDR, Microsoft<br>Microsoft Azure<br>United States | 1,123,200 | 561.20 | 846.84 | |
| 4 | **Supercomputer Fugaku** - Supercomputer Fugaku, A64FX 48C 2.2GHz, Tofu interconnect D, Fujitsu<br>RIKEN Center for Computational Science<br>Japan | 7,630,848 | 442.01 | 537.21 | 29,899 |

***SUPERCOMPUTING***

***Current platforms of HPC (High Performance Computing),***
***(unit of arithmetic speed = 1 TFLOPs = $10^{12}$ ops/s)***

1. **CPU** (central processor unit, x86, soon: ARM?)
   ~1 TFLOP/s dp (double precision)

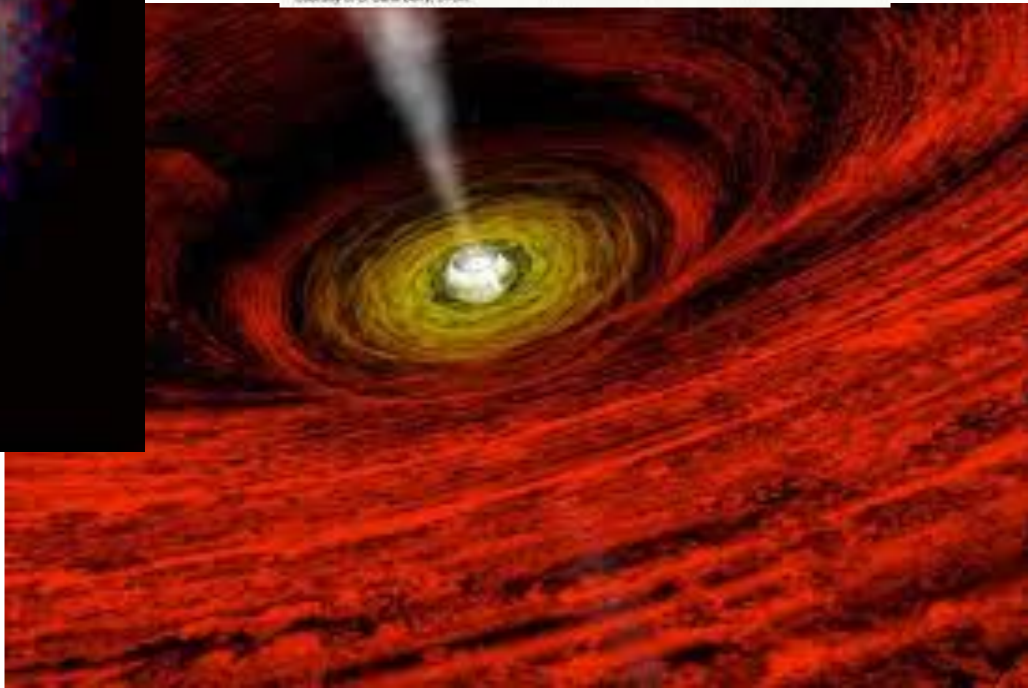
- clock freq. ~3 GHz no longer grows (physical limitations)
- up to 70 cores – can run 140 independent programs simultaneously
- bandwidth of order 250 GB/s to DDR5 operating memory
- bandwidth 20 GB/s to peripheral devices like GPU
- bandwidth to other nodes on supercomputer (cluster) using Infiniband or 10G Ethernet interconnect at ~10 GB/s

2. **GPU** (graphics processing unit) ~$10^1$ TFLOP in dp

- typically Nvidia graphics cards, ~40 multiprocessors
- has to run (to avoid latency) > $10^4$ threads: massively parallel computation
- bandwidth to DDR5 up to 500 GB/s

All platforms typically use 250 W per processor while computing.

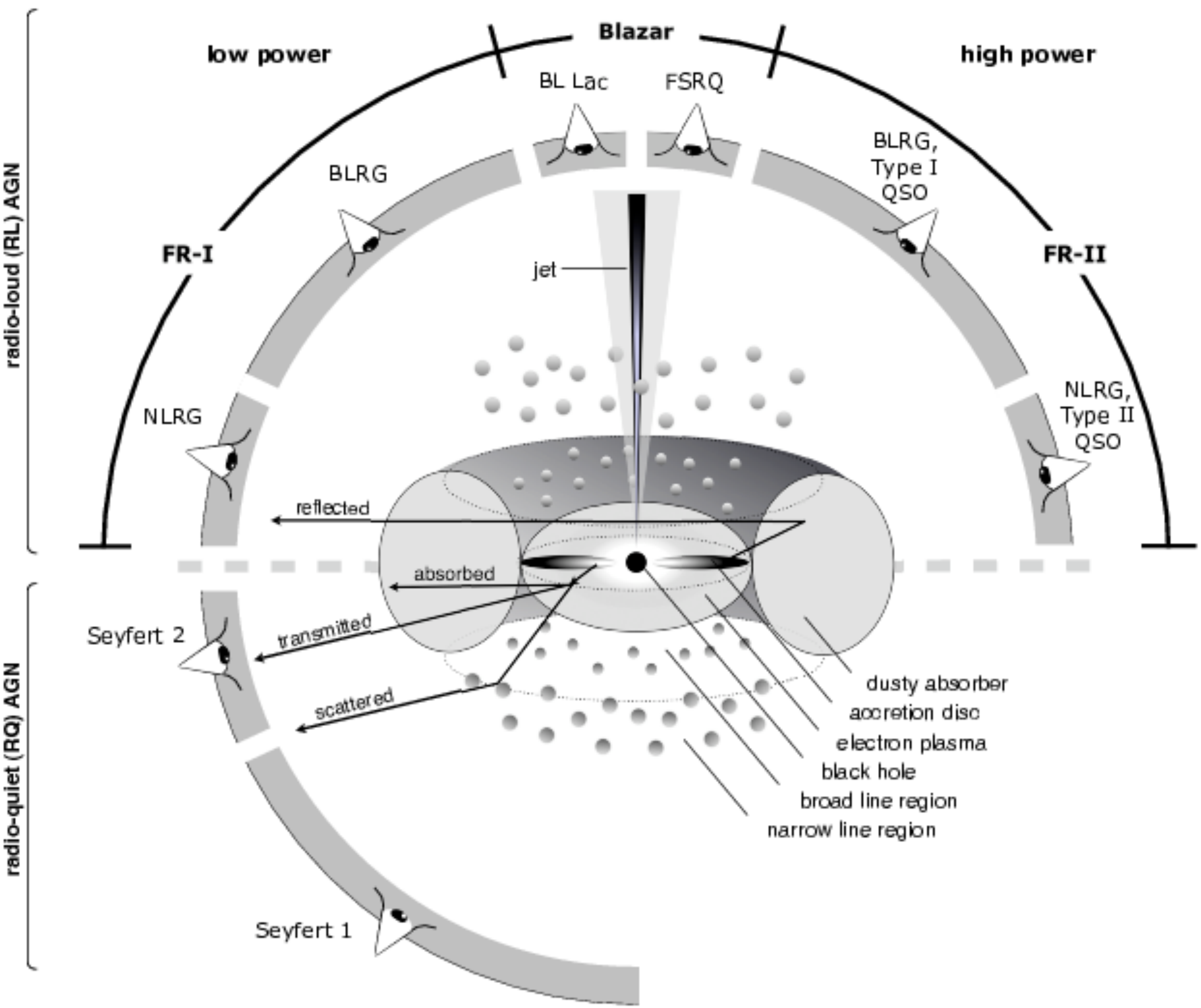
# Superluminal motions in **Active Galactic Nuclei**

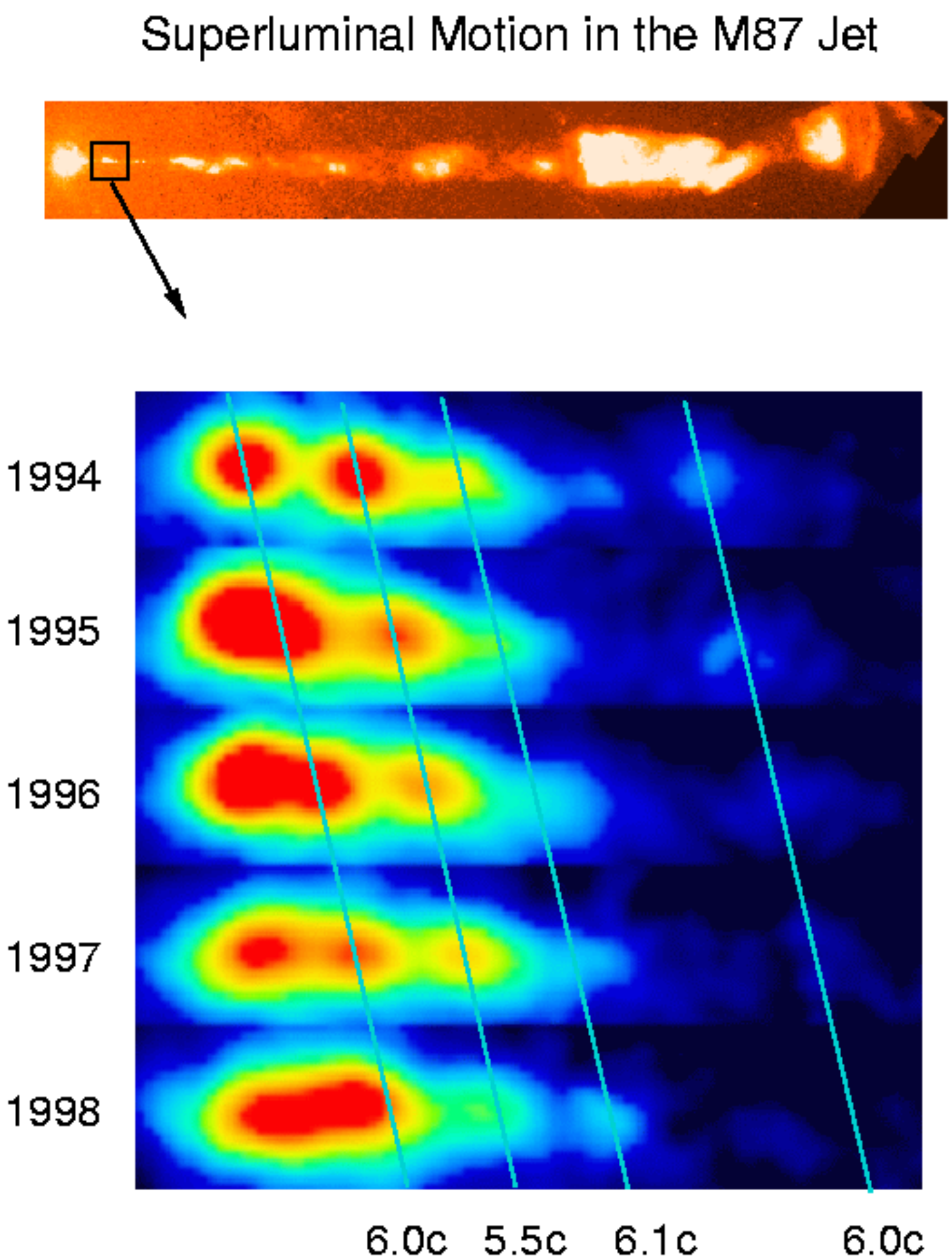


AGN

schematic

# The unified scheme of AGN/Quasar






Can any object move faster than light? NO. Not according to physics.

This trick of nature is connected with special relativity theory and the relativistic gamma factor, but it really just  follows from the geometry of the problem involving a small angle (orientation of the jet to the line-of-sight) and a finite speed of light.
*Read about it on p. 330 of the textbook!*

Relativistic effects, as already mentioned,  boost  the jet luminosity by a large factor, while dimming the image of the receding jet to a point where we can’t see it at all!

An explanation of apparent **superluminal motions** in BL Lac objects and quasars' jets



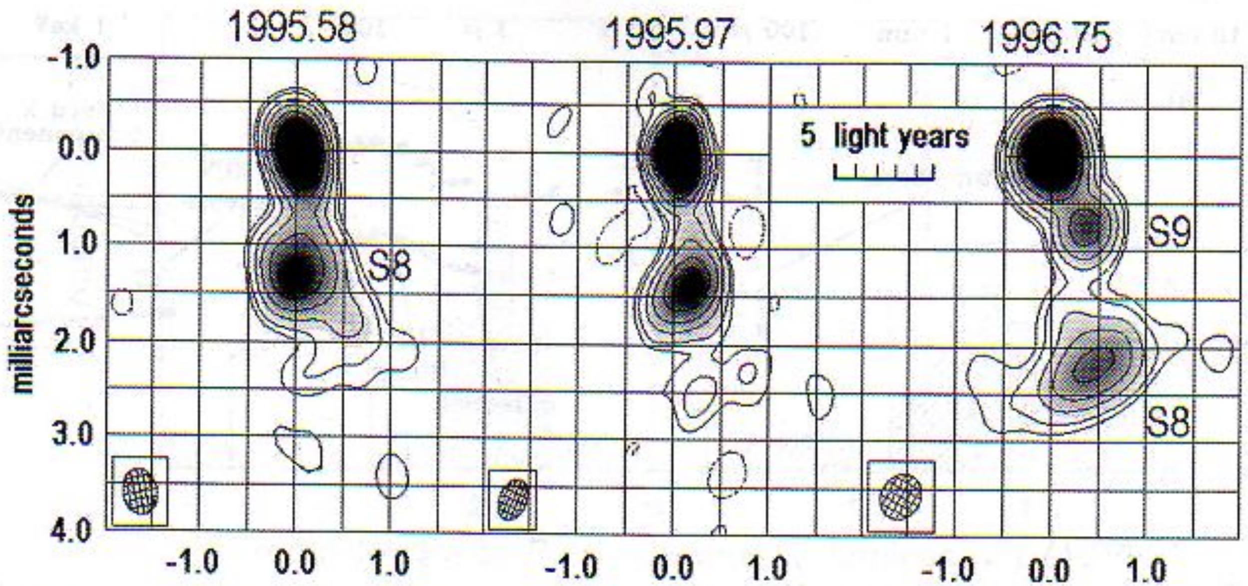



**Figure 8.10** Radio maps at 22 GHz of the active galaxy BL Lac; the scale bar is 5 light-years long, assuming $H_0 = 67\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Blob S8 moves in a corkscrew path away from the core at apparent speed $\sim 3c$. The hatched ellipse shows the telescope beam; a pointlike source would appear with roughly this size and shape – G. Denn.

Apparent speed of blob S8 is 3 times the speed of light in vacuum (!)
Was Einstein wrong about physics? Do we see tachions? No.

# The explanation is relatively simple: time delays plus trigonometry

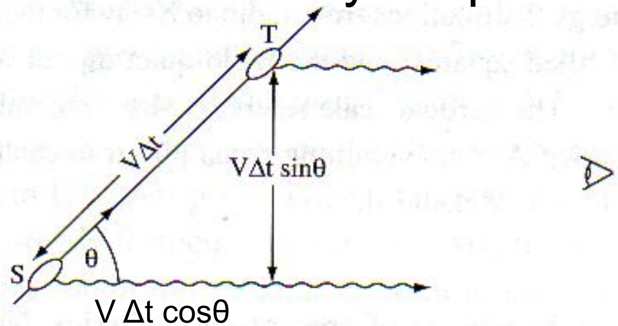



**Figure 8.11** Luminous blobs ejected at angle $\theta$ to the line of sight can appear to move superluminally across the sky if their speed $V \approx c$.

shows the jet of the blazar BL Lac. In about half the well-studied cores, motion is *superluminal*: the blobs appear to move away from the core with transverse speeds of 3–50$c$. These high apparent speeds arise because the emitting gas is moving toward us at speeds close to that of light.

superluminal = faster than light

To understand these apparently superluminal speeds, consider an observer who sees a blob of jet material approaching at speed $V$, on a course making an angle $\theta$ with the line of sight (Figure 8.11). The blob passes point S at time $t = 0$, and point T at a time $\Delta t$ later. Radiation emitted at T reaches our observer later than radiation from S; but because T is closer, the interval between the two arrivals is only

$$\Delta t_{\text{obs}} = \Delta t(1 - V\cos\theta/c). \qquad (8.15)$$

In this time, the blob has travelled a distance $V\Delta t \sin\theta$ across the sky, so its apparent transverse speed is

$$V_{\text{obs}} = \frac{V\sin\theta}{1 - V\cos\theta/c}. \qquad (8.16)$$

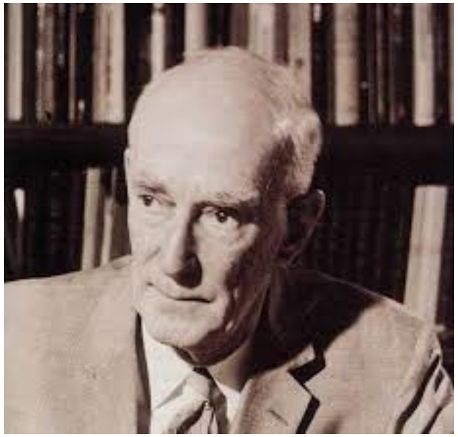
As $V \to c$, the blob's motion can appear faster than light.

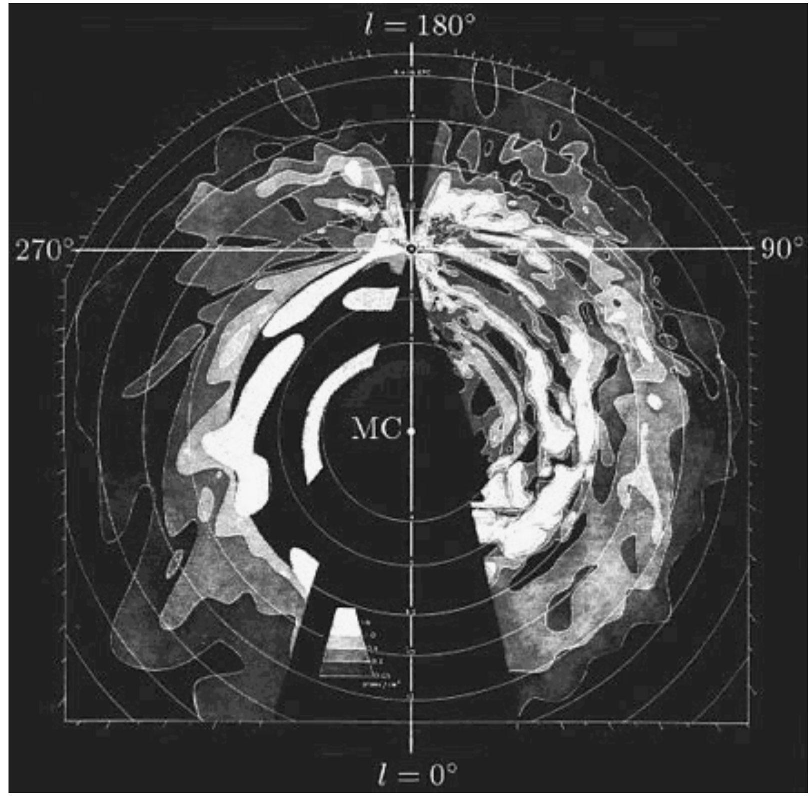
Demonstrate that:

$$V_{obs}^{\max} = \frac{V}{\sqrt{1-(V/c)^2}}$$

more precisely, when $V > \dfrac{c}{\sqrt{2}}$

# DARK MATTER






Galaxy

In 1932, Dutch astronomer Jan Oort measured streaming of stars & found that the orbital velocities in the Milky Way do not decrease with increasing distance from the Galactic center as expected. This was the first evidence for Dark Matter, however, he did not speculate at the time that the cause is some new kind of invisible matter.
Instead, he felt that the apparent anomaly would be cleared up by better observations of normal matter in the Milky Way.

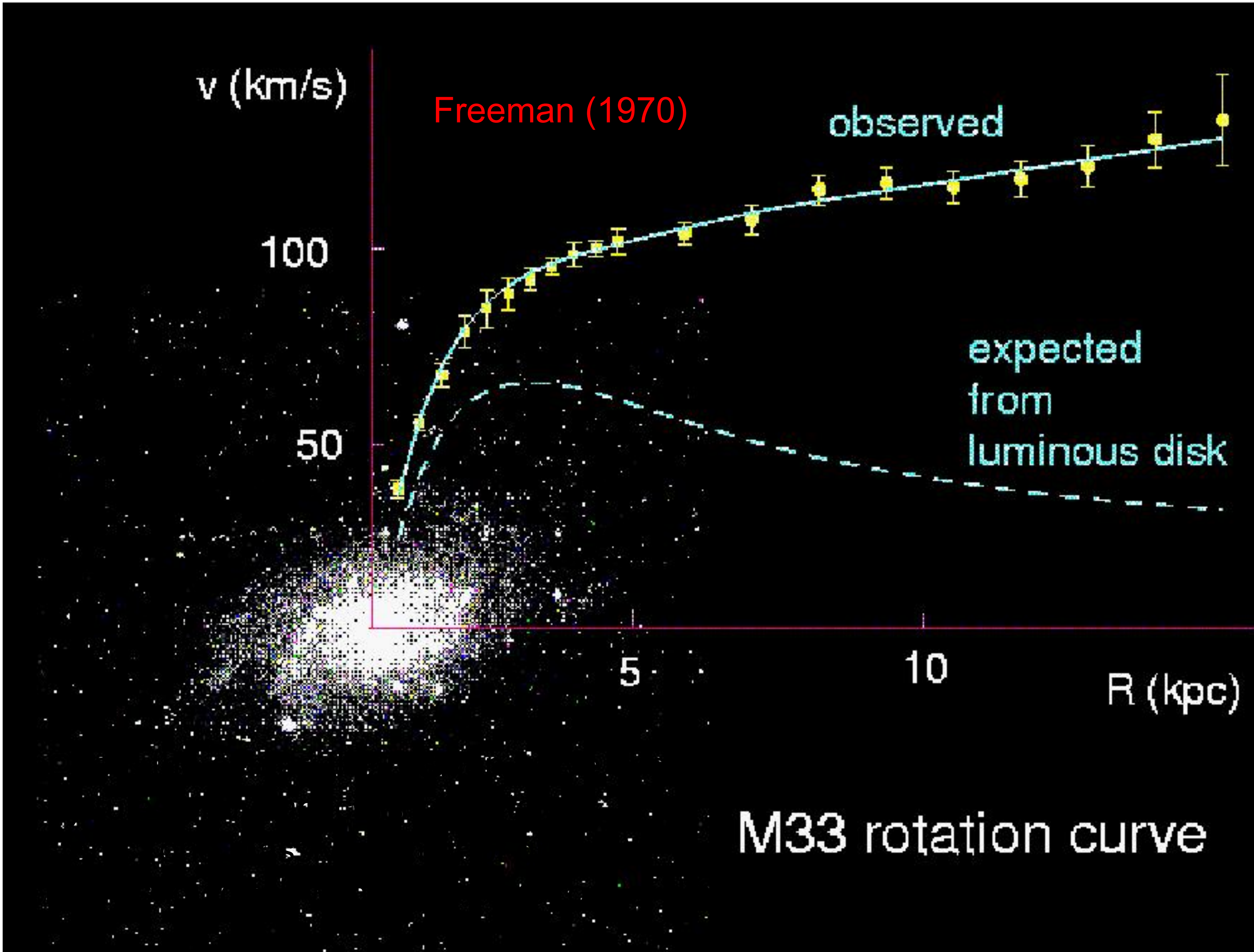
In 1933, Bulgarian-American observer Fritz Zwicky suggested that 'dark matter' might exist, based on the velocities of galaxies in galaxy clusters.

Zwicky initially called the conundrum "missing mass" (though a better name would be "missing light"). Later he proposed 'dark matter". (He also proposed the term 'supernova').

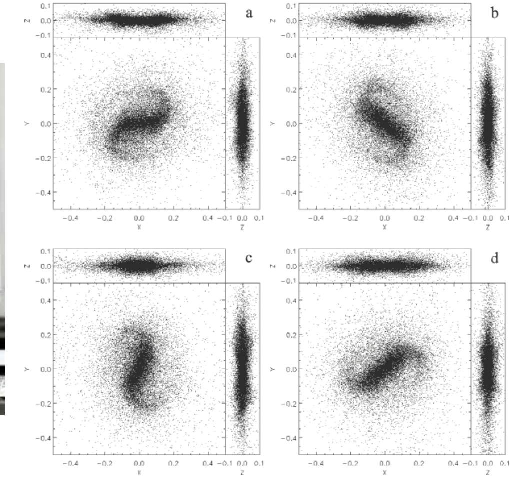
The modern interpretation of rotation curves of spiral galaxies as evidence of Dark Matter halos was probably first proposed by Kenneth Freeman (1970) – he noticed that the expected decline of rotational speed V(R) with radius R in galaxy M33 does not happen. Instead, the rotational speed keeps increasing out to the outermost detectable hydrogen gas clouds

# Dark matter

M33


v (km/s)
Freeman (1970)
observed
100
expected
from
luminous disk
50
5
10
R (kpc)
M33 rotation curve

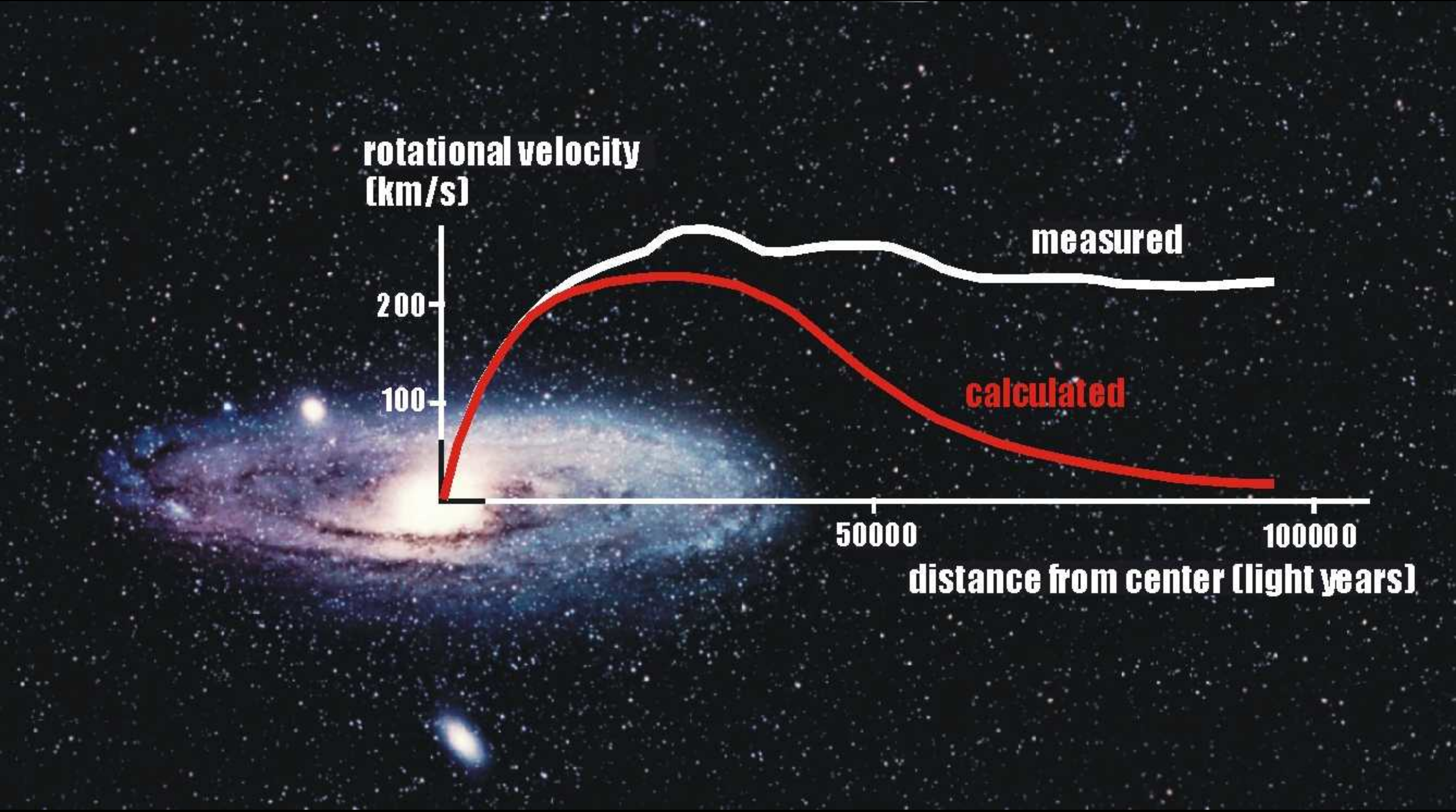
# DARK MATTER

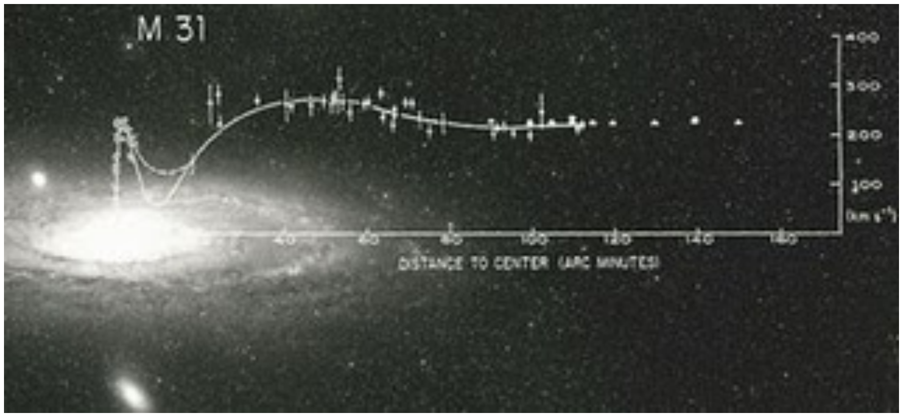


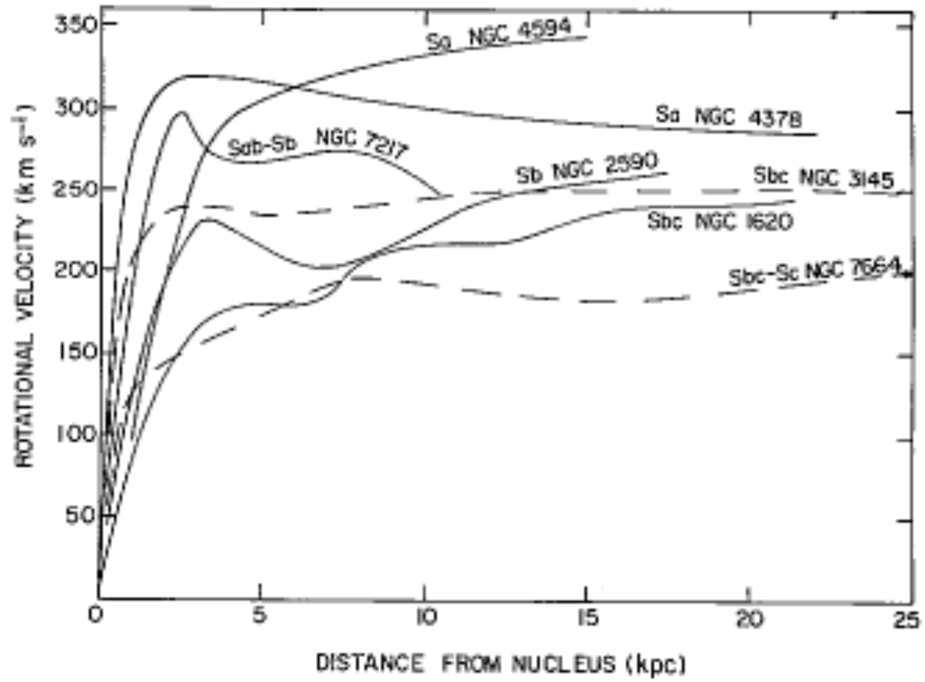

- In 1973 James Peebles and Jeremiah Ostriker provided a theoretical argument for the presence of dark matter in galaxies: if dark halos exist, they would explain dynamical stability of disk galaxies against bar-forming instabilities.

- Vera Rubin's work in the 70s was the first clear, observational evidence for dark matter as an important and *general* feature in galaxies.
She popularized the subject.

Evidence of dark matter –
not made of the usual matter but of weakly-interacting elementary particles (called WIMPs)



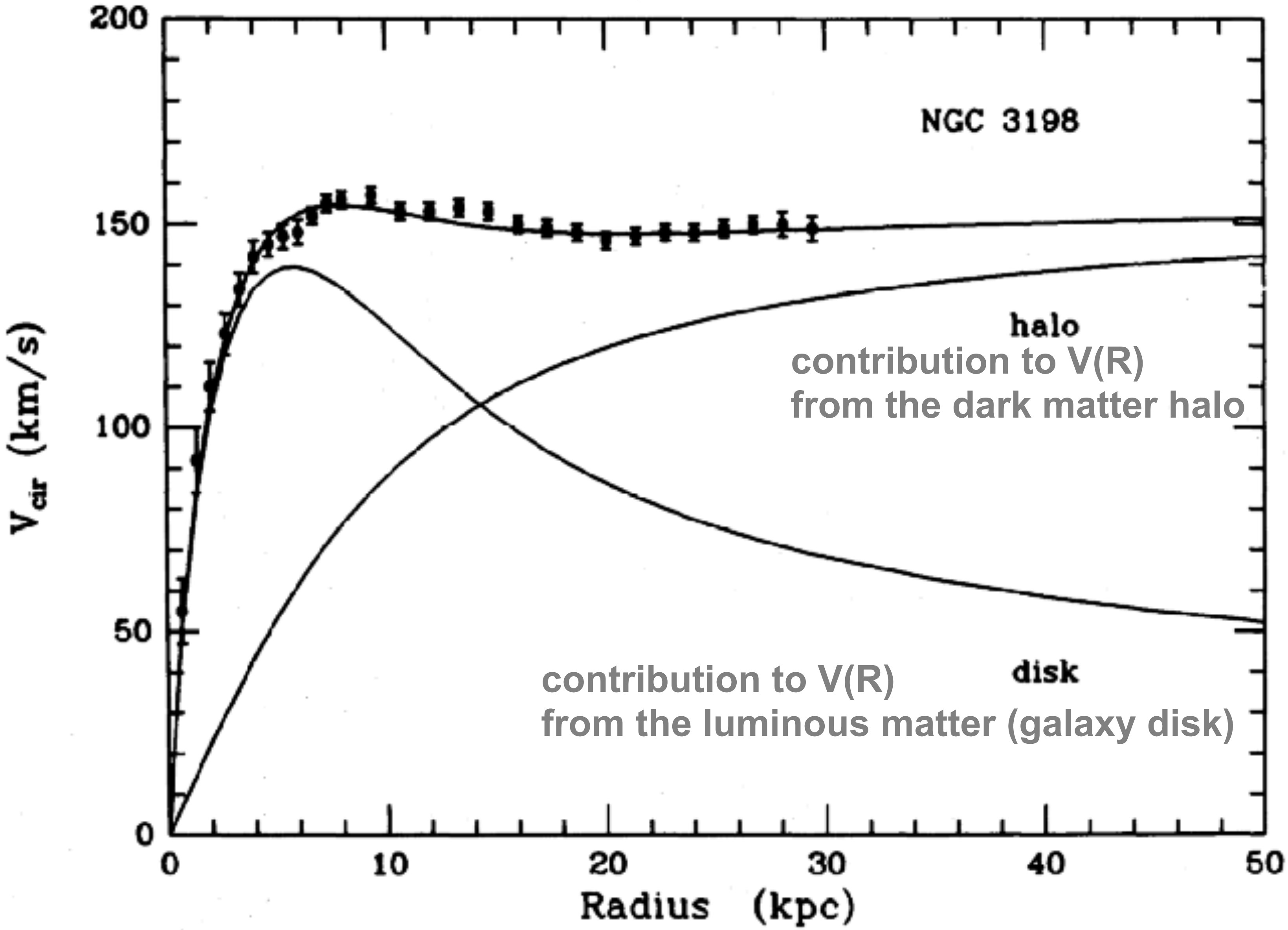



Detailed observations were first made by V. Rubin and W.K. Ford of the Andromeda galaxy and published in *The Astrophysical Journal* (1970).

They then made observations of ***over 60 other*** spiral galaxies, which confirmed that the presence of dark matter is a general phenomenon.

DISTRIBUTION OF DARK MATTER IN NGC 3198



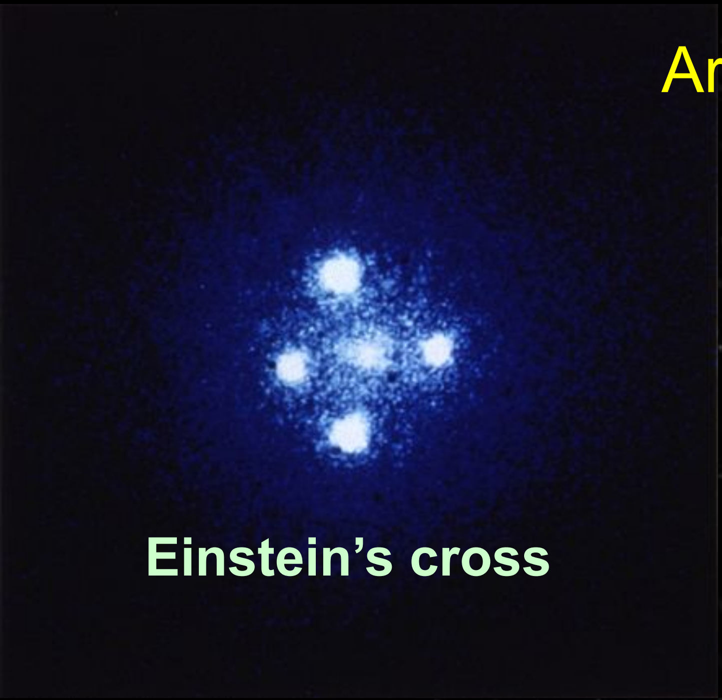
But rotation curves is not the only proof of dark matter.
FURTHER EVIDENCE OF DARK MATTER includes:

Rapid motion of galaxies in their pairs and clusters

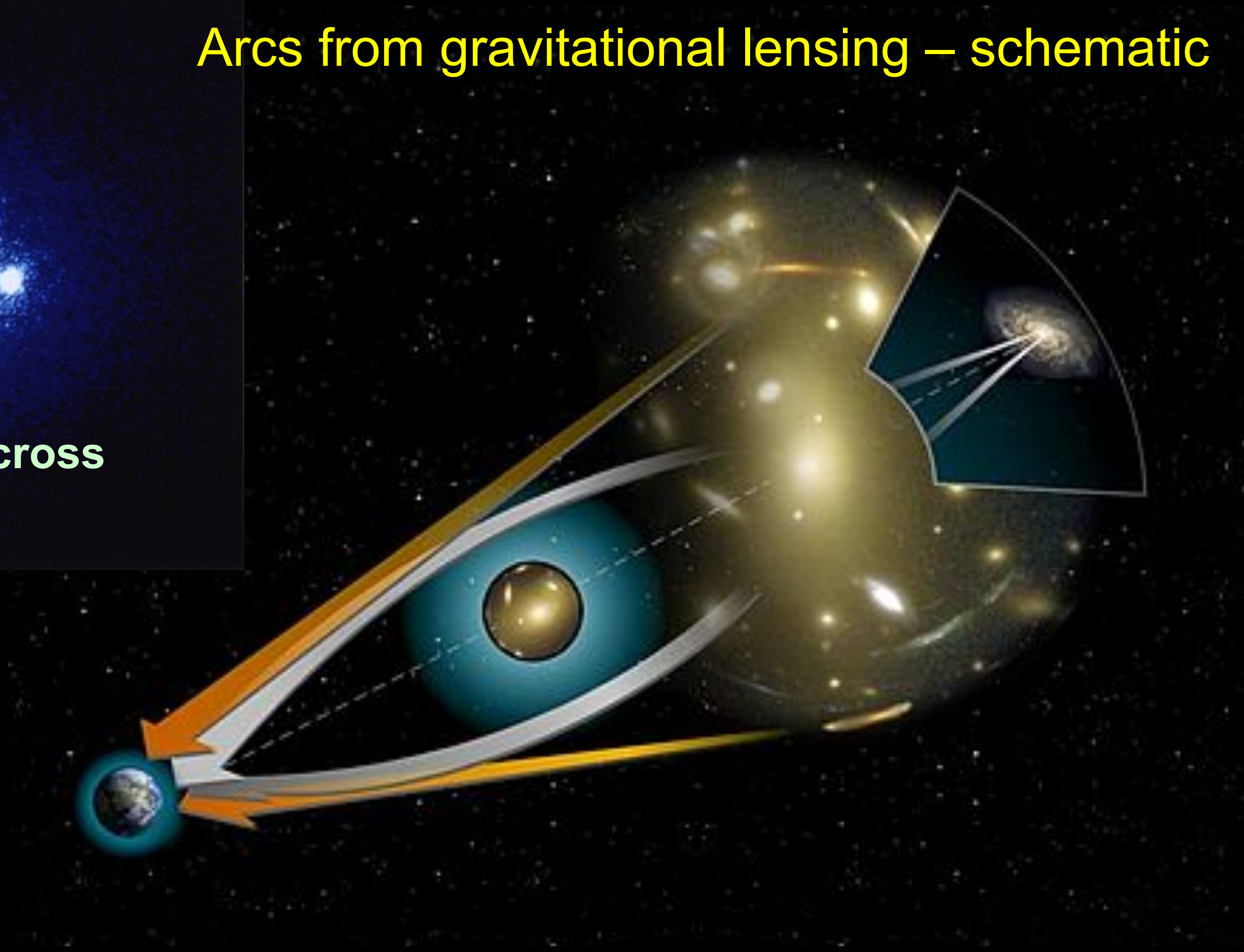
Speeds in pairs and clusters produced by luminous matter are insufficient to explain the observed speed V

$$V^2 \sim G M / r$$

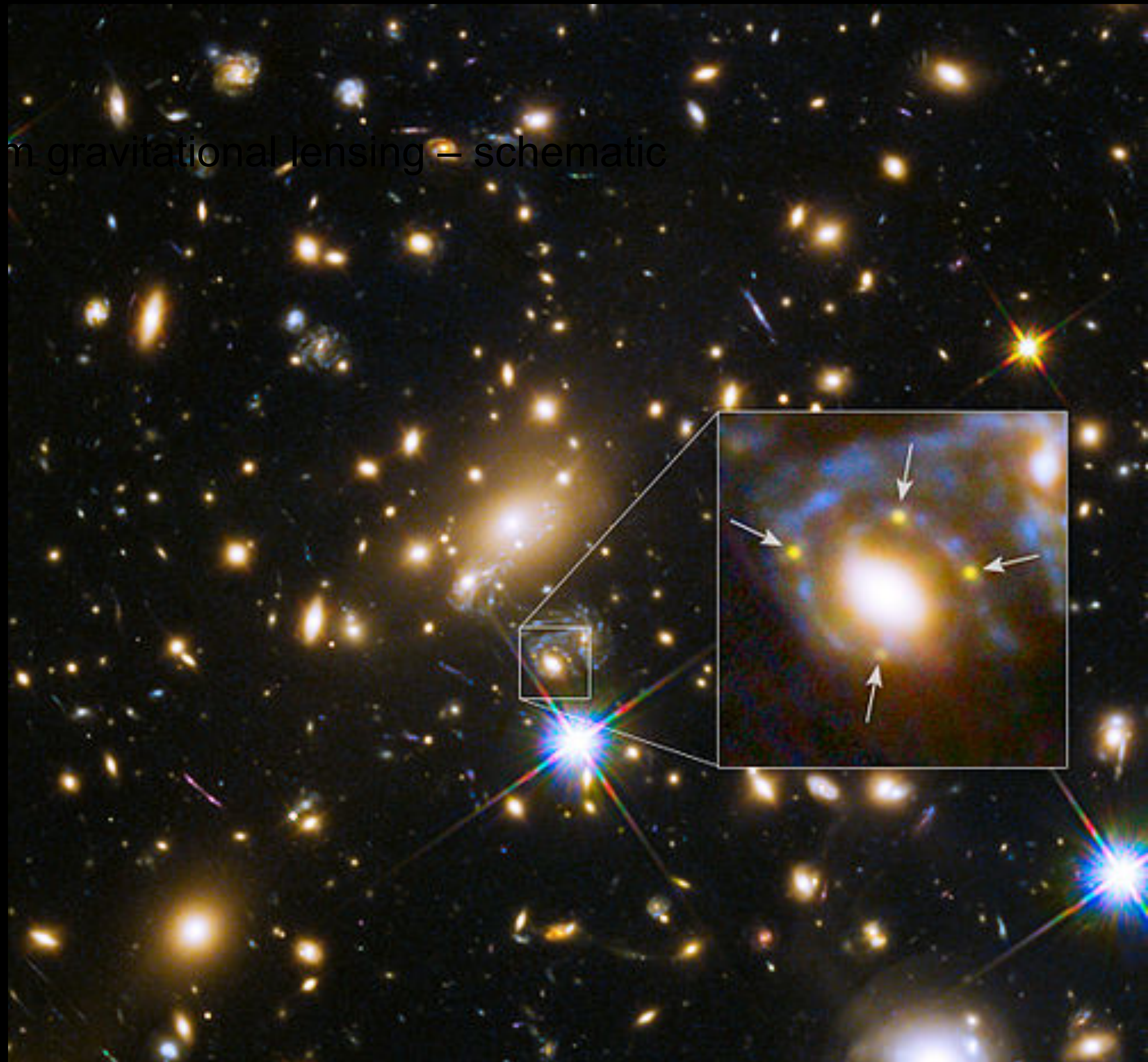
Arcs and crosses from gravitational lensing




# Arcs from gravitational lensing – schematic



m gravitational lensing – schematic

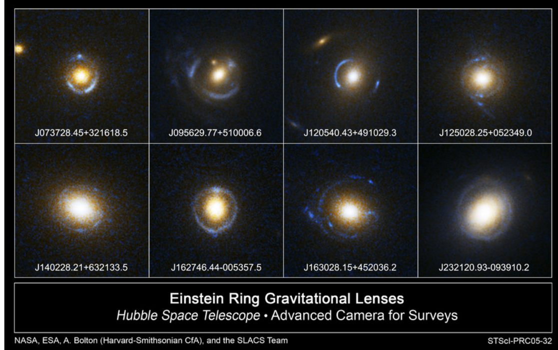


**The host galaxy of the lensed supernova Refsdal is at a distance 14.4 Gly**

# EVIDENCE OF DARK MATTER – LENSING EFFECT, EINSTEIN RINGS

LRG 3-757, a gravitational mirage object was discovered in Sloan Digital Sky Survey in 2007, and pictured by HST.
The blue galaxy has redshift z=2.4 defined as $z = \frac{\lambda - \lambda_0}{\lambda_0}$
it emitted light 3 Gyr after Big Bang,
11 Gyr ago.

HST:




Cosmic Horseshoe is one of the most beautiful Einstein rings

galaxy cluster
SMACS 0723

Deep Field 1
(2022)

JWST robotic telescope
(J. Webb Space Telescope, NIR+MIR successor to HST which observed optical+NIR light)

NIR = near infrared
MIR = mid-infrared

# ASTB23. Lecture L24.
# Universe is mostly dark energy

---

Galaxy clusters and galaxy evolution

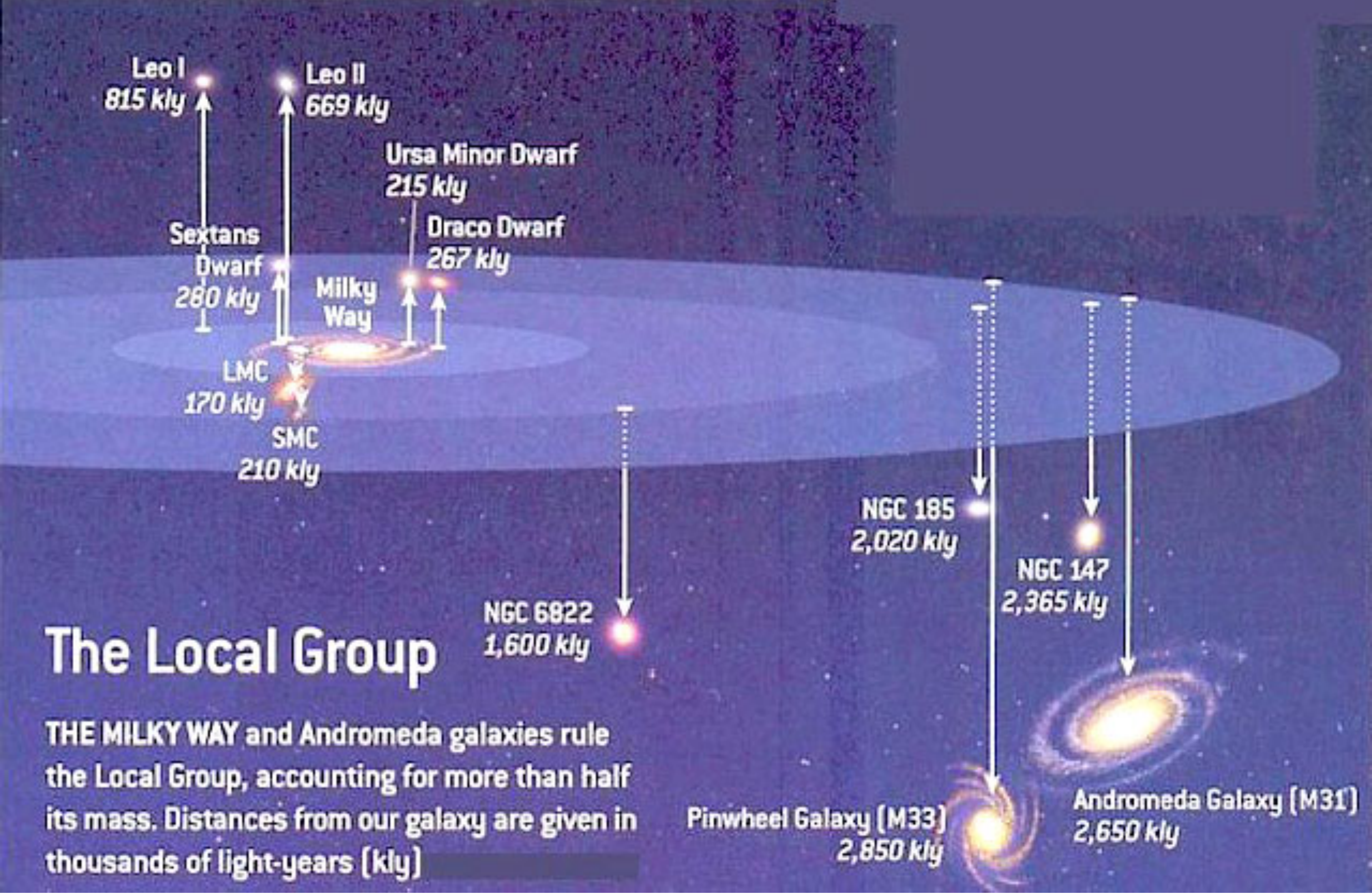
Distribution of galaxies in space. Google map of the universe?

CMBR = Cosmic Microwave Background Radiation
observed by COBE, BOOMERANG and WMAP, and other satellite and sub-orbital observatories

Boomerang, WMAP and the flatness of the space-time (Omega=1, k=0)

Expansion accelerates: Lambda model (a.k.a. Lambda-CDM model)

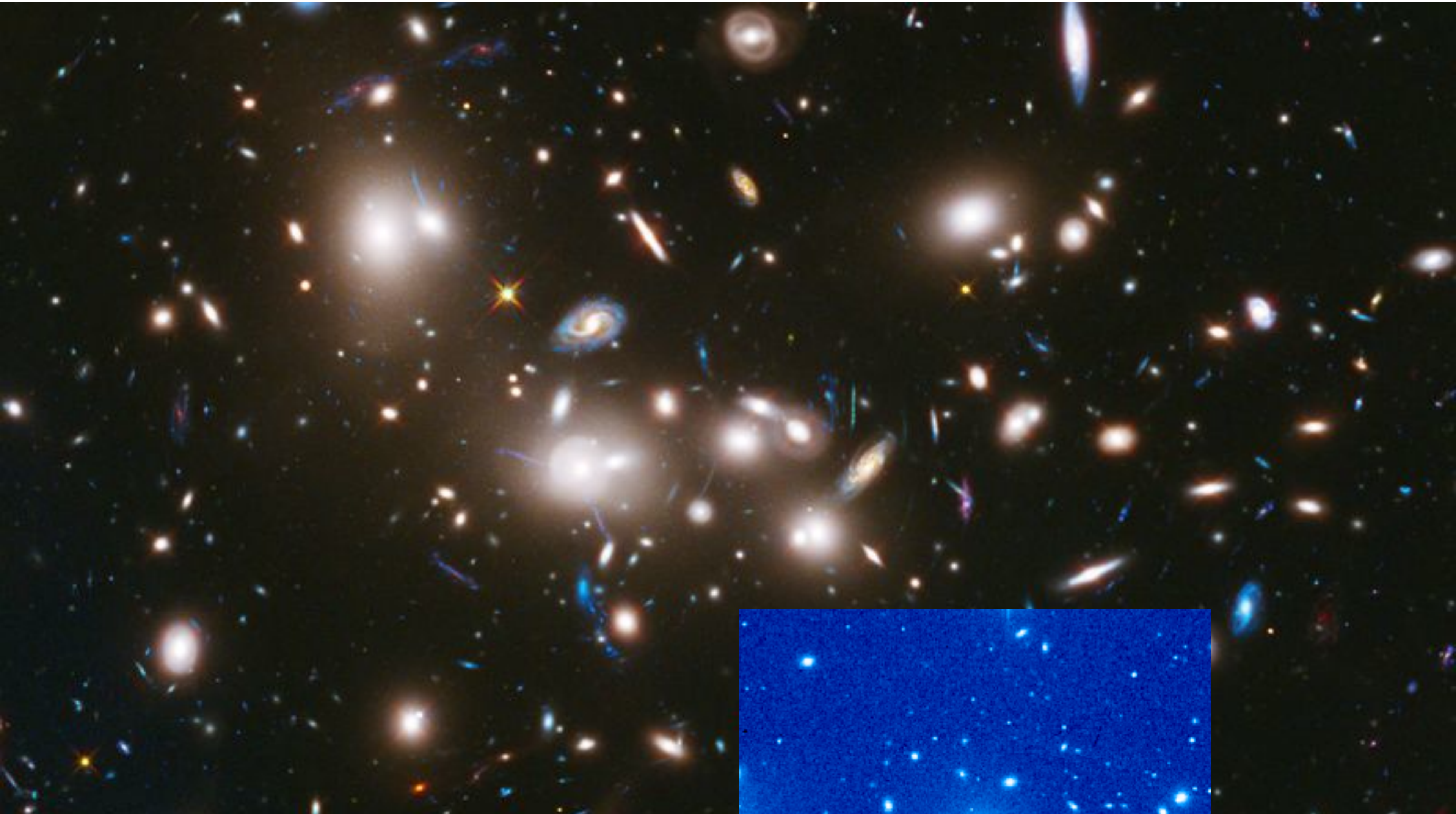
What is dark energy?

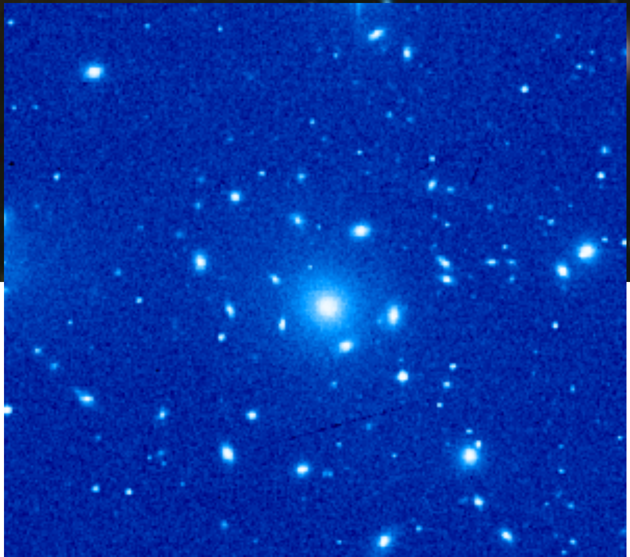

Leo I
815 kly
Leo II
669 kly
Ursa Minor Dwarf
215 kly
Sextans Dwarf
280 kly
Draco Dwarf
267 kly
Milky Way
LMC
170 kly
SMC
210 kly
NGC 185
2,020 kly
NGC 147
2,365 kly
NGC 6822
1,600 kly
The Local Group
THE MILKY WAY and Andromeda galaxies rule the Local Group, accounting for more than half its mass. Distances from our galaxy are given in thousands of light-years (kly)
Pinwheel Galaxy (M33)
2,850 kly
Andromeda Galaxy (M31)
2,650 kly

The nearest (~20 Mpc) cluster of 1500 galaxies in Virgo constellation spans 8 degrees on the sky

Perseus cluster. $10^3$ galaxies observed by ESA's Euclid space telescope (2023): Not a large telescope but dedicated to survey covering 1/3 sky.
Other observatories observe individual objects most of the time. Euclid will make inventory of galaxies out to 10 Gly (most of the universe).

Galaxies cluster on different scales, from Local Cluster (30 glx)





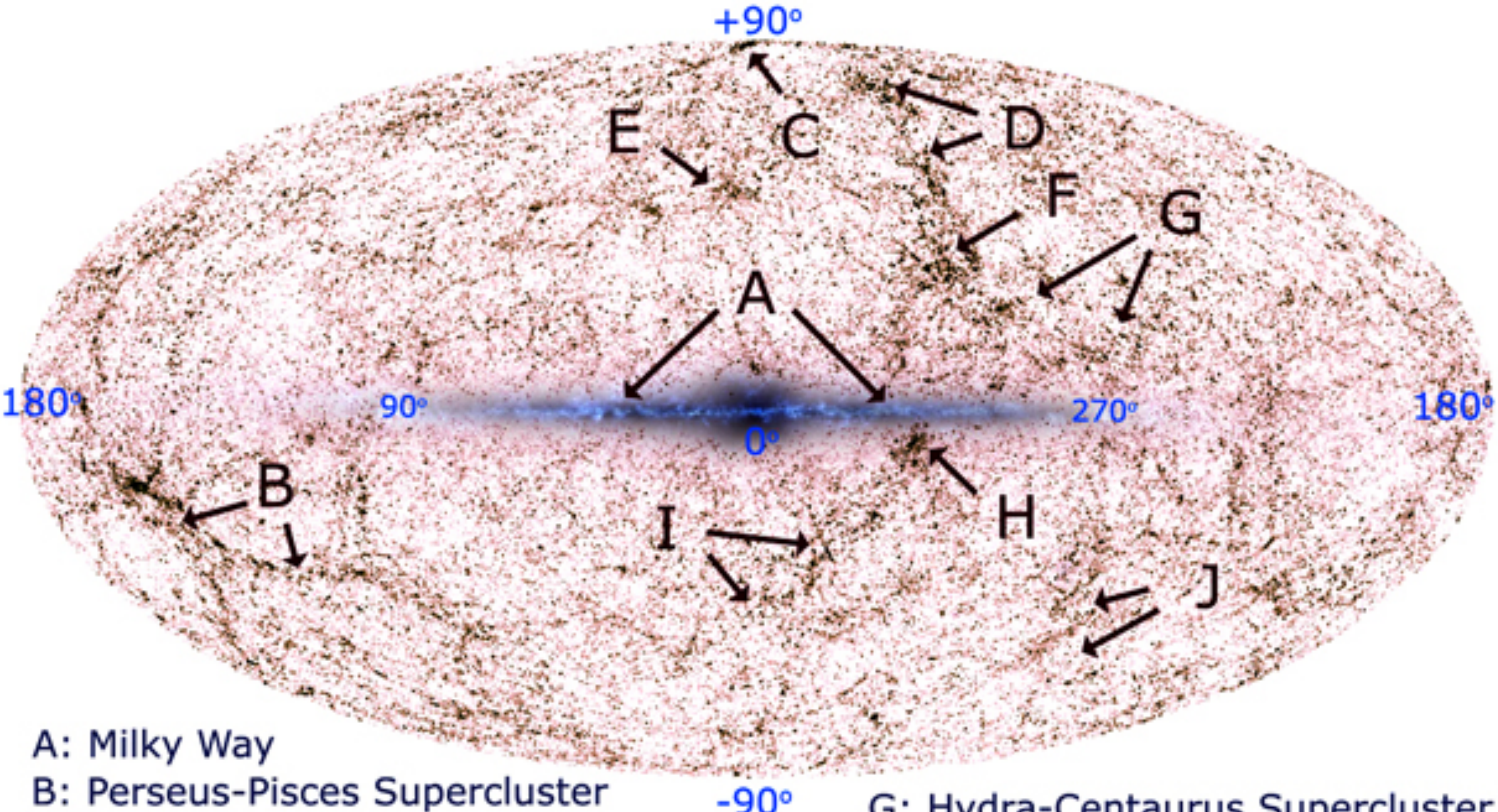
to this cluser in Coma, 100 Mpc away (~1000 galaxies, elipticals in the center)

90% of mass is dark matter


see this movie announcing the future JWST

https://www.youtube.com/watch?v=gu_VhzhlqGw

# Some of the large-scale clusters ans superclusters of galaxies



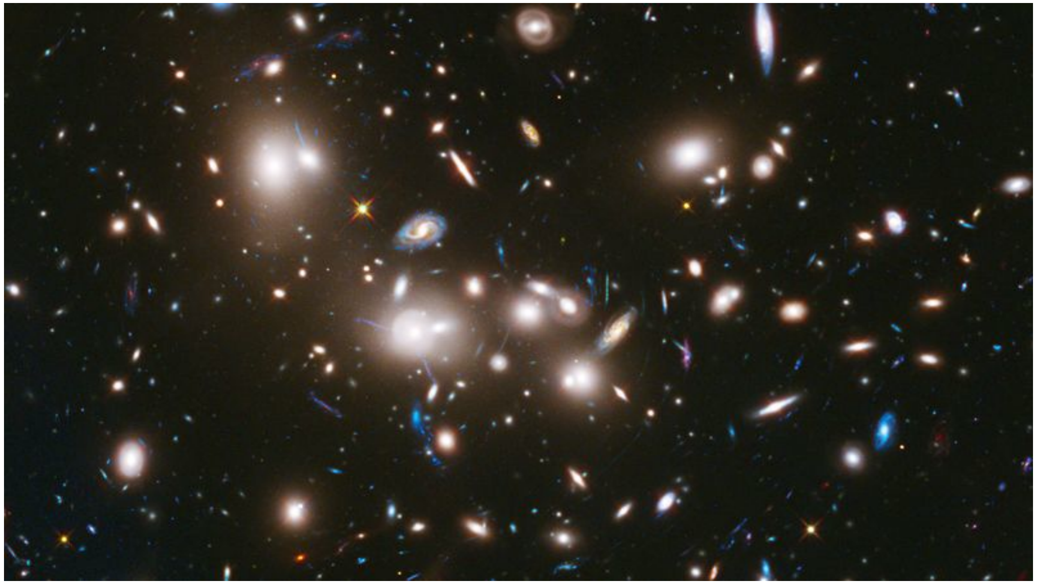

A: Milky Way
B: Perseus-Pisces Supercluster
C: Coma Cluster
D: Virgo Cluster/Local Supercluster
E: Hercules Supercluster
F: Shapley Concentration/Abell 3558
G: Hydra-Centaurus Supercluster
H: "Great Attractor"/Abell 3627
I: Pavo-Indus Supercluster
J: Horologium-Reticulum Supercluster

83% of all the stars formed since the Big Bang were initially located in a disc-shaped galaxy. However, only 49% of stars that exist in the Universe today are located in these disc-shaped galaxies--the remainder are located in oval-shaped galaxies.

The Cardiff survey results suggest a massive transformation in which disc-shaped galaxies became oval-shaped galaxies.

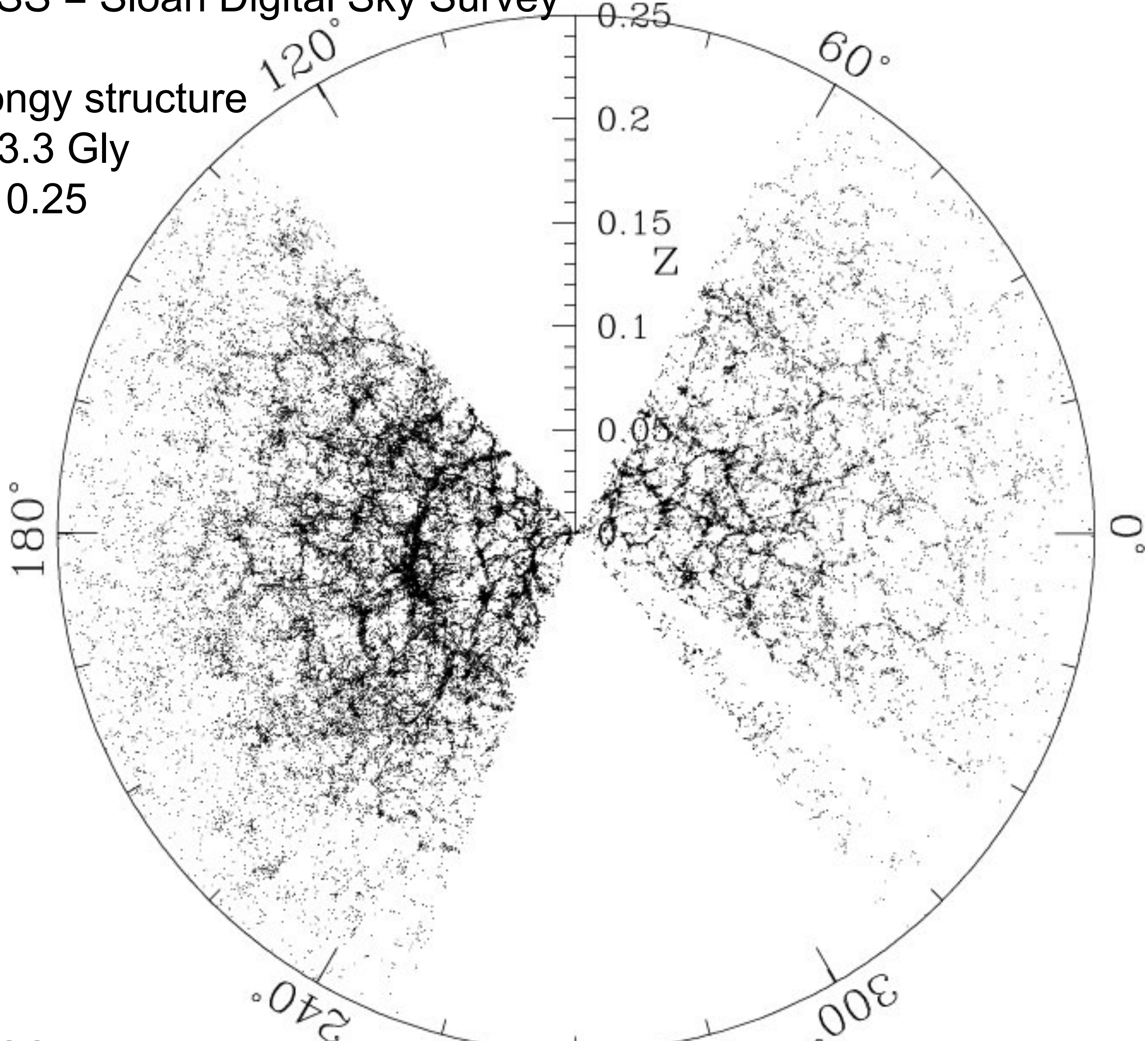
A popular theory is that this transformation was caused by many cosmic catastrophes, in which two disk-dominated galaxies, straying too close to each other, were forced by gravity to merge into a single galaxy, with the merger destroying the disks and producing a huge pileup of stars. (An opposing theory was that the transformation was a more gentle process, with stars formed in a disk gradually moving to the centre of a disk and producing a central pile-up of stars.)

Cardiff U. galaxy survey



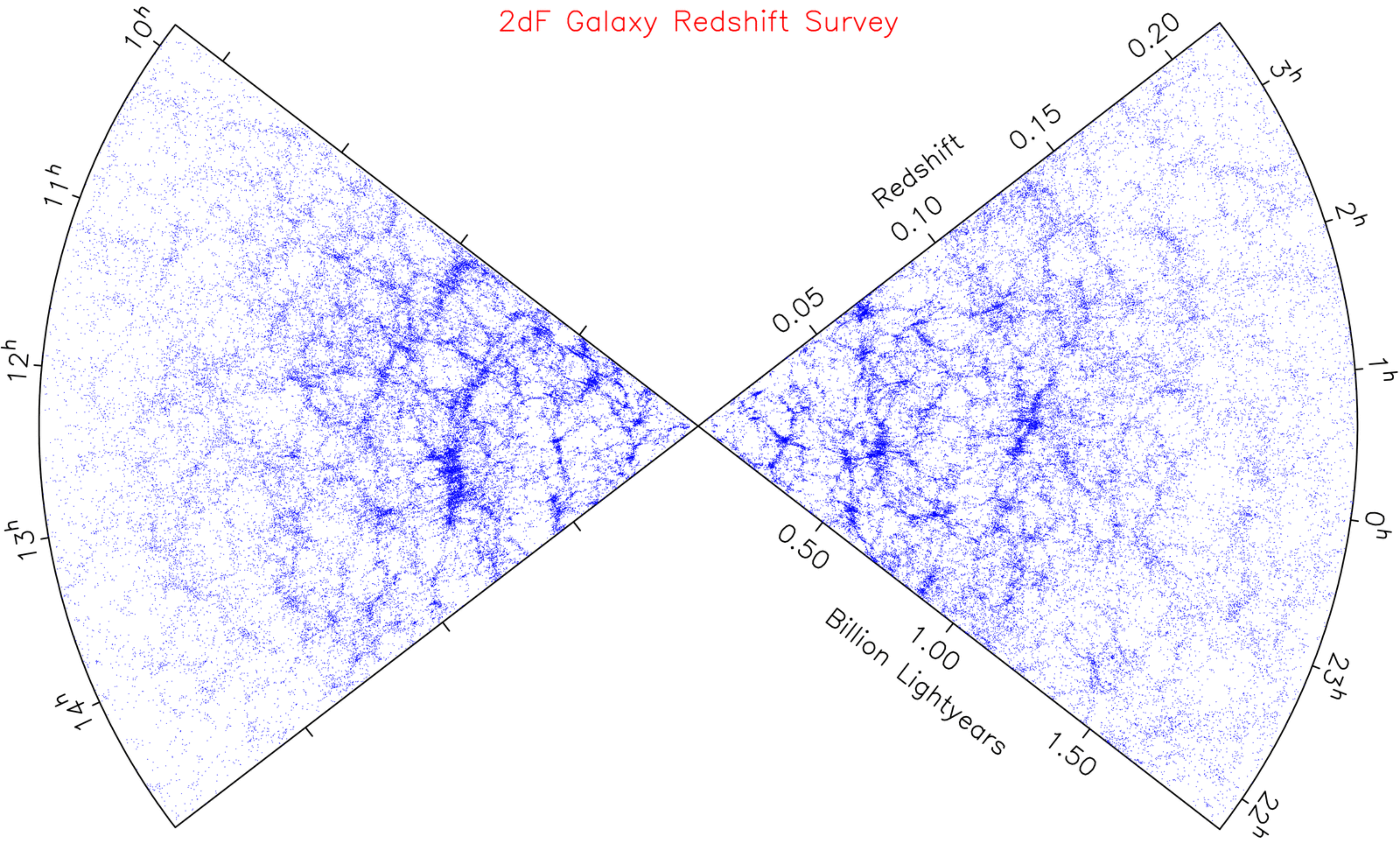
SDSS = Sloan Digital Sky Survey

spongy structure
d <3.3 Gly
z < 0.25



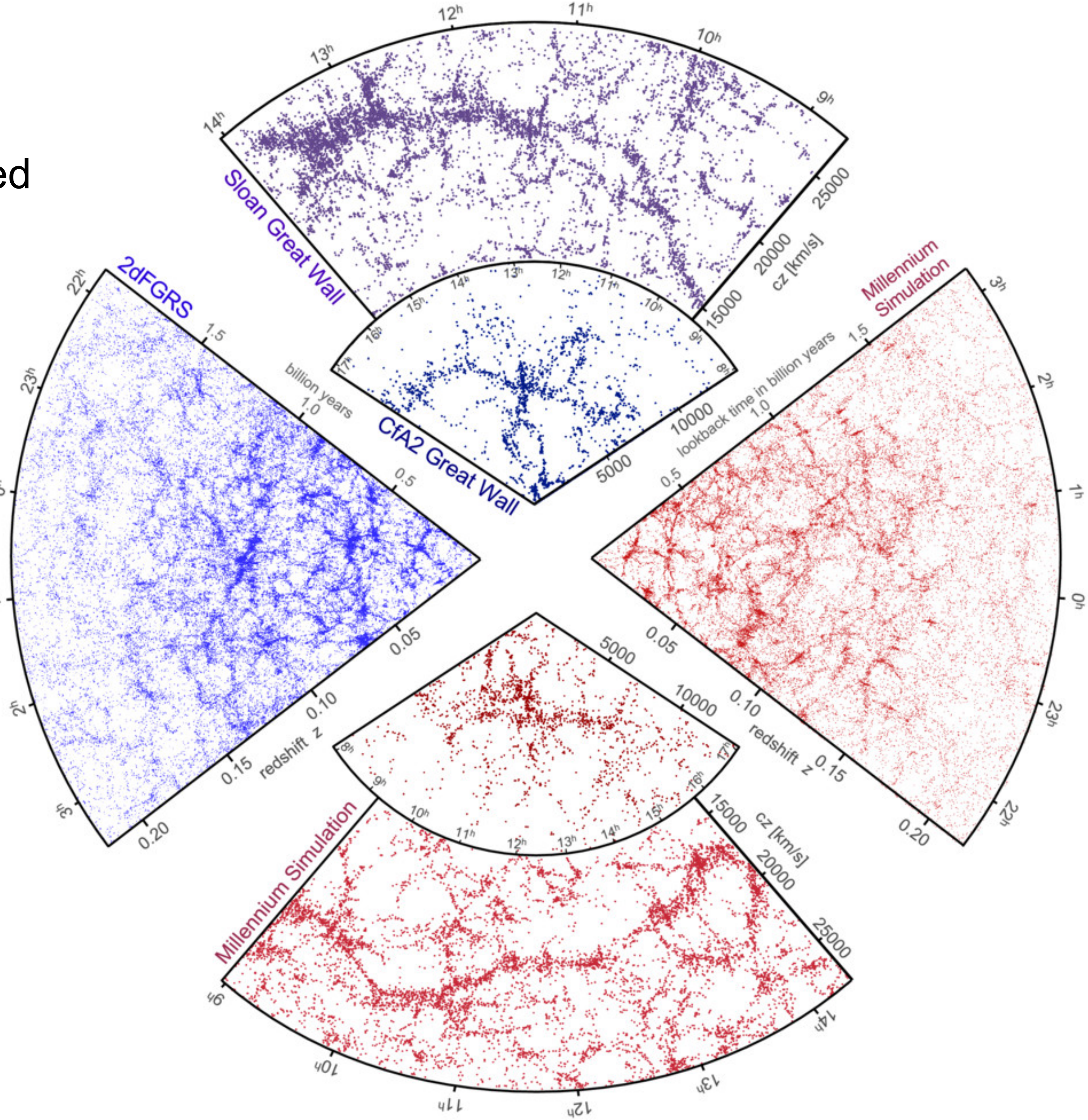

SDSS: 5-color sky images + Doppler radial speed measurement

Lage-scale structure of our Universe a look at two opposite directions in the sky

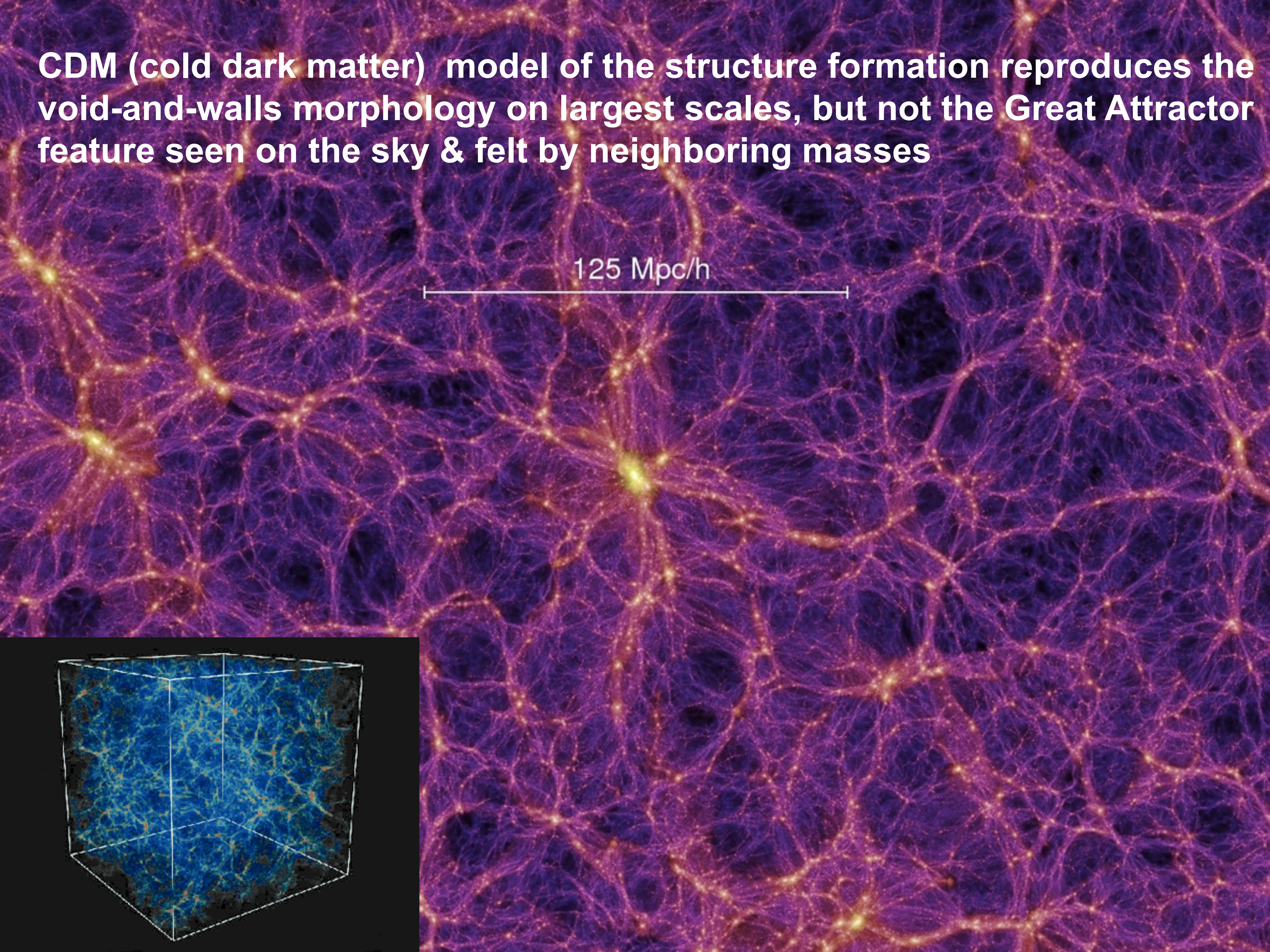


The Universe

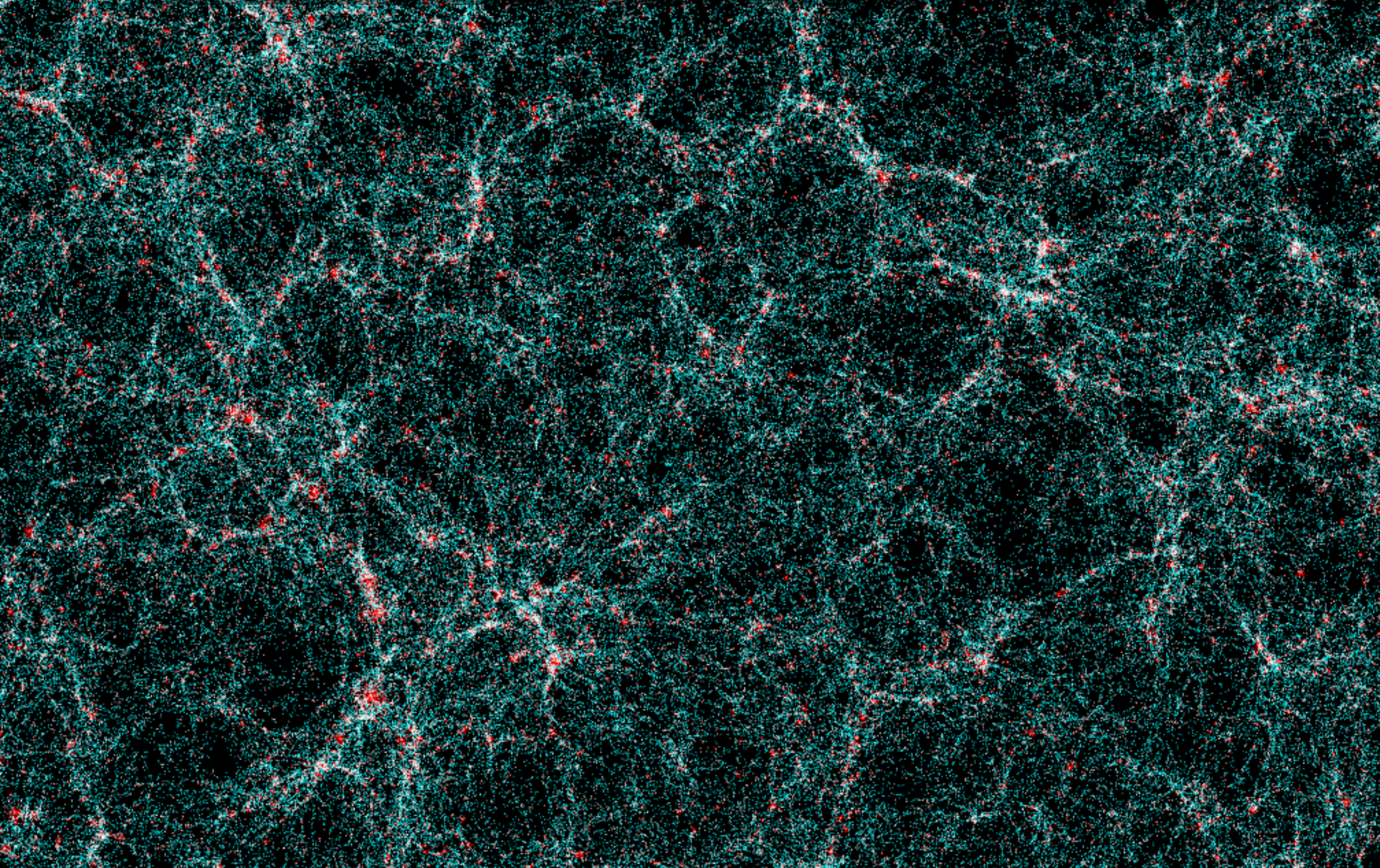
It's a bit....complicated



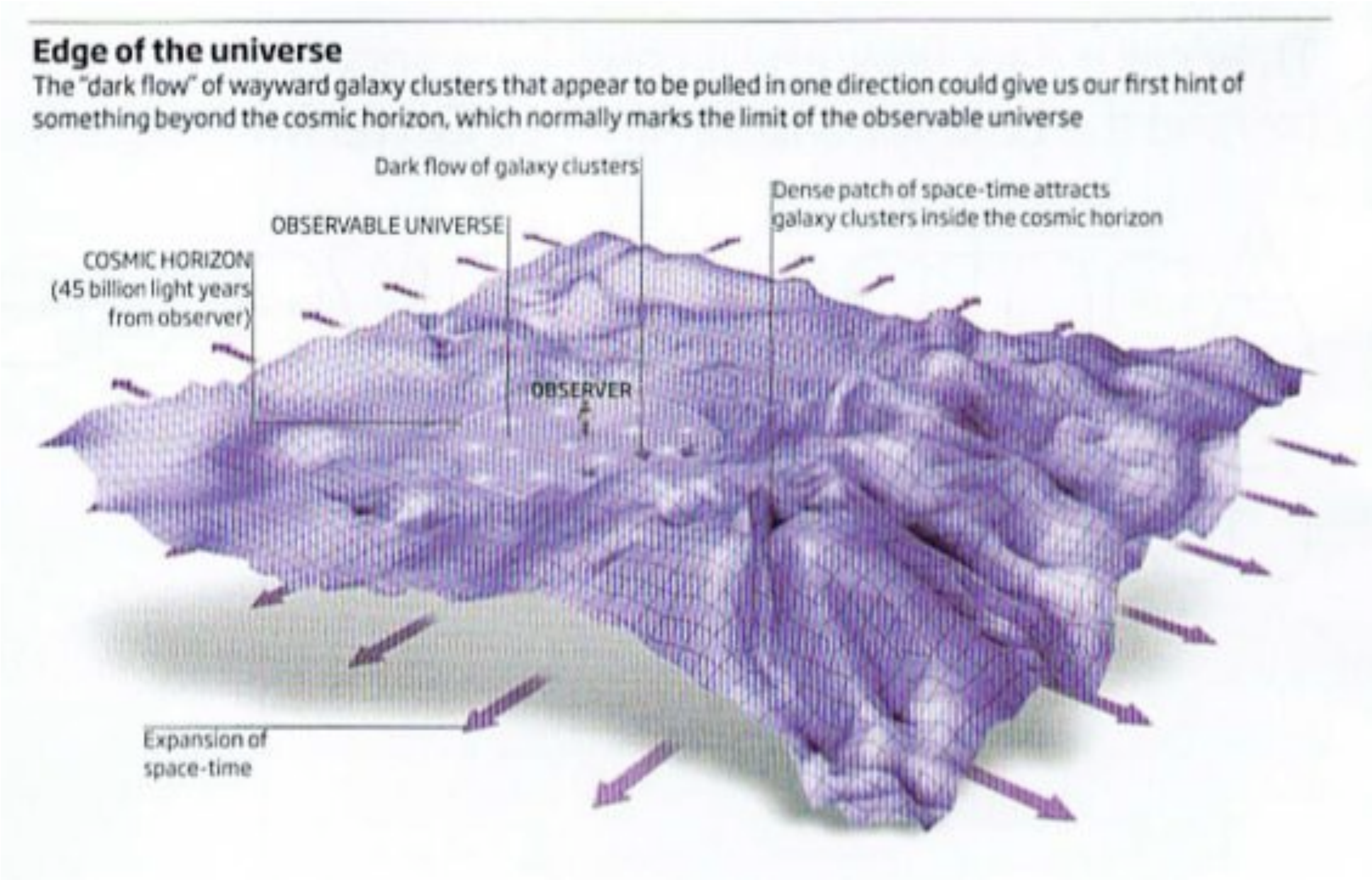
CDM (cold dark matter)  model of the structure formation reproduces the void-and-walls morphology on largest scales, but not the Great Attractor feature seen on the sky & felt by neighboring masses



Euclid Flagship Simulation (2023) of expected structure in the 3-D map of the Universe that Euclid telescope begins to create, as it orbits the sun near L2 point next to JWST & Gaia observatories



One day we may be able to study the World beyond our Universe



# Discovery of Cosmic Background Radiation – remnant of Big Bang

In 1964-1965, American radio astronomers Robert Wilson and Arno Penzias discovered the noise from the sky, which turned out to be **CMB (cosmic background radiation)**. This was a great moment of serendipitous, i.e., accidental, discovery.

The 6 m Holmdale Horn Antenna was built to communicate on Earth via Echo baloon satellites (passive reflectors).


Bell Labs' horn in New Jersey picked up an odd buzzing sound that came from all parts of the sky at all times. The noise puzzled the researchers, who did their best to deeply cool the detector and eliminate all possible sources of interference, even removing some pigeons nesting in the antenna & leaving a lot of ~~sh..~~ droppings on it, which Penzias and Wilson suspected as source of the antenna noise. But the signal was coming from cosmos.

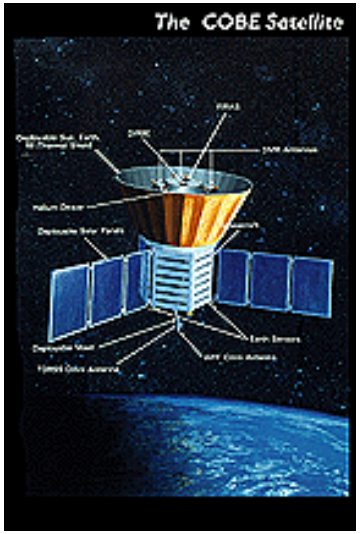
# CMB

The CMB is the oldest light in the universe, dating to the epoch 380,000 yr after Big Bang. Temperature of the universe dropped then to ~3000 K, so neutral atoms started forming from the previous hot plasma (separate electrons, protons, and alpha particles=nuclei of helium). The universe became, and remains, essentially transparent.

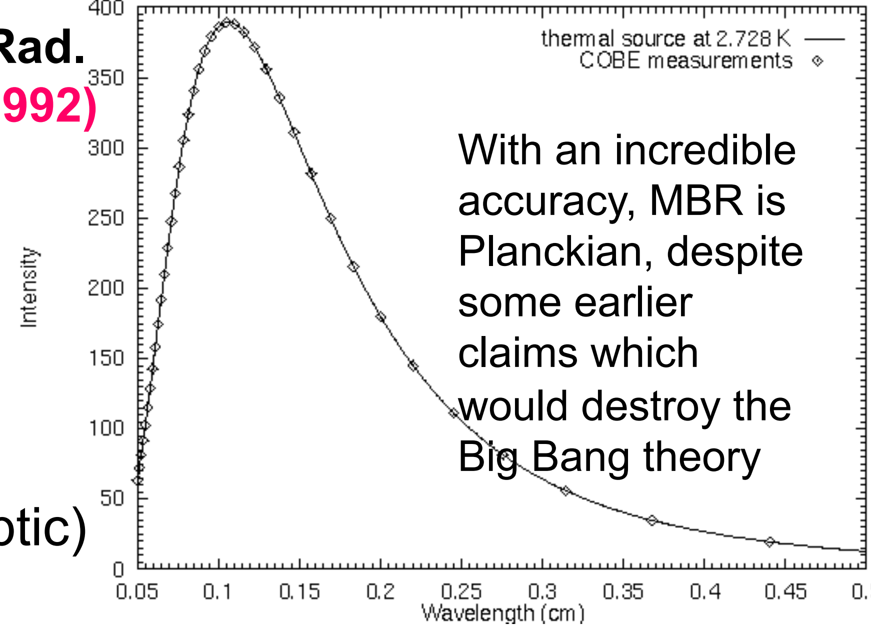
The CMB is markedly uniform, lending support to the theory of cosmic inflation, which says that the vacuum of the universe expanded much faster than the speed of light just a few tiny fractions of a second after the Big Bang (< 0.0000000000...01 s) "Why the Cosmic Microwave Background temperature is the same at different spots in the sky would be a mystery if it was not for inflation. Our whole sky, the visible universe came from a tiny (pre-inflation) region of “something bigger”.

BTW, we don’t have a good name for that Whole Thing (the *actual* Universe).

# CMBR = Cosmic Microwave Background Rad.

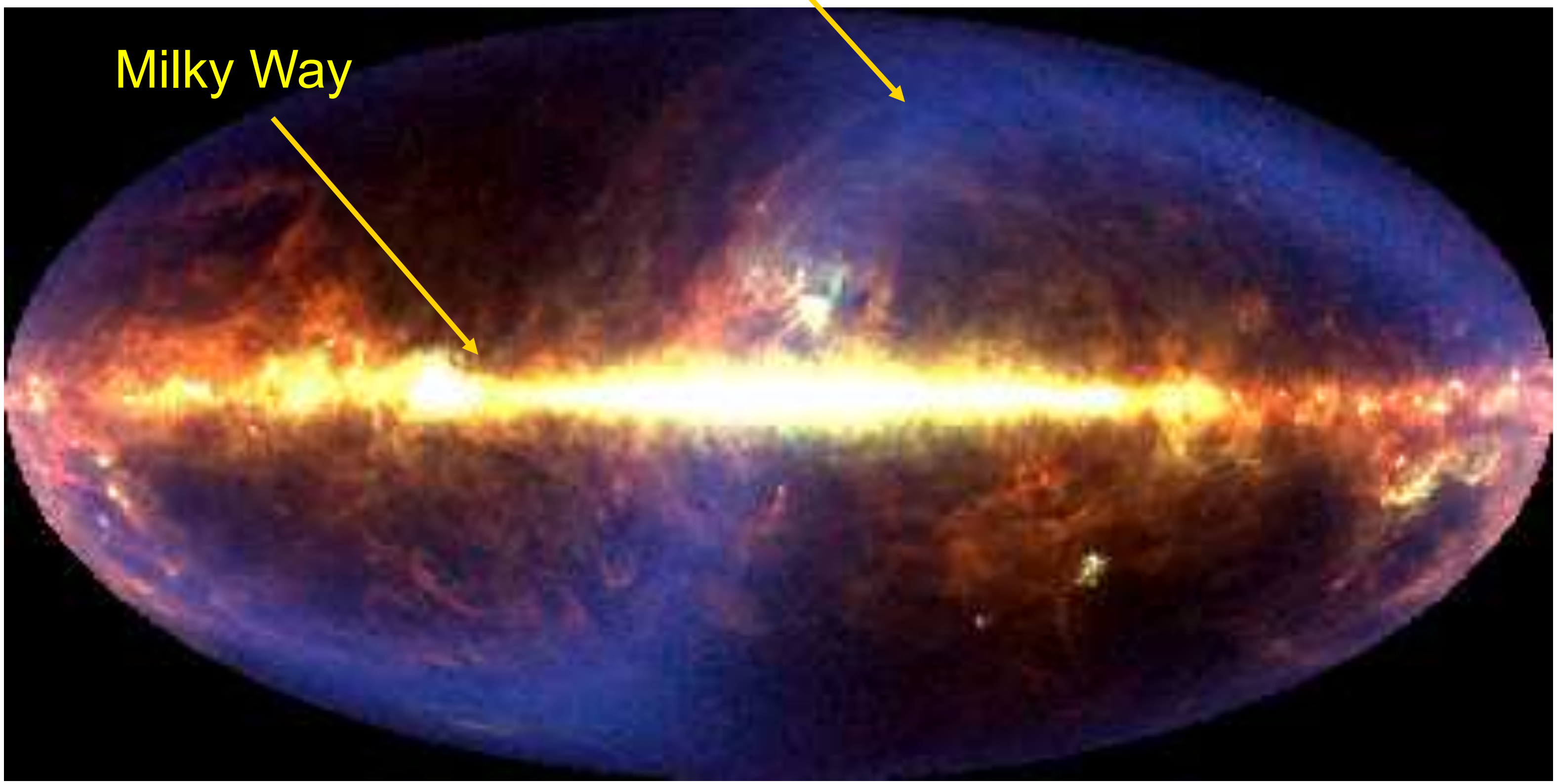



**observed by COBE (1989-1992) satellite observatory**

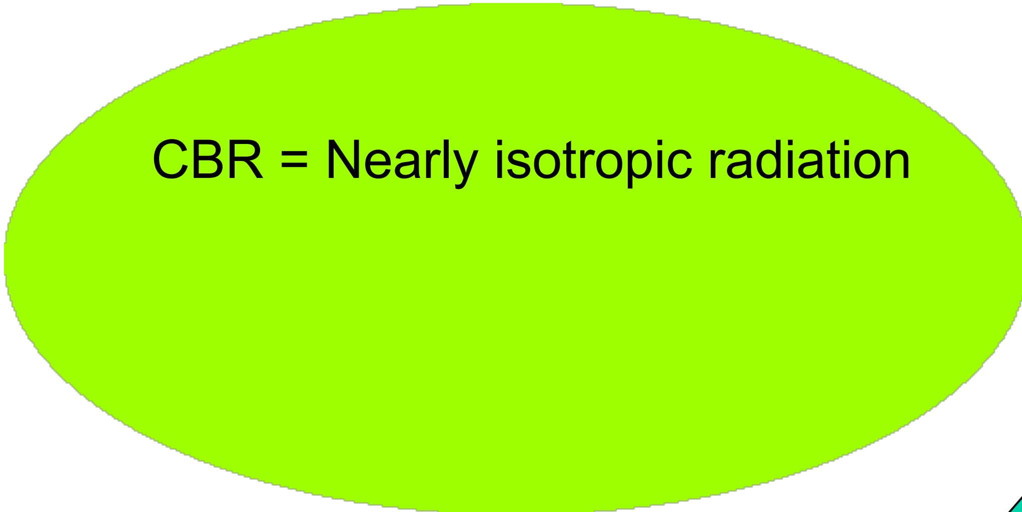


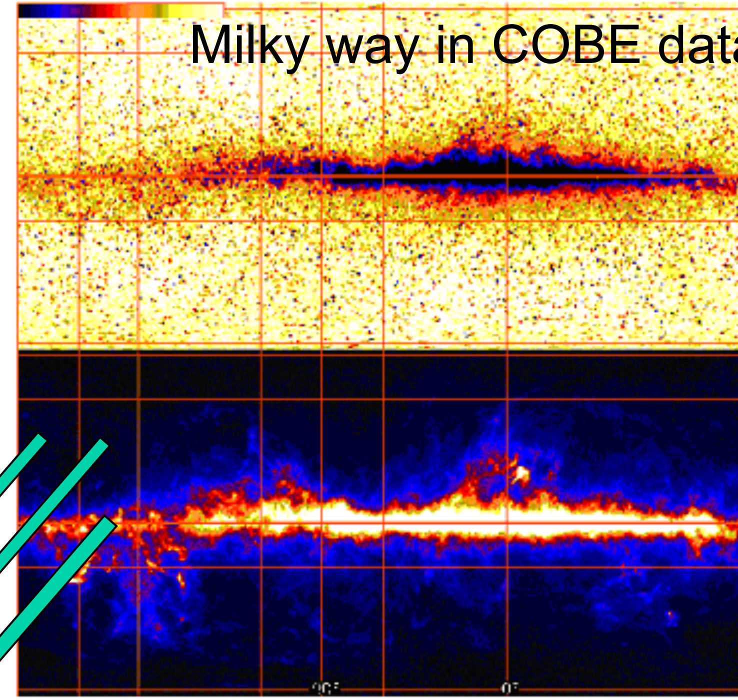

With an incredible accuracy, MBR is Planckian, despite some earlier claims which would destroy the Big Bang theory

Zodiacal light disk (solar system ecliptic)

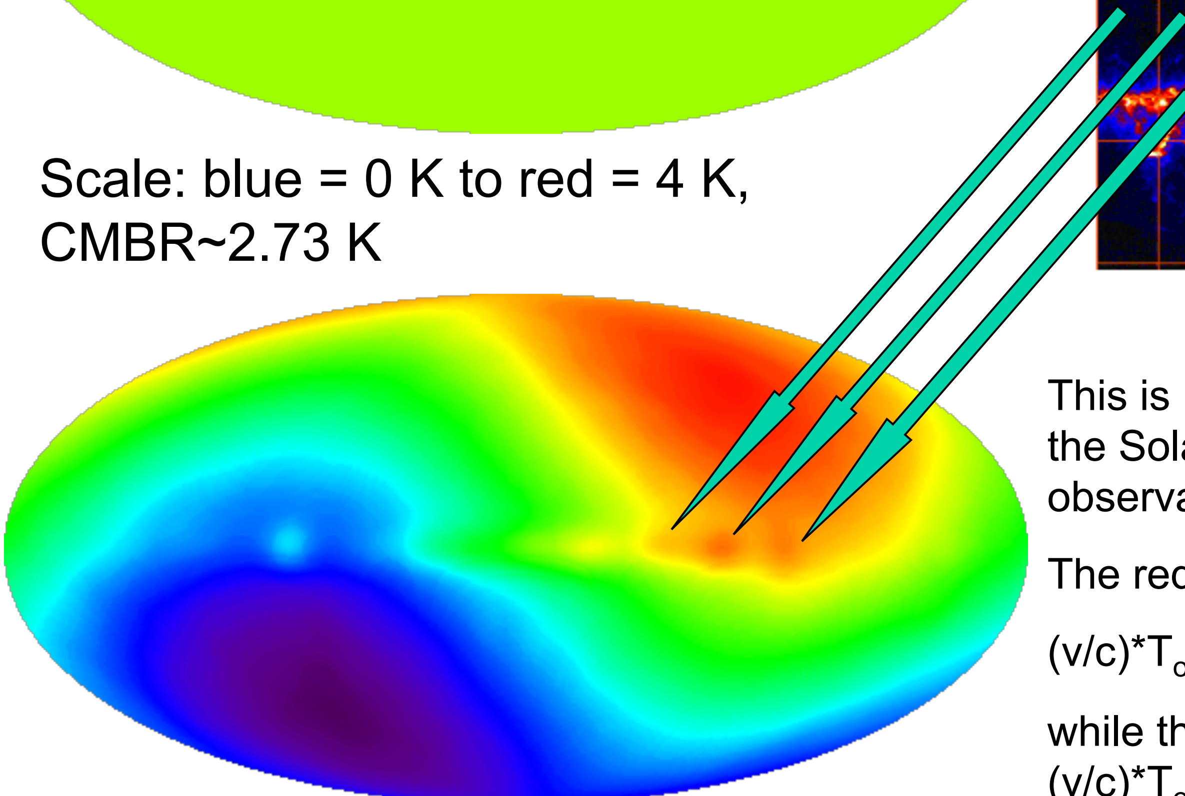


Milky Way




Scale: blue = 0 K to red = 4 K,
CMBR~2.73 K



Blue 2.724 K to red for 2.732 K.

This is the dipole component of CBR.

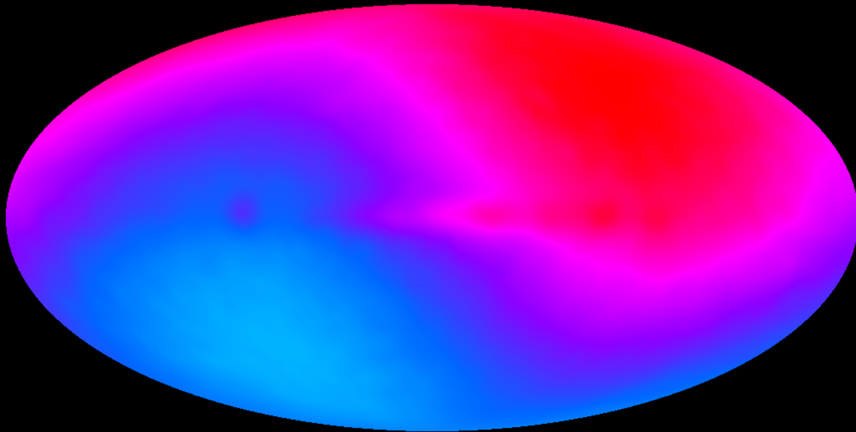
This is how we measure the velocity of the Solar System relative to the observable Universe (CMBR).

The red part of the sky is hotter by

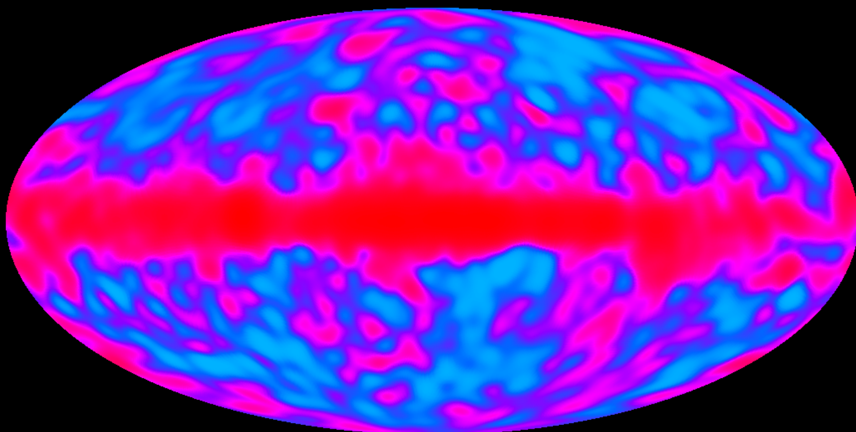
$(v/c)*T_o$,

while the blue part of the sky is colder by $(v/c)*T_o$. The inferred velocity is **v = 368 km/s**.

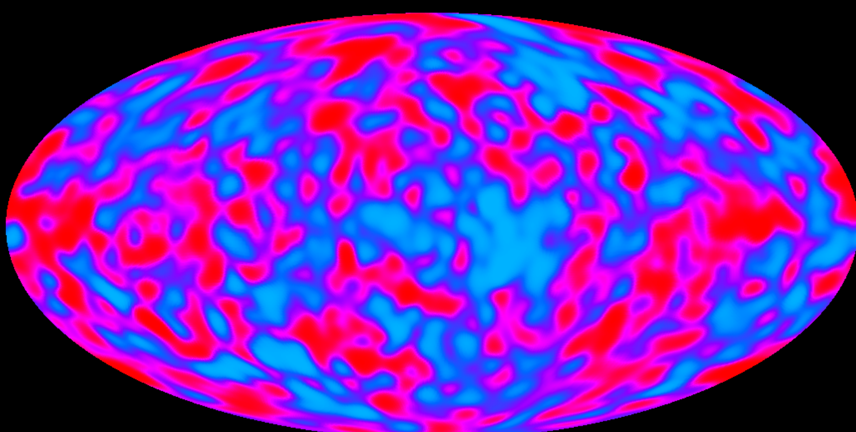
This picture of COBE data subtraction appeared on the cover of Physics Today in 1992



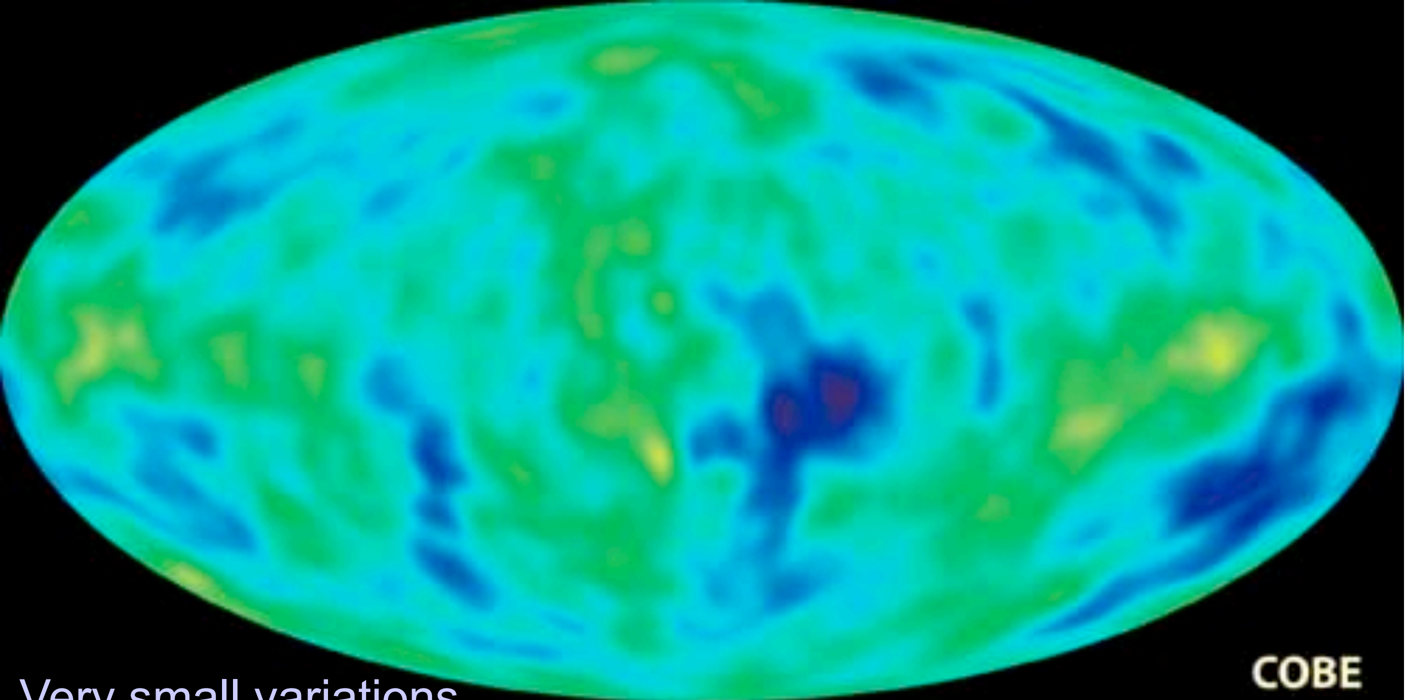
Dipole due to the peculiar motion



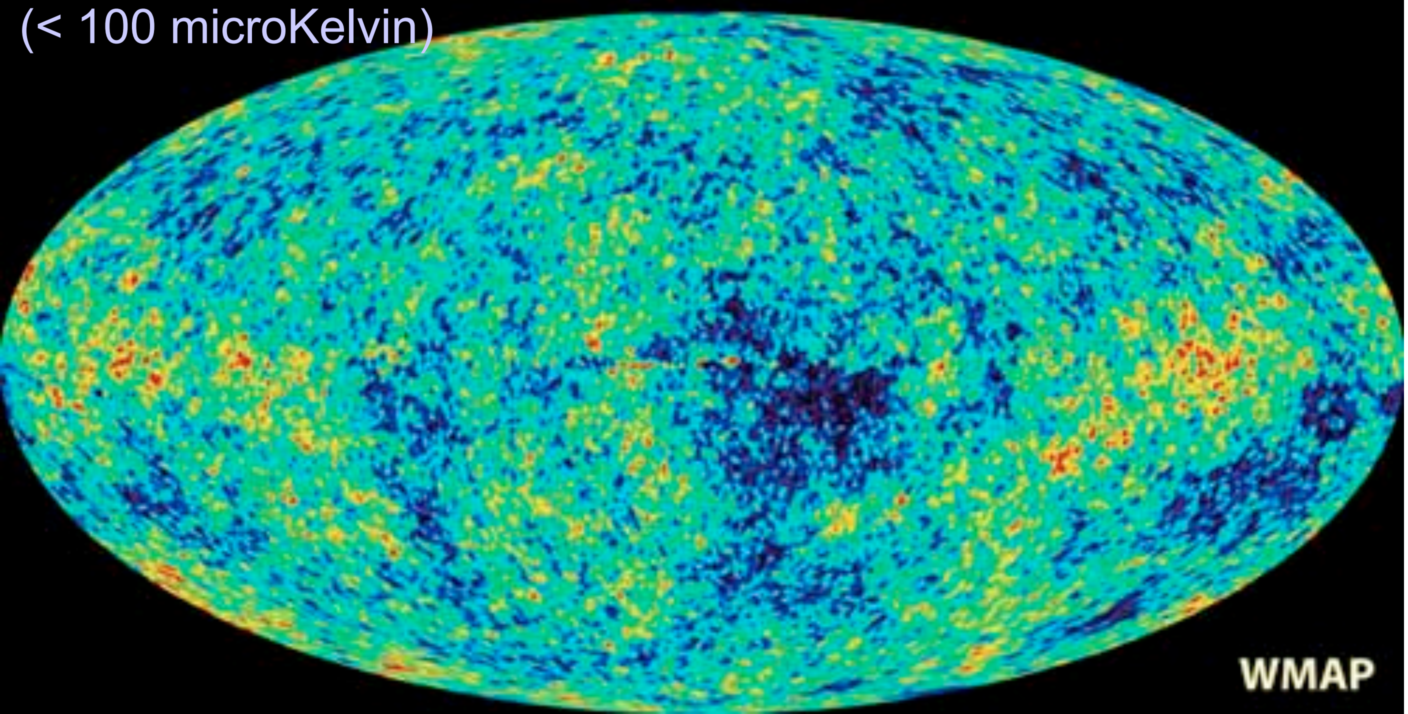
Milky Way background (warm dust)



+-0.00001 variations of CMBR temperature




Very small variations
(< 100 microKelvin)




Cosmic Microwave Background Radiation: what remains after the dipole *and* zodiacal light *and* the Milky Way subtraction.

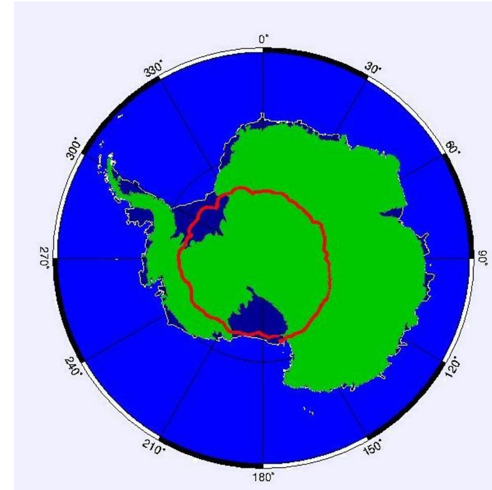
Spatial resolution poor, ~ 7 degrees

spatial resolution 0.25 degree or better was achieved by Boomerang and WMAP experiments the **Boomerang Project (1998-2003)**

a microwave telescope flown first for 10 days in 1998 under a baloon over Antarctica; surveyed 2.5% of the sky with an angular resolution of 0.25$^{o}$; the 1st experiment to show flatness of the space-time.

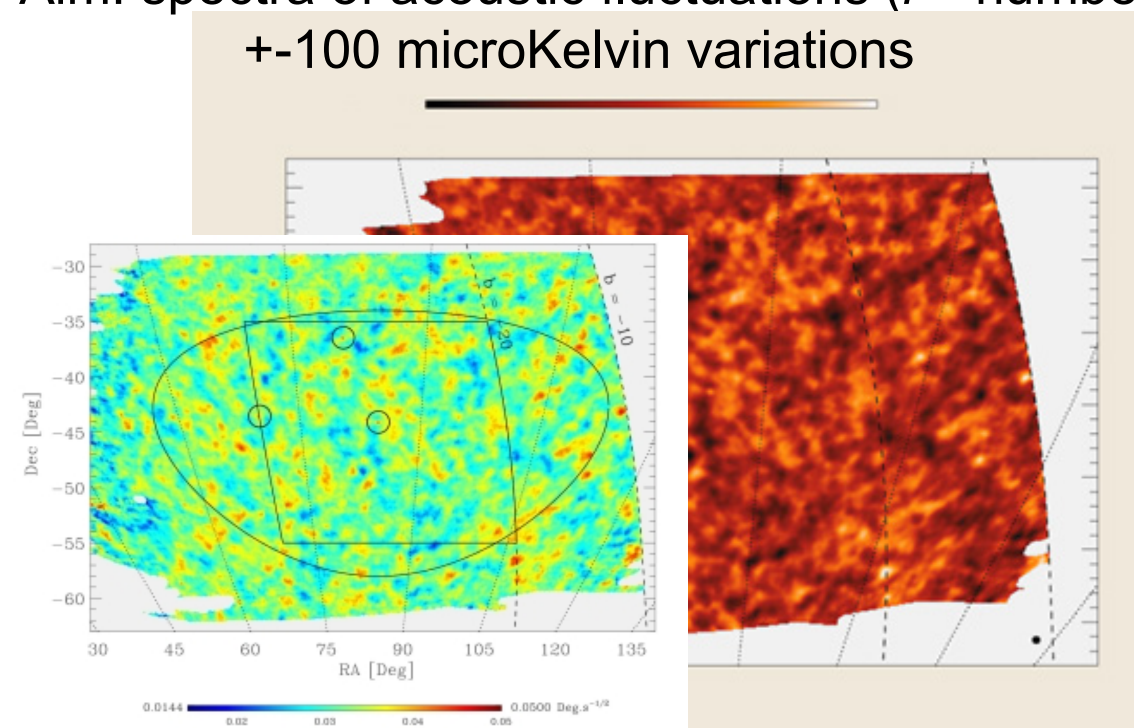
Principal Investigator

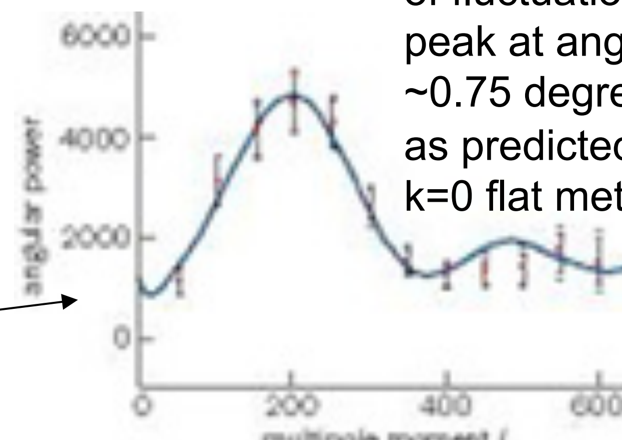
1.3m telescope with cryostat cooled to T=0.28 K




track map

Aim: spectra of acoustic fluctuations (*l* = number of wavelengths over a circle)



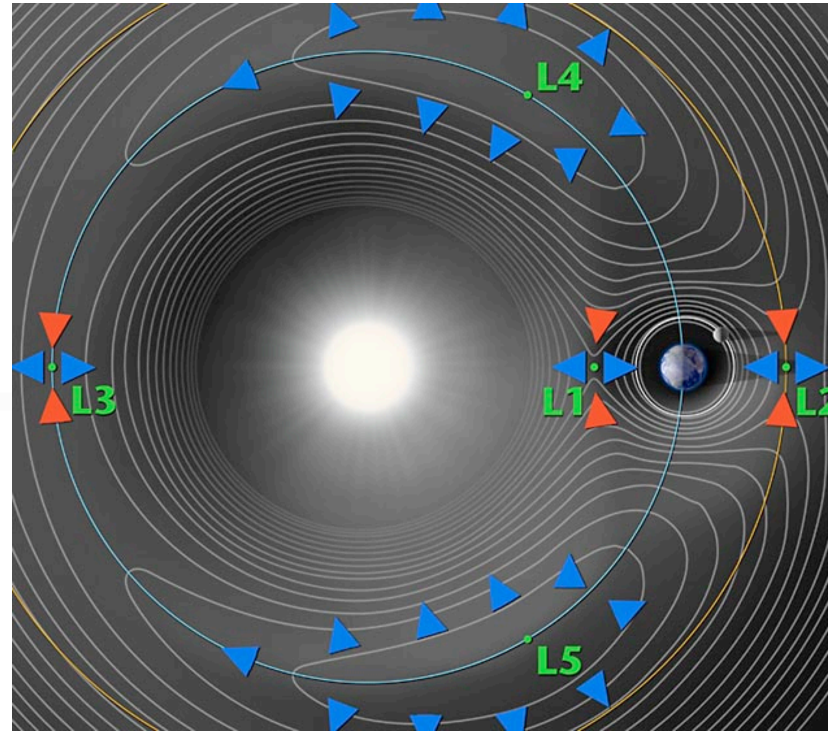

Multipole moment *l*

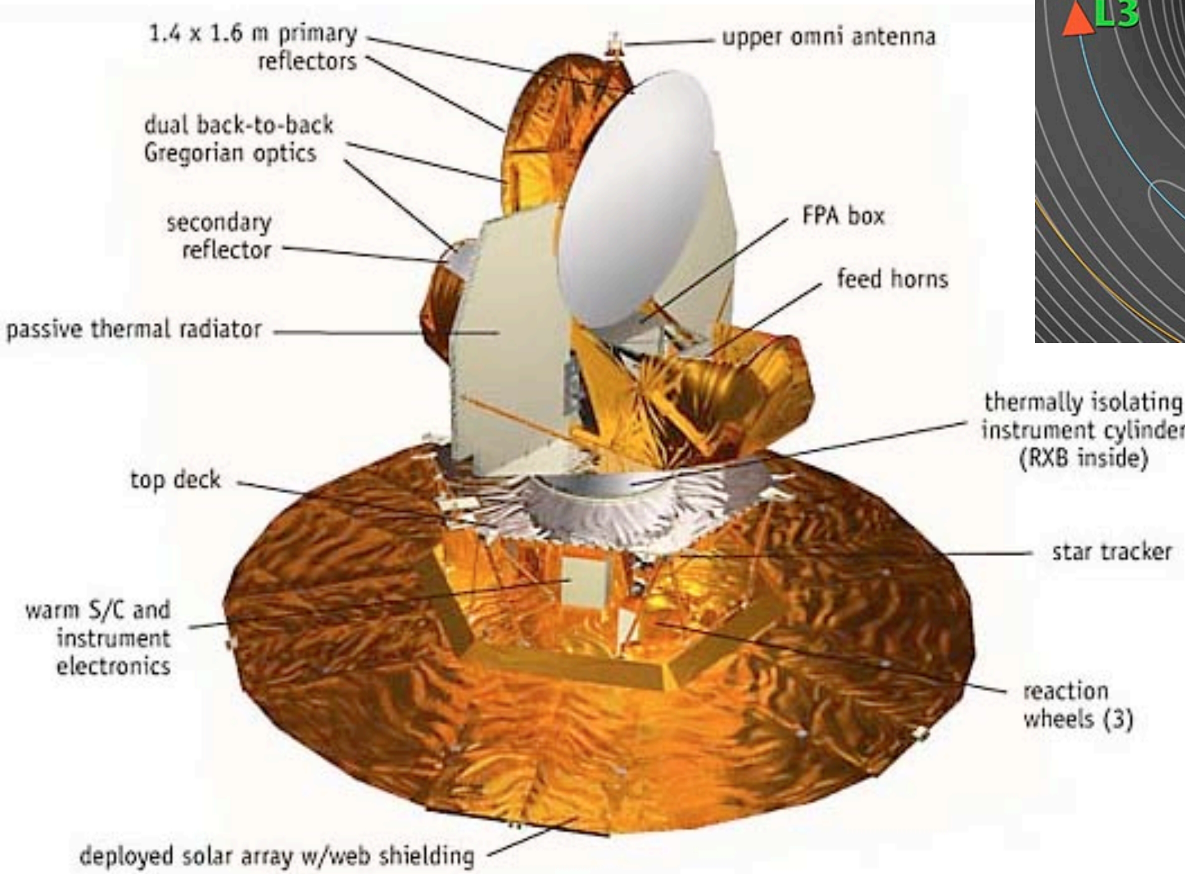
Spatial spectrum of fluctuations, peak at angle= ~0.75 degree as predicted for k=0 flat metric

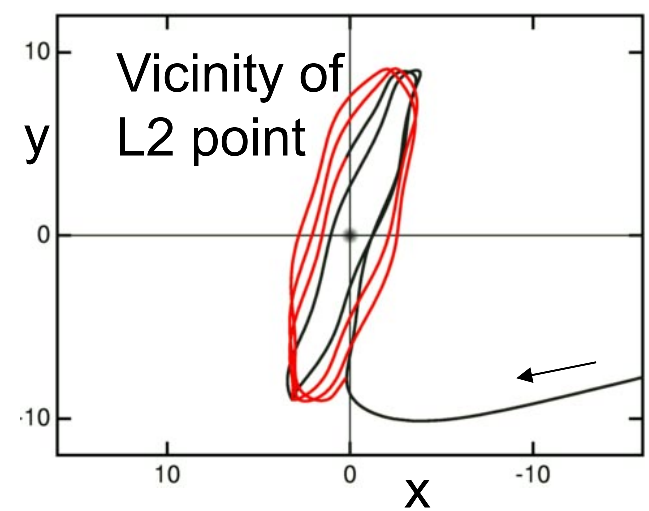
## WMAP = Wilkinson Microwave Anisotropy Probe

Launched by a Delta II rocket in 2001, results in 2003-2008.

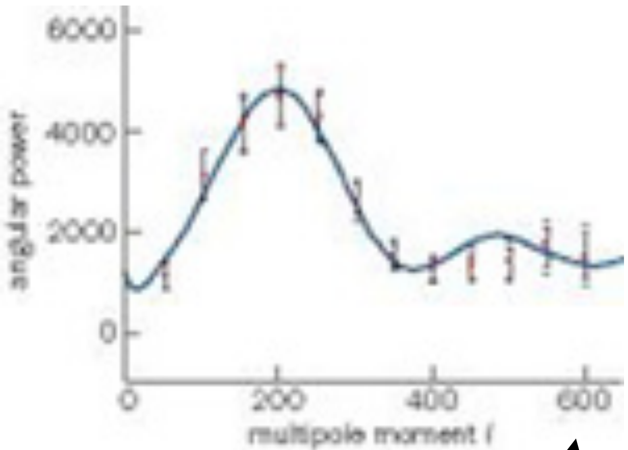
L2 point of the Sun-Earth system is unstable if trajectory not corrected, but very useful, because the instability is slow.



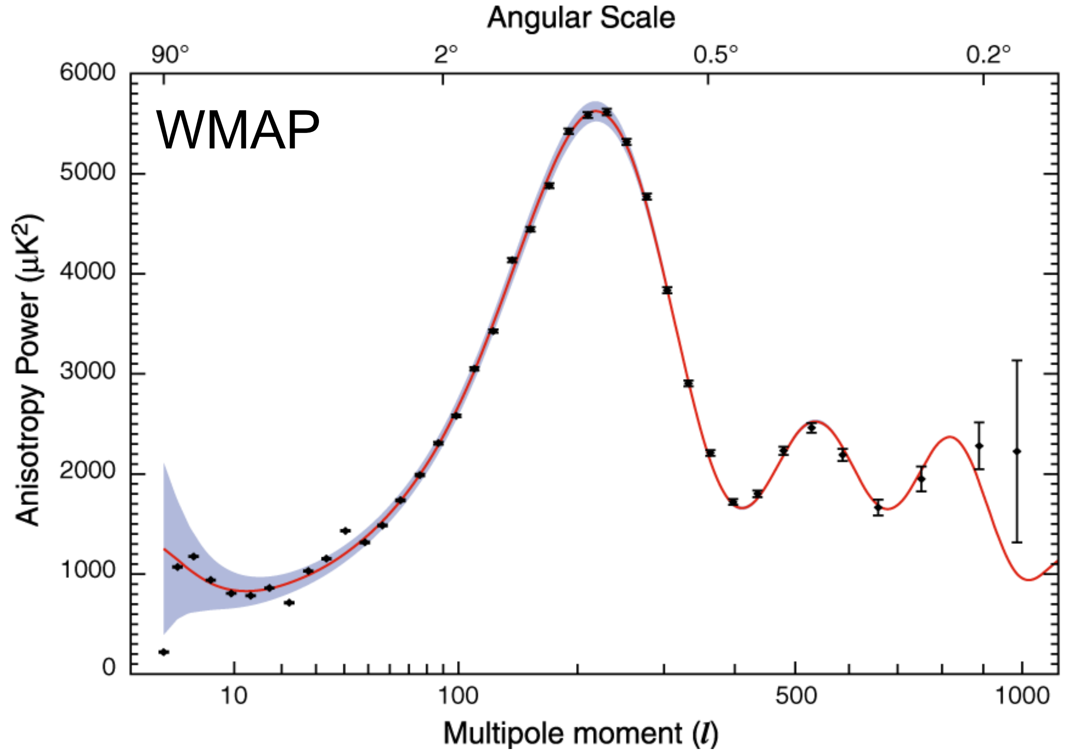



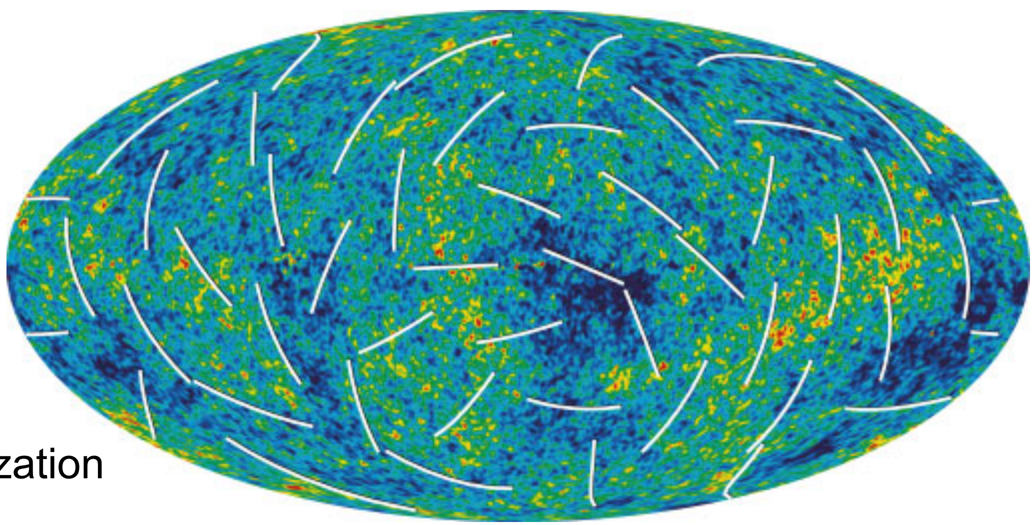



Boomerang vs. WMAP



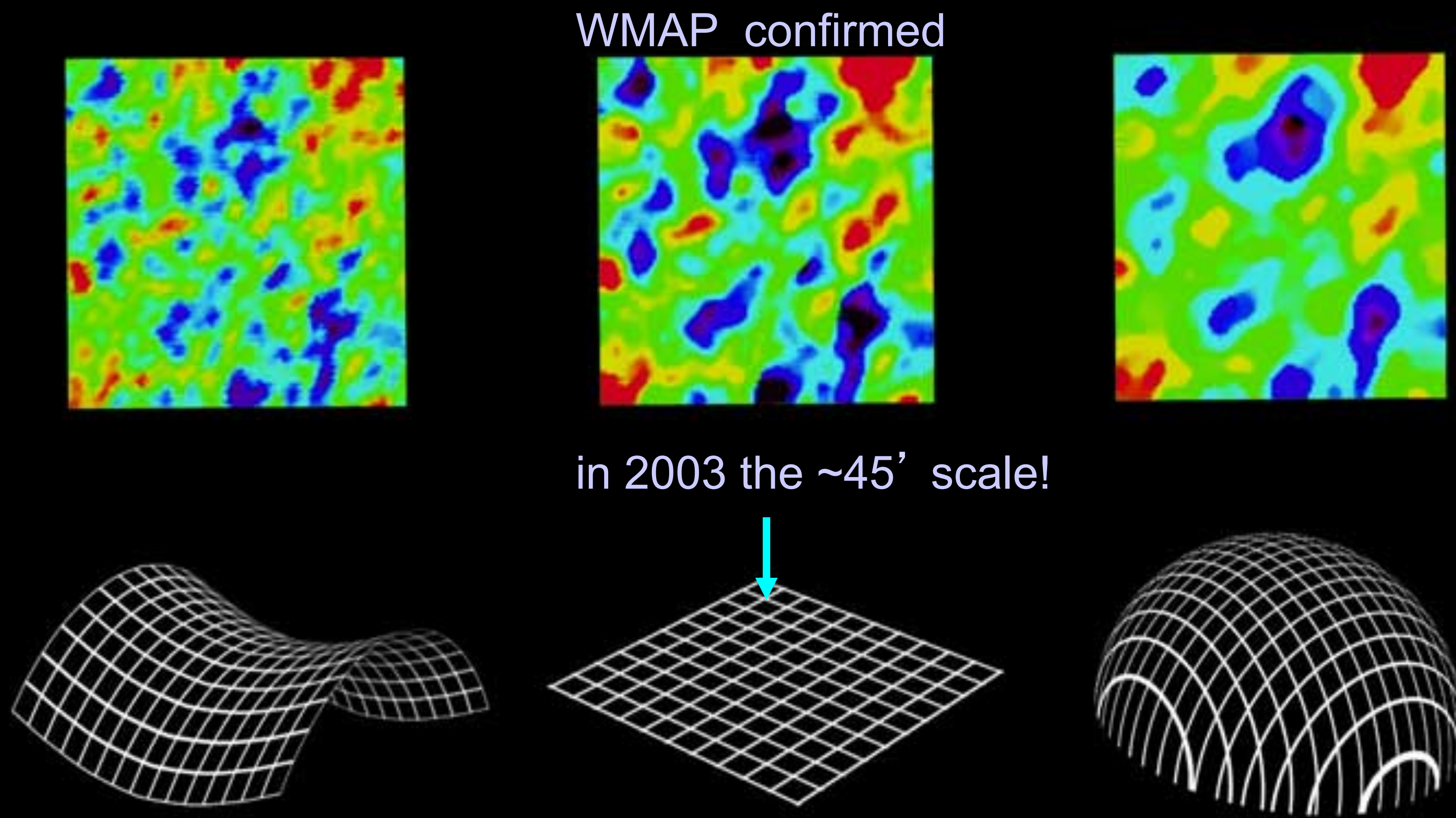

(lower resolution, hence lower multipole numbers)



Polarization
+ map

# GEOMETRY OF THE UNIVERSE




**OPEN**

Fluctuations largest on half-degree scale

**FLAT**

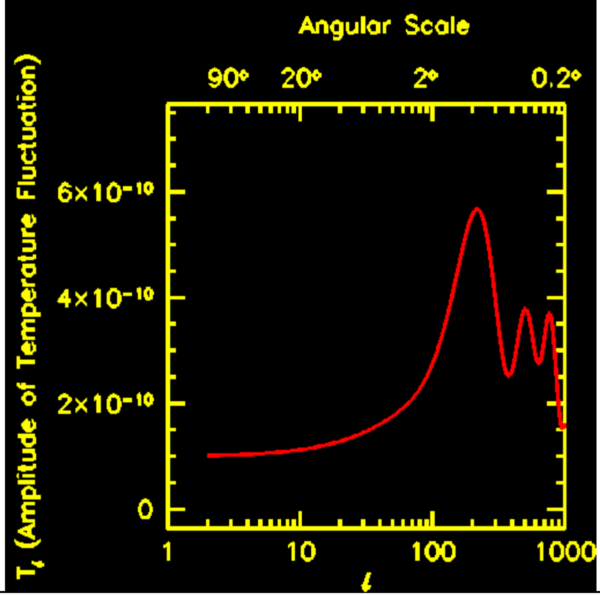
Fluctuations largest on 1-degree scale

**CLOSED**

Fluctuations largest on greater than 1-degree scale

**WMAP Project** **is like a sonar making *pictures* of ancient *sound* (pressure waves in plasma) from the universe at the recombination epoch (z~1000)**

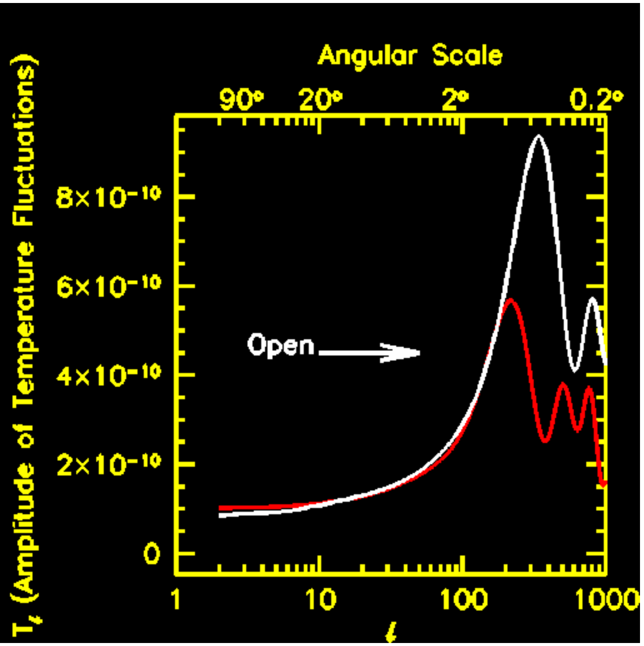
CMBR map → analysis

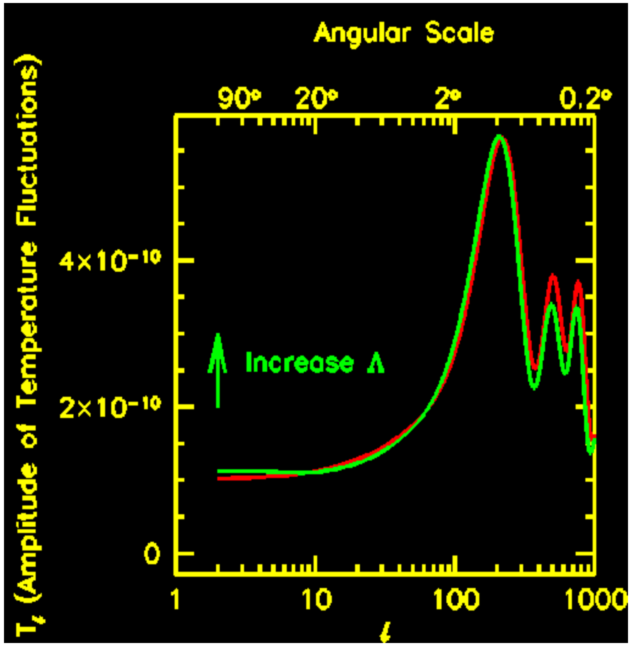


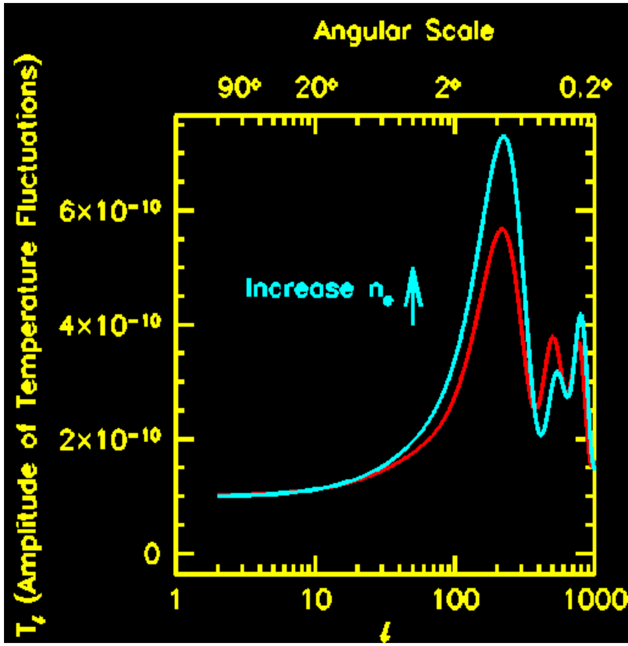

Spatial spectrum of fluctuations,

*l* = multipole moment = wave number

spectra of acoustic fluctuations (l = number of wavelengths over a circle)



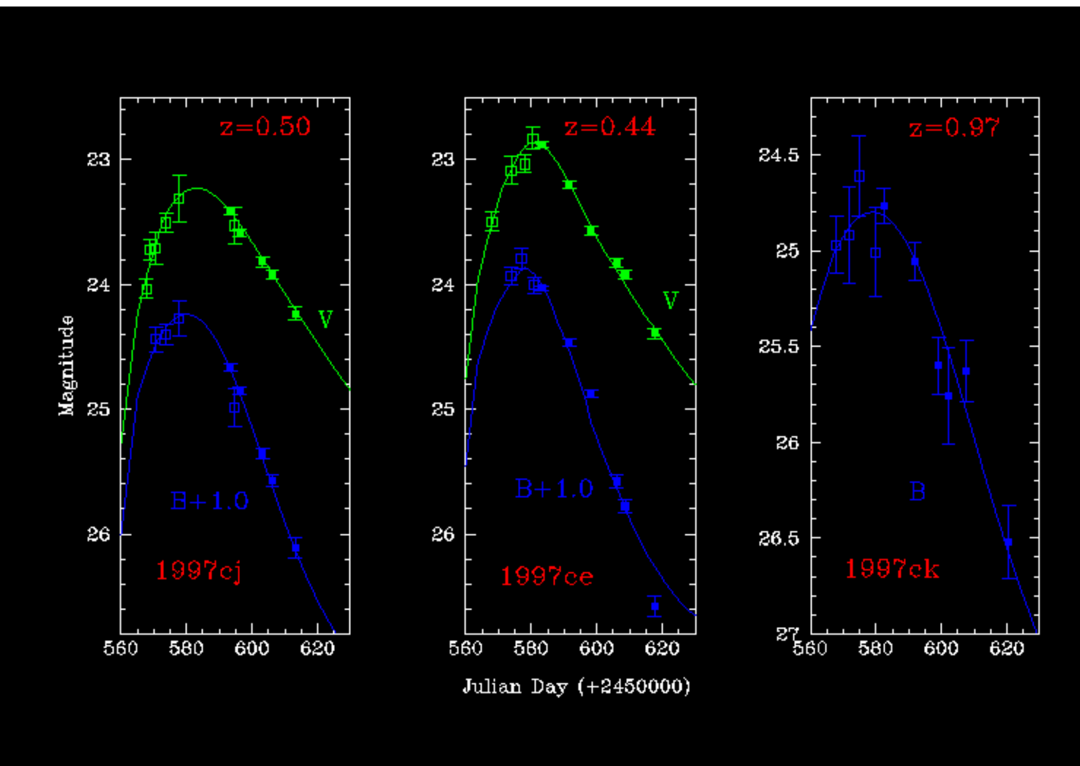




Red = measured, other colors show effects of physical parameters' variation

## reminder: Supernovae type Ia

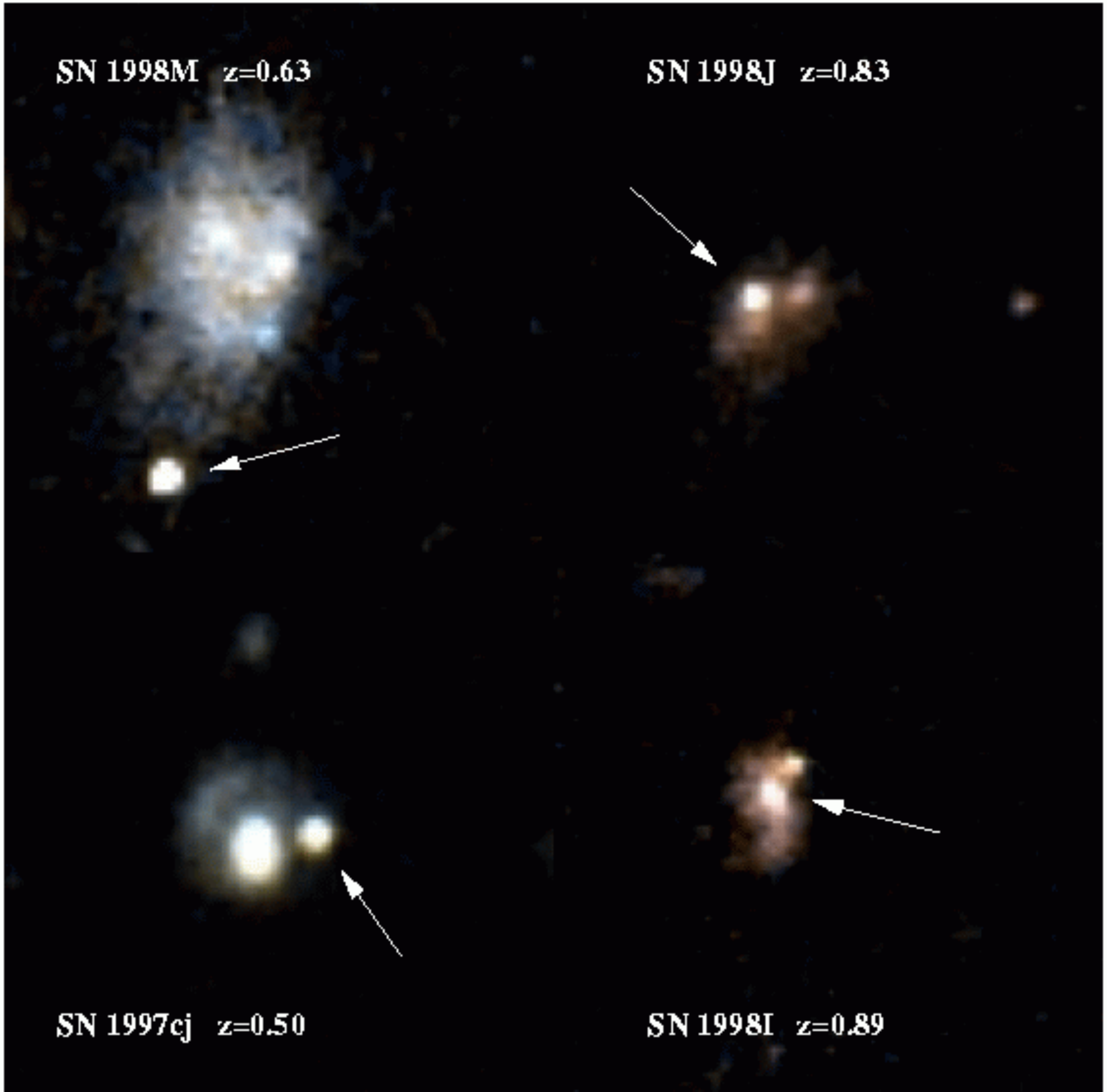
like these (SN 94D, 99el, 99eb) are, after an additional calibration due to a correlation of how steeply their brightness grows & falls with the absolute magnitude, good standard candles (+-7% distance error).
SN Ia can be used to construct the diagram of the rate of expansion of the universe as a function of time (redshift).

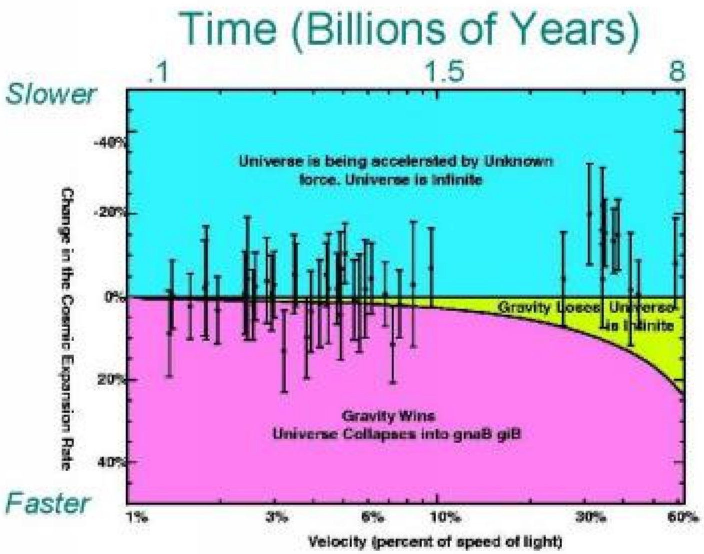


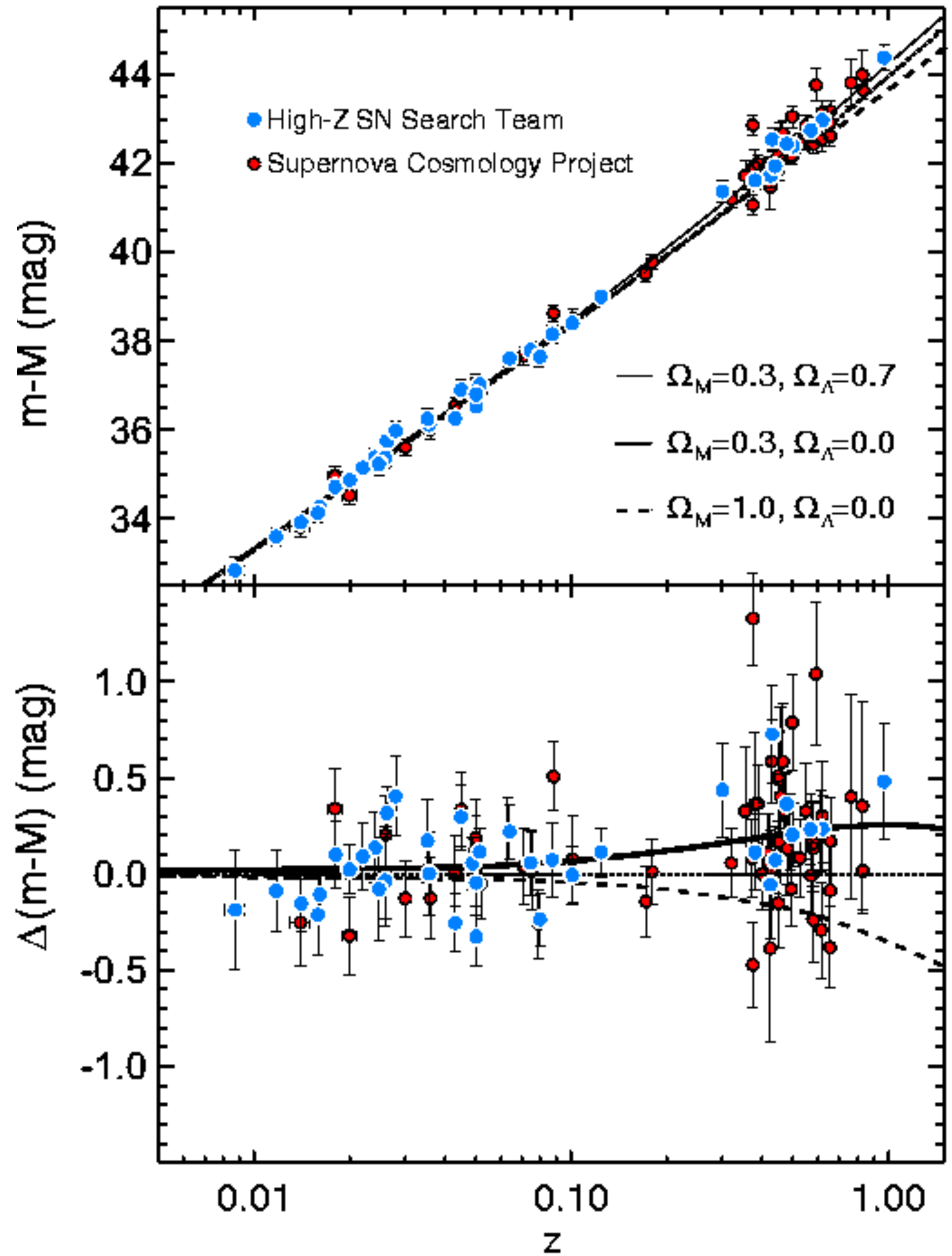

SN 94D

SN 99eb

SN 99el


SN 1998M z=0.63
SN 1998J z=0.83
SN 1997cj z=0.50
SN 1998I z=0.89



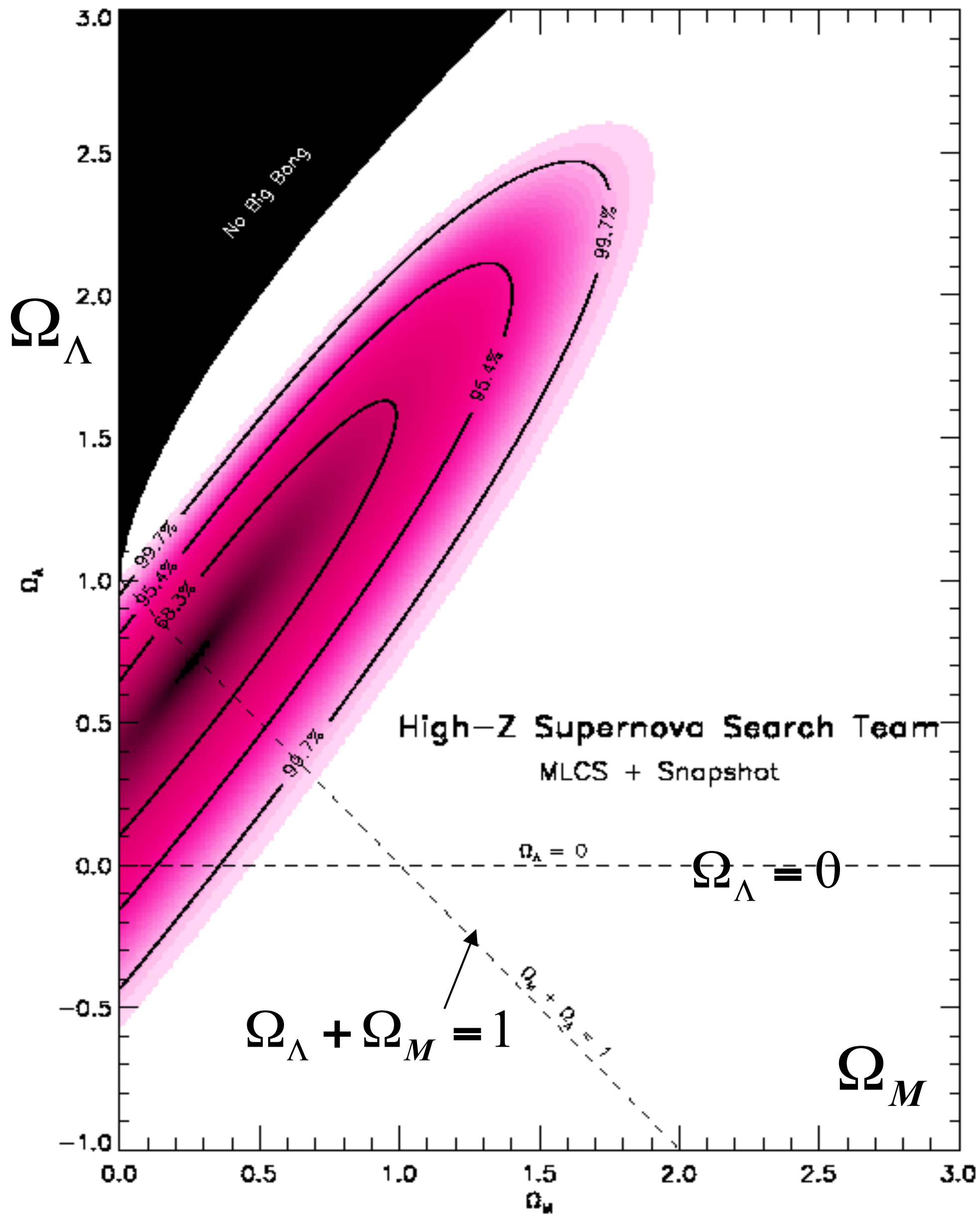




Hubble diagram (with distance modulus m-M replacing the distance) should look different in universes with different mean densities of:

all matter (symbol M), including dark matter!

dark energy (Λ).

Mean density is close to critical, in physical units about 10 atoms

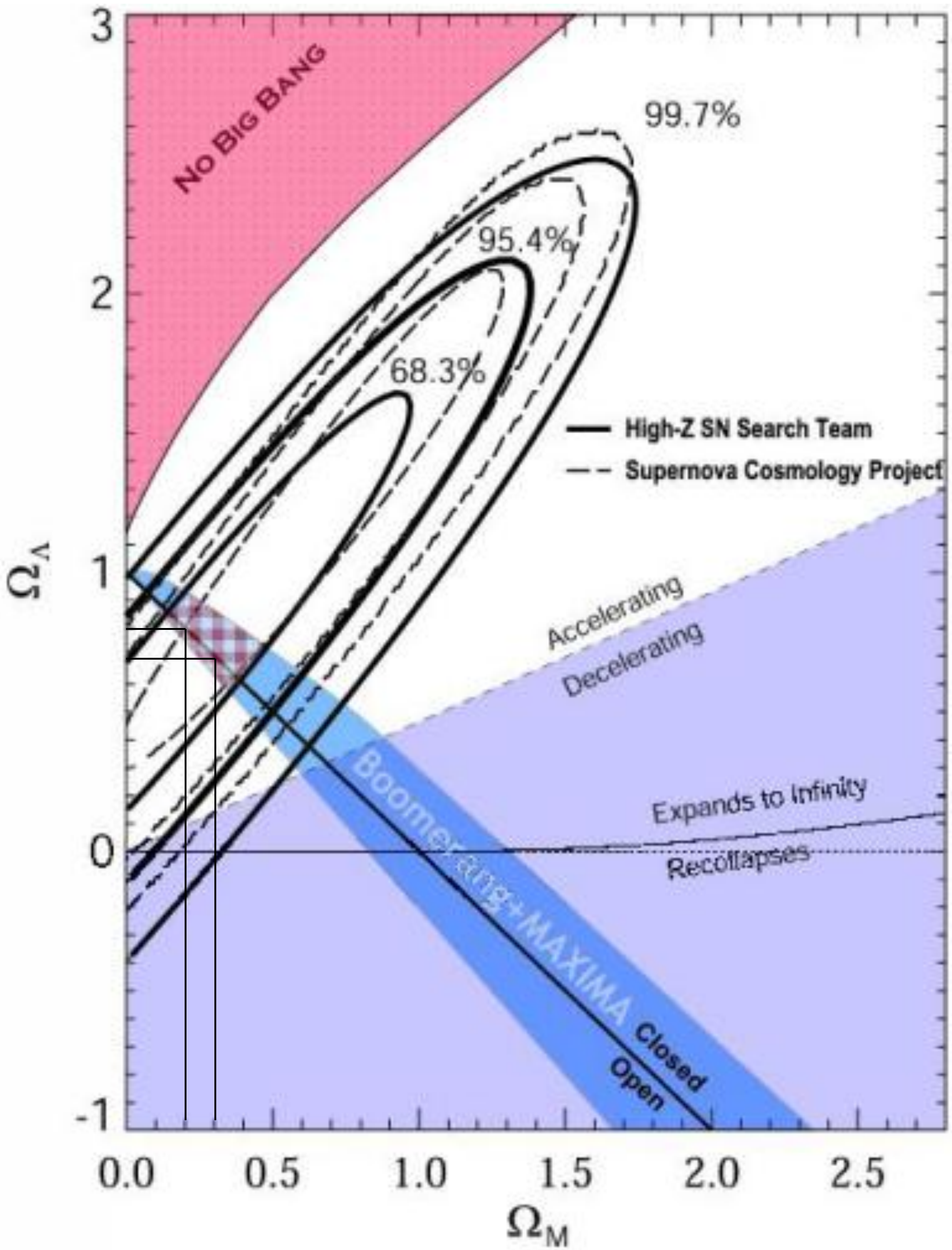



Although the error bars are considerable, SN distribution in space points toward the same conclusions as the CMBR mapping:

**$\Omega_\Lambda$~0.73** of the universe is a mysterious **dark energy**

**$\Omega_M$~0.27** of the matter-energy density is **normal (barion) (0.04)** + unknown **dark matter (0.23)**

Their sum is, however, quite well constrained, and corresponds to **critical density** At such mean density, Universe is barely able to expand to infinity and has flat space-time




SN Ia research together with Boomerang data show that...

a critical density of the universe fits the observations best:

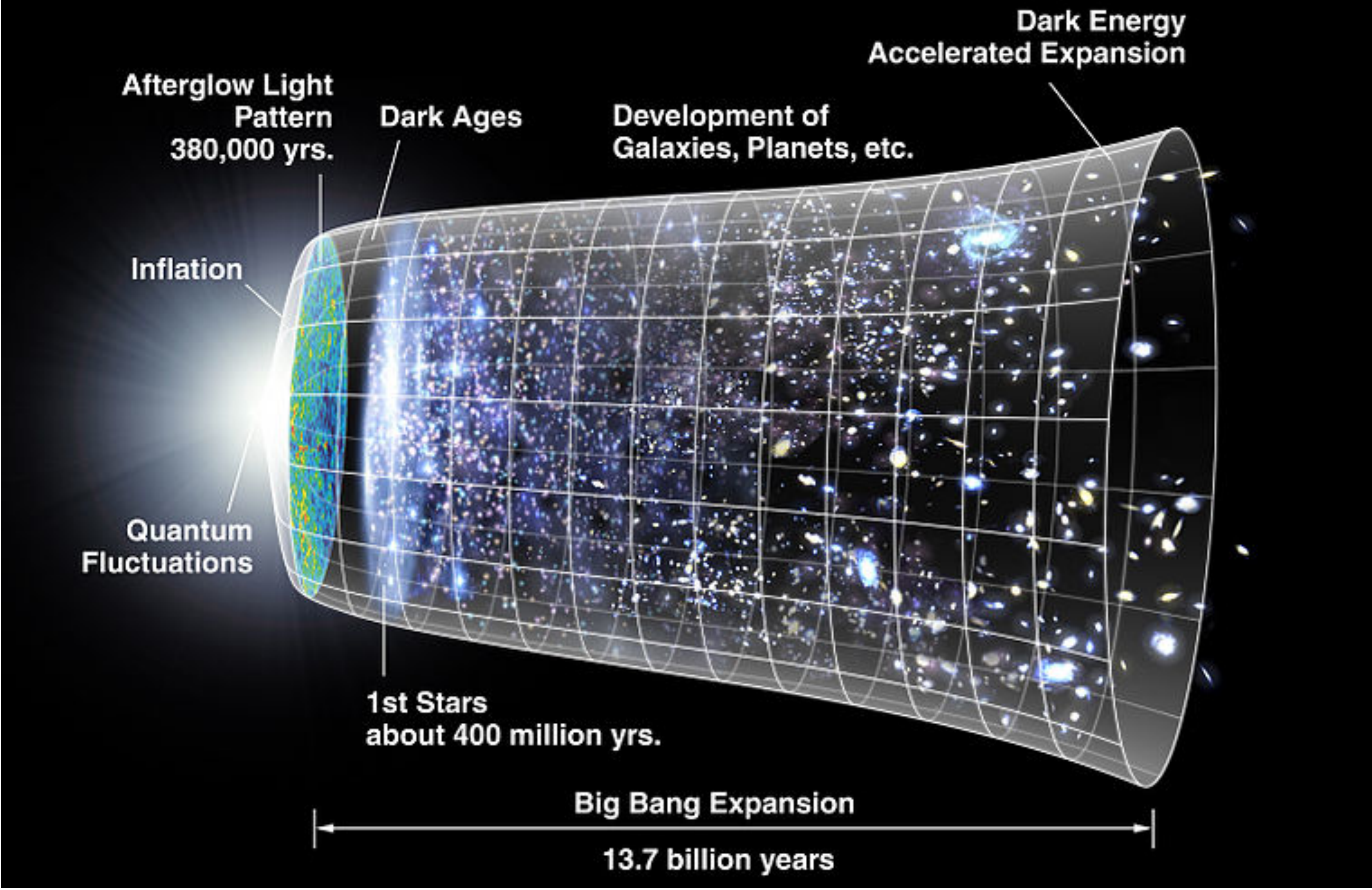
$$\Omega_\Lambda = 0.7...0.8$$

$$\Omega_M = 0.2...0.3$$

$$\Omega_\Lambda + \Omega_M = 1$$

~25% from normal and dark matter
~75% from "dark energy"

WHY is the space-time flat?
We think it's because in the first $10^{-31}$s after Big Bang there was a brief period of rapid exponential ***inflation*** (growth) of the universe. Inflation predicts the sum of omegas = 1 to with a very good precision.

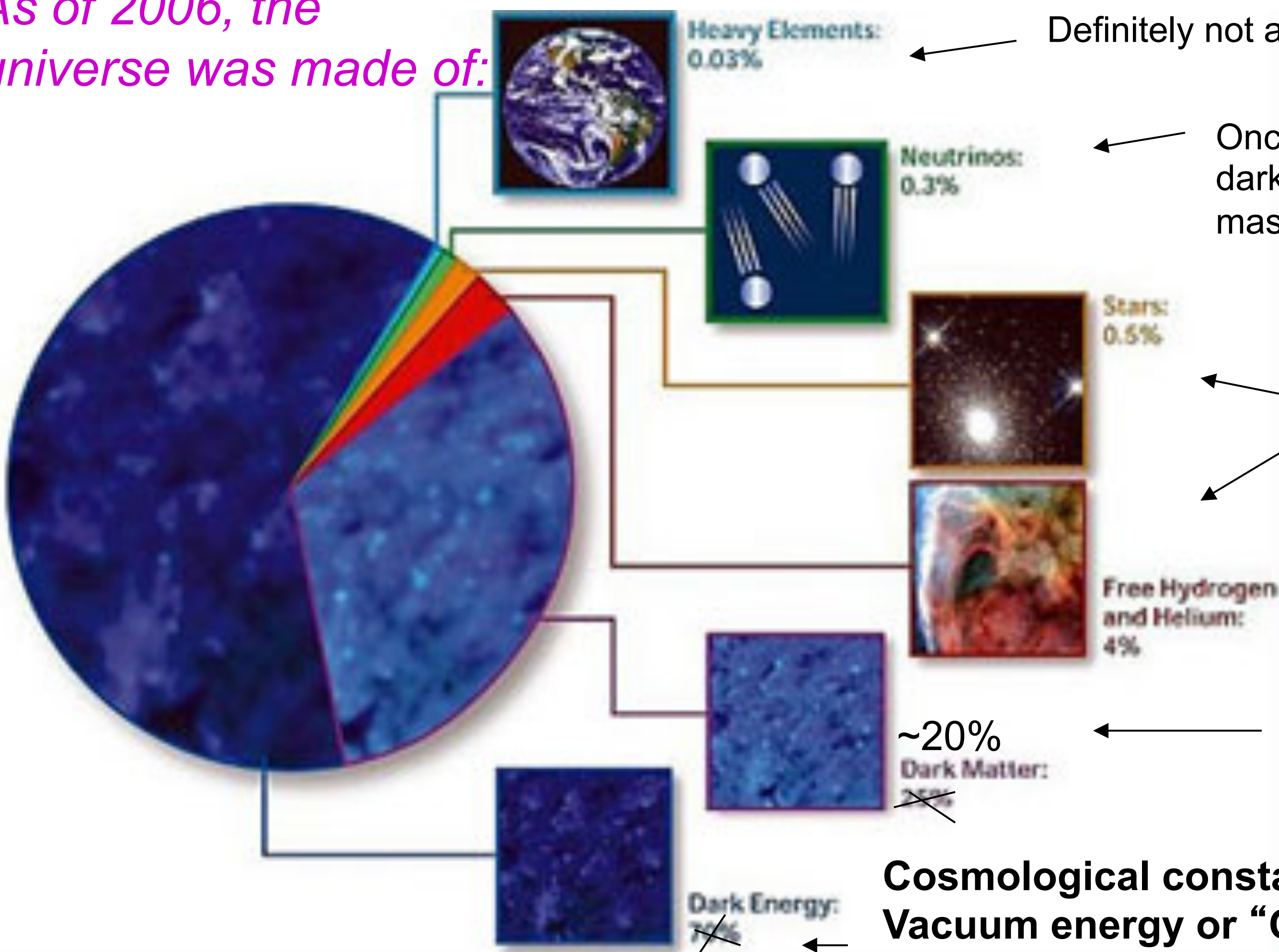


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

Unless Einstein's & Newton's theories of gravity (GR, $F \sim 1/r^2$) breaks down on scales larger than galaxy superclusters, which has recently been excluded (MOND=MOdified Newtonian Dynamics), we have to accept that there is both **dark matter** (attractive force) and **dark energy** (repulsive force), and that we still have little understanding of either one.

And that's just the beginning of the unknowns in cosmology...

*As of 2006, the universe was made of:*




~80%

Definitely not a major player

Once supposed to be the dark matter because of non-zero mass, but not massive enough...

Not enough either

The latest results show that dark halos of galaxies end at distances ~300 kpc; the **dark matter is inhomogeneous (cold; as in CDM theory)**

**Cosmological constant?** $\Lambda \approx 10^{-29}\ g/cm^3$ ?

**Vacuum energy or "Quintessence"**

**(a very homogeneous sea of very light, weakly interacting elementary particles)**

## Cosmological constant - an engine of the accelerated expansion?

The simplest explanation for dark energy is that it is simply the "cost of having space": that is, that a volume of space has some intrinsic, fundamental energy. This is the cosmological constant, sometimes called Lambda after the mathematical symbol used to represent it, the Greek letter $\Lambda$. Since energy and mass are related by $E = mc^2$, Einstein's theory of general relativity predicts that it will have a gravitational effect. It is sometimes called a vacuum energy because it is the energy density of empty vacuum. In fact, most theories of particle physics predict vacuum fluctuations that would give the vacuum exactly this sort of energy. The cosmological constant is of order $\Lambda \sim 10^{-29}$g/cm$^3$.

The cosmological constant has negative pressure equal to its energy density and so causes the expansion of the universe to accelerate. The reason why a cosmological constant has negative pressure can be seen from classical thermodynamics. The work done by a change in volume $dV$ is equal to $-p\, dV$, where $p$ is the pressure. But the amount of energy in a box of vacuum energy actually increases when the volume increases ($dV$ is positive), because the energy is equal to $\rho V$, where $\rho$ is the energy density of the cosmological constant. Therefore, $p$ is negative: $p = -\rho(c$^2$)$.

A major outstanding problem is that most quantum field theories predict a **huge** cosmological constant from the *energy of the quantum vacuum fluctuations (creation and annihilation of virtual particles)*, up to 120 orders of magnitude too large. This would need to be cancelled almost, but not exactly, by an equally large term of the opposite sign. Some supersymmetry theories of elementary particles require $\Lambda = 0$, which does not help. This is the **cosmological constant problem**, the worst problem of fine-tuning in physics: there is no known natural way to derive the tiny $\Lambda$ from physics.

Some physicists invoke the *anthropic principle*. (= only a specific fine-tuning leads to life and intelligence. Universes with large $\Lambda$ may not have stars, planets, and life)

Others think the *quintessence* is *the* answer (‘hot’, weakly interacting particles)...

*Appendix to the last lecture and a reveal of the solution to one of the problems in your assignments set #4.*

**Why the standard model of the expansion of the universe with critical density ($\Omega$=1) and Euclidean flat space-time (k=0) fails: The oldness of stars in globular clusters.**

**How the addition of $\Lambda > 0$ (dark energy) makes the universe safe for globular clusters.**

# The oldness of stars paradox, and its solution

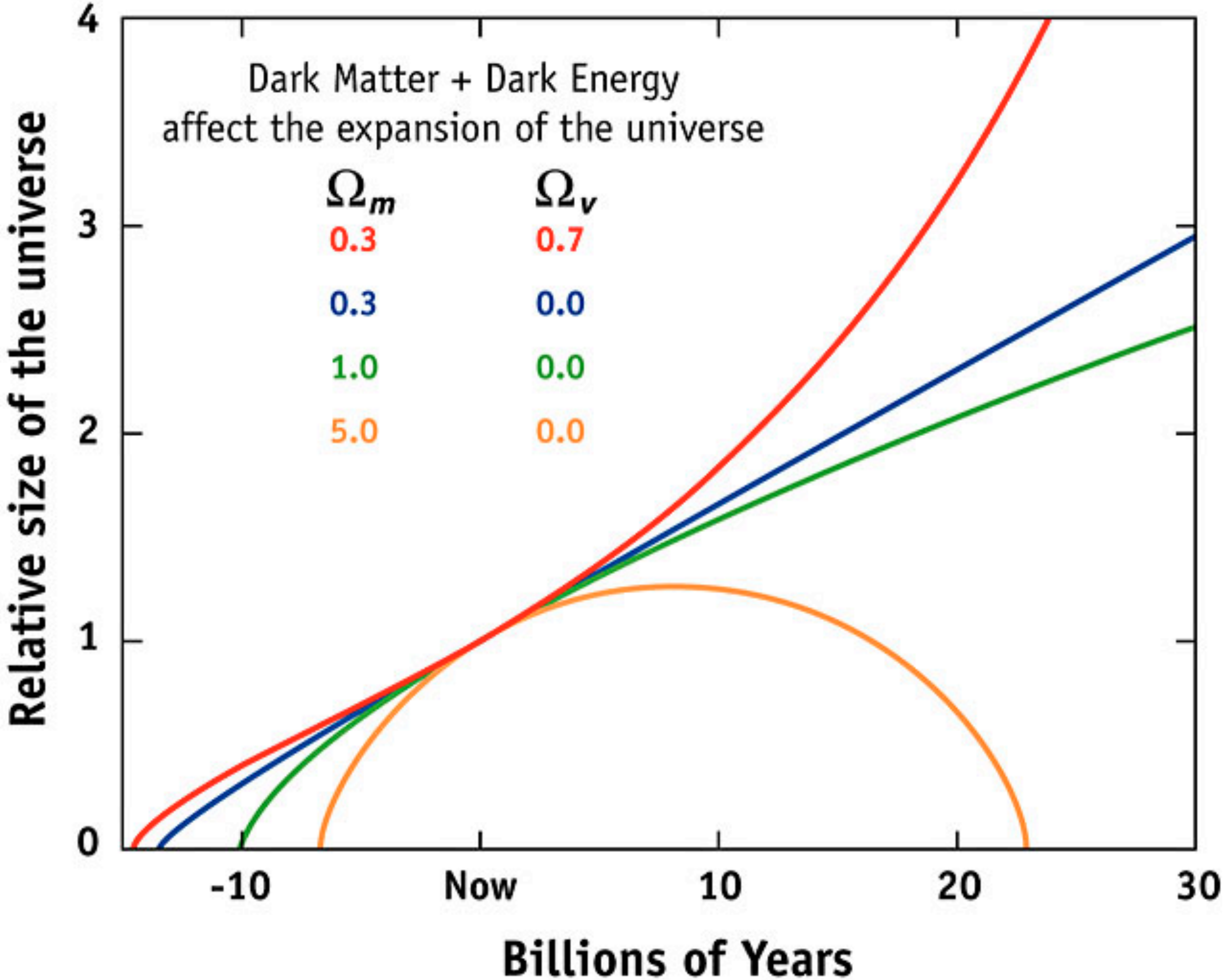
Axiom: *You can't be older than your mother*

The age derived from the value of the Hubble constant known in 1990s (~66 km/s/Mpc) and the standard model of expansion (flat space-time or k=0; with critical density or Omega=1) is **~10 Gyr**, that is too short to accommodate the oldest objects in it.

Some globular clusters contain stars which are **12-13 Gyr** old (as determined by their H-R diagram and the theory of stellar structure and evolution.)

Are the stars "too old" or the universe "too young" in our theories?

# Expansion of the Universe



We will see here how the **cosmological constant** helps us solve the **oldness problem** of objects in our world, unexplained in the standard model of a critical density universe. (We use Newtonian dynamics, which is ~OK, cf. Sect.7.2 of textbook)

The expansion of a homogeneous, isotropic universe

[Point "O" and radius R are arbitrary] Expanding shell encloses


$$M(<R) = M = \frac{4}{3}\pi R^3 \bar{\rho} = \text{const.}$$

Test particle at radius R is subject to acceleration

$$\ddot{R} = -\frac{GM}{R^2} + \frac{\Lambda}{3}R$$

($\Lambda > 0 \Rightarrow$ accelerated expans.)

$\Lambda$ = Einstein's cosmological constant. He introduced $\Lambda$ before the discovery of Hubble flow (expansion of the universe). If $\Lambda = 4\pi G\bar{\rho}$ then $\ddot{R} = 0$, which Einstein presumed. (Later, he called it his biggest blunder, to assume R=const)

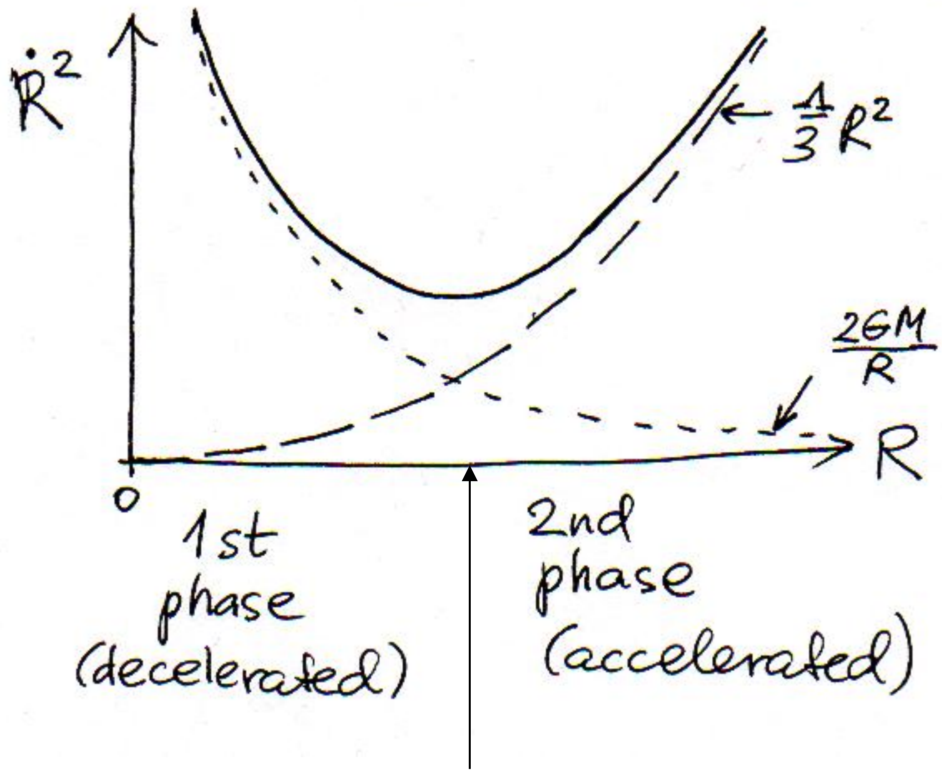
a.k.a. steady-state universe

We can write the equation of motion as

$\ddot{R} = -\nabla\Phi_{eff}$, where $\Phi_{eff} = -\frac{GM}{R} - \frac{\Lambda R^2}{6}$, and the energy integral is $E = \frac{\dot{R}^2}{2} + \Phi_{eff}(R) = 0$, if we want to model the flat space-time ($k=0$). Otherwise we would have to set $E = -kc^2$, $c$ = speed of light.

Expansion with dark energy ($\Lambda > 0$)

$$\dot{R}^2 = -2\Phi_{eff}(R) = \frac{2GM}{R} + \frac{\Lambda}{3}R^2$$



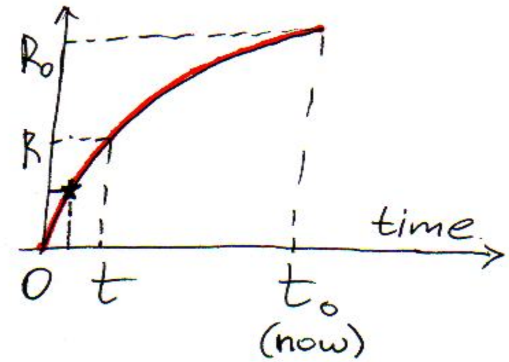

*At this radius, there will be an inflection point in the curve R(t), where the second time derivative of R changes sign and the R(t) curve switches between convex and concave.*

here we get the **standard model** of the universe, as in the textbooks:

First phase of expansion ($\Lambda \approx 0$ compared with gravity pulling back)

$$\dot{R} = \sqrt{2GM} \cdot \frac{1}{\sqrt{R}} \Rightarrow \int_0^R R^{\frac{1}{2}} dR = \sqrt{2GM}\, t$$

$$\frac{t}{t_o} = \left(\frac{R}{R_o}\right)^{\frac{3}{2}}, \qquad \boxed{\frac{R}{R_o} = \left(\frac{t}{t_o}\right)^{\frac{2}{3}} = \frac{1}{1+z}}$$




dR/dt = H R
Hubble's law

e.g. $z = 3$ (*)

$R_o = (1+3) R = 4R$

$t = t_o \left(R/R_o\right)^{3/2} = (1/8)\, t_o$

Current rate of expansion, measured by $H_o = h \cdot 100 \frac{km}{s \cdot Mpc}$, determines $t_o$ (the age of the universe):

$H = \dot{R}/R$, $H_o = \left(\dot{R}/R\right)_o$, let's differentiate $R(t) \Rightarrow \dot{R} \Rightarrow$

$$\Rightarrow H = \frac{2}{3} \frac{t^{-1/3} R_o}{t_o^{2/3} R}, \qquad H_o = \frac{2}{3 t_o} \quad \text{or} \quad \boxed{t_o = \frac{2}{3} H_o^{-1}}$$

The age of the universe

$$H_o \cong h \cdot 100 \left(\frac{pc}{Myr}\right) \frac{1}{Mpc}, \quad h \approx \frac{2}{3}.$$

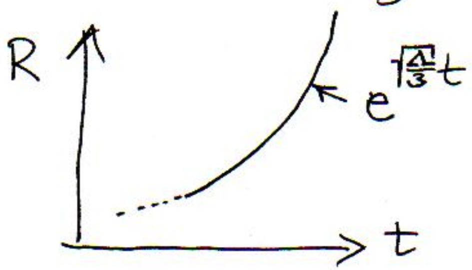
$$t_o = \left(\frac{2/3}{h}\right) 10\ Gyr \approx \mathbf{10}\ Gyr \rightarrow$$

This age is too short!
Some stars are ~14 Gyr old.

Second phase of expansion, dominated by $\Lambda$

$$\ddot{R} \approx + \frac{\Lambda}{3} R$$

$$\Rightarrow \quad R \approx e^{+\sqrt{\Lambda/3}\, t}$$

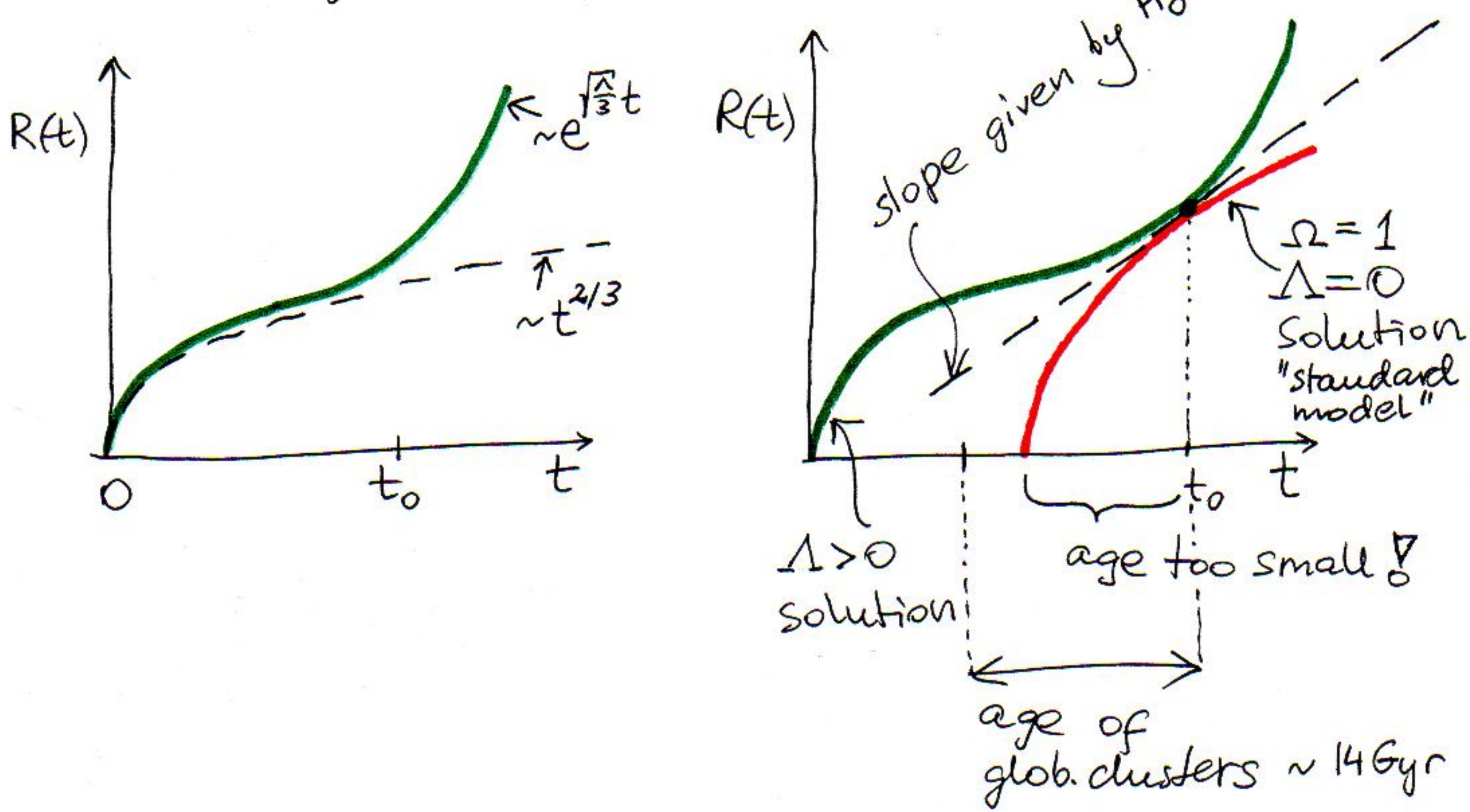



There were two such epochs (most probably)

① at $t \lesssim 10^{-31}$ s, before the ordinary elementary particles existed, the universe may have undergone a phase transition from a vacuum of higher energy to the present vacuum of lower energy. The released energy powered expansion by a factor $e^{100}$ or so. (This is thought to flatten spacetime to $k=0$.). This epoch is called INFLATION EPOCH

② not long ago (several Gyr ?) the mean density of the matter (ordinary and dark) dropped below a threshold beyond which the $\Lambda$-term in the eq. of motion dominates. This "dark energy" corresponding to $\Lambda = \text{const} > 0$ is probably a property of the vacuum, but its value is not understood.

How the $\Lambda > 0$ cures the age problem of stars in globular clusters:



# Expansion of the Universe