ASTB23 PREPARATION SET FOR THE FINAL EXAM (ASTROPH. OF GALAXIES AND UNIVERSE ONLY. FOR STELLAR ASTROPHYSICS, PLEASE SEE THE SOLVED ASSIGNMENTS FROM THE EARLIER PART OF THE COURSE AND TUTORIAL NOTES). MOST PROBLEMS BELOW ARE SOLVED.

Points in the square brackets give the relative weight with which the problems count toward the final score. This writtem problem part of the exam is worth **25% of the total course score**. Various physical constants are given at the end of this part. Please write legibly and explain what you are doing. Explicitly check units. Describe any obstacles you encounter - this may help you get a partial credit.

1 [0.5 p.] Triangulum Galaxy M33

Some claim that you can see it with the naked eye. Using telescope, its visible magnitude is 5.72 mag. Is the claim believable? Find out by researching the subject of a limiting magnitude we can see on the web.

2 [0.5 p.] Triangulum Galaxy M33

The distance to this disk galaxy is 840±105 kpc. The angular extent on the sky is 71 by 42 arcminutes. Use simple geometry arguments to estimate the radius of the galaxy in kpc.

HINT

For small angles, sin(x) and tan(x) are very nearly equal to x (in radians). One arcminute is 1/60 of one degree angle.

3 [0.5 p.] If you send a text message to somebody in M33

When will you or your descendants get an immediate answer? Use the distance data above.

SOLUTION

In approximately 6 million years.

4 [1 p.] How long

How many years would it take to travel to the nearest star with the maximum speed that the most efficient chemical fuel burning can provide? Find appropriate data, present the sources. Assume payload weight 5 tons (5000 kg) and fuel weight 100 tons. Assume all chemical energy of fuel converts to the kinetic energy of payload (not really achievable, but good assumption to get an upper limit of the speed achievable). Also assume that the rocket engine is ignited already far from the sun and Earth, i.e., don't bother with subtracting potential gravitational energy from the fuel energy.

HINT: If your calculation results in $V \ll 40$ km/s or $V \gg 100$ km/s, then it's probably wrong, since man-made probes called Pioneer and Voyager travel at the first speed given in space outside the solar system, and their speed is not orders of magnitude below the one achievable theoretically with chemical fuel.

5 [1 p.] Fraction of dark matter

Consider a model of a rotation curve of our Galaxy consisting of luminous and dark matter (DM). In the model, the DM is distributed according to density law $\rho(r) = \rho_0/(1+r^2/R_C^2)$, and for small radii $r \ll R_C$ (much less than the core radius of the DM halo, which is of order 15 kpc) the DM-induced velocity grows from 0 km/s at zero radius to 120 km/s at radius $r = 3$ kpc. Compute the constant value of density ρ of DM at $r \ll R_C$ in units of solar mases per cubic parsec. What is the mass (in solar masses) of DM contained in the volume of: (i) one cubic parsec and (ii) one cubic AU? Compare these values with the estimate of the mass of luminous matter (all stars and planets) in the same two volumes. For convenience, use this version of units of G: $G = 4.302e-3$ (km/s)^2 pc/$M_\odot$.

SOLUTION outline:
Compute the value of circular velocity due to dark matter at $r = 3$ kpc.

6 [1 p.] Hubble law

Assuming negligible random motion w.r.t. vacuum, how many Mpc away is a galaxy showing redshift $z = 0.75$, i.e. in whose spectrum all spectral lines have wavelengths $(1+z) = 1.75$ times longer than in laboratory? Assume Hubble constant equal to $H = 67$ km/s/Mpc.

SOLUTION OUTLINE

E.g., page 46, (1.26) and (1.34) in Spare+Gallagher textbook. z replaces the ratio V_r/c in the lowest order approximation (non-relativistic) Doppler formula, we can use this to infer appropriate V_r.

7 [2 p.] Star SO-2 near the Galactic black hole

See: http://www.galacticcenter.astro.ucla.edu/animations.html several times, and try to follow the motion of SO-2, or "source 2", a massive star that was observed to make more than one full orbit around

Sagitarius A*, the massive black hole in the central parsec of our Galaxy (8 kpc away from us).

Compute from a generalization of one of Kepler's laws the mass of the black hole, assuming for simplicity that the orbit of a star, which is much less massive than the black hole (BH), lies in the plane of the sky (is perpendicular to the line of sight). The scale bar on the animation will be helpful in finding the semi-major axis of the orbit base on known distance to Galactic Center; the text informs about the orbital period.

The generalization is needed because the mass of the BH is not one solar mass, but much larger, therefore you can't meaningfully expect the original Kepler's laws to hold in this situation. However, they still work, you just need to consider appropriate scaling of the periods with the central mass: start from the balance of forces in a circular orbit, as always.

SOLUTION

From force balance it is easy to conclude that $\Omega = \sqrt{GM/a^3}$, so $P = 2\pi/\Omega$ is equal to

$$(P/yr) = (M/M_\odot)^{-1/2}(a/AU)^{3/2}$$

It is straightforward to check that this formula reduces to Kepler's 3rd law in case of the Solar System.

According to the cited web page of the Galactic Center group at UCLA, $P = 16.1$ yr, and a is the distance to galactic center times the angular semi-major axis in radians. From the animation or from a wiki page on SO-2, it can be estimated that a (a half of long axis of ellipse) subtends about 0.12 arcsec, or 5.818e-7 rad. Hence, it is approximately 8 kpc * 5.818e-7 rad, or $a \approx 955$ AU.

Therefore,

$$M \approx (P/yr)^{-2}(a/AU)^3 M_\odot = 16.1^{-2} 955^3 \approx 3.4 \cdot 10^6 M_\odot.$$

8 [1.5p.] Why?

Look at arbitrary 3 nearby galaxies on wikipedia. Choose one with redshift below 0.05, and find its radial velocity predicted by Hubble's law, based on the distance (from wiki). Then compare the result with the actual radial velocity in the wiki article. Did you get a good agreement? Why?

9 [1.5p.] Density from rotation

In a spherically symmetric system, if you know one of these functions: $\rho(r)$, Φ, $F = -d\Phi/dr$, $V_c(r)$, then you should be able to derive all the other. Most often, the derivation will be connected with the force balance equation. Prove that $\rho(r)$ is derived from rotation curve using one differentiation.

10 [2.5p.] Relax

The two-body relaxation described in Lecture notes and textbook applies not only to real stellar systems, but also to their numerical simulations, e.g. of the structure of merger remnants - remaining after two galaxies merge. Simulations have their own struggle with the issue of accuracy, but let's assume for a moment that they are conducted very accurately.

Suppose an elliptical-like galaxy resulting from a merger has characteristic core radius $R \sim 10kpc$ and mass $M \sim 10^{12} M_{\odot}$. How long is the dynamical timescale (inverse of orbital angular speed on a circular orbit, at the mean radius, Ω^{-1})? How many dynamical timescales will occur in the next Hubble time (i.e., 10^{10} yr)?

What minimum number of stars should an accurate dynamical simulation follow, in order for the 2-body relaxation time of the simulated object to be longer than Hubble time?

Hint: $\Omega R = V$, and V can be estimated from Virial Theorem.

11 [5p.] A visit to Dr. Hubble

Brent Tully and J. Richard Fisher meet Edwin Hubble over tea. (Tully-Fisher empirical law is stated in the Lecture Notes and section 5.3.3 of optional texbook 3). They are interested in the Coma cluster, especially in one spiral galaxy to the right of the giant elliptical NGC 4881.

http://www.seds.org/hst/NGC4881.html

It's a 16-th magnitude object in the visible, having solar-type spectrum and colors at all wavelengths. It moves at a speed 7300 km/s away from us. They would very much like to know how far the galaxy is, so they can compute its luminosity in units of $L_{\odot}$. Their ultimate goal is to determine the maximum rotation speed V_{max} of the galaxy, based on an empirical law they have discovered and compare it with the results obtained with radio telescopes by their colleagues. They wonder what value of his constant Hubble would recommend. Hubble says he believes that $H \approx 60$ km/s/Mpc but has to leave the party early and cannot help them find V_{max}. Can you?

12 [4 p.] Maximum superluminal illusion

Solve problem 9.6 from the Sparke and Gallagher textbook in section 9.2.1.

Essentially: how fast can an object appear to us to move in the plane of the sky, if it moves at speed V through space? What is an optimal viewing angle θ? Also, how fast must the object move through space, in order to appear to move faster than light to a distant observer? All you need is our Lecture notes...

SOLUTION:

Using eq. 9.16 in the (2007 edition) textbook, we can first find the angle θ which maximizes the apparent velocity $V_{\rm obs}$ by the usual route (zero derivative over θ). This results in the condition $\cos\theta = V/c$. Then substituting this angle into 9.16 we find that maximum apparent speed is equal $V_m = \gamma V$, where $\gamma = 1/\sqrt{1-V^2/c^2}$ is known as the relativistic gamma factor. Finally, this maximum V_m is equal or larger c if $\gamma V \geq c$, or, after brief algebra, $V/c \geq 1/\sqrt{2}$.

13 [3 p.] Resonances in NGC0912

NGC0912 is a spiral galaxy, where an $m = 2$ pattern of arms is visible between about 4 kpc and 10.2 kpc from the center. Morphology of gas rings in the galaxy suggests that at 3 kpc there is an Inner Lindblad Resonance, and that the 10.2 radius is very close to the Corotation with the pattern. Astronomers suspect that the rotation curve is flat throughout the observed region. Does the dynamical theory support such a presumption? If yes, where would the Outer Lindblad Resonance be located (so that observers might try to obtain direct confirmation of the arrangement of resonances)?

HINT

ILR/OLR criteria in constant $V = \Omega R$ disks, written in terms is Ω's are (ILR – upper sign):

$$m(\Omega - \Omega_{CR}) = \pm\sqrt{2}\Omega$$

The corotation happens when $\Omega = \Omega_{CR}$.

14 [5 p.] Cluster M4

M4 is a globular cluster in our Galaxy. Measurement of radial velocities of 29 stars in the cluster gave the following root-mean square (dispersion) of radial velocities relative to the mean velocity of the whole cluster: $\sqrt{< v_r^2 >} = 70.7 \pm 0.8$ km/s (an actual measurement published in 1991 by Rastorguev and Samus).

The light and thus the density distribution (assuming constant M/L ratio) are known in this cluster. Using fig. 3.7 from the 2007 edition of Sparke and Gallagher, reproduced in the lectures notes (find it!), as well as the virial theorem, or an appropriate theoretical potential-density pair you choose (with justification!), please *estimate* the mass of the core of the cluster M4 (an order of magnitude estimate is ok, but any additonal precision would be appreciated. However, don't strive for precision higher than two accurate digits!)

Estimate the mean distance between the stars in the core of M4.

Estimate the crossing (or dynamical) time scale t_{cross}, i.e., the radius divide by the typical speed, and the relaxation time t_{relax} of M4. Assuming that it takes 20 relaxation time for a cluster to approach the core collapse stage, is M4 a plausible candidate for a star cluster with a massive black hole inside?

SOLUTION

The main results that we can read off the figure of brightness is that the there is a flat-density core of radius about 0.3 pc, followed by a rapid (perhaps r^{-2} (?) fall-off) of the density. Several potential-density pairs could be used, but the simplest is the estimate based on the virial theorem, stating that minus potential energy is equal to twice the total kinetic energy of all stars. Neglecting constants of order unity, the following scaling must hold (it even follows from dimensional analysis!)

$$GM_c/r_0 \approx \sigma^2$$

where M_c is the core mass (characteristic mass within the core radius r_0), and σ is the 3-D velocity dispersion in M4.

We don't really know anything about the proper motions of stars, but for the purpose of estimation, we can assume that the velocity distribution is isotropic, and thus σ is $\sqrt{3}$ times larger than $\sqrt{< v_r^2 >}$, because $\sigma^2 = < V_r^2 > + < V_\phi^2 > + < V_\theta^2 >$. So, in the end we conclude that $\sigma \sim 1.7 \cdot 70$ km/s $= 120$ km/s.

From the virial equation divided side by side by an equation for the circular velocity of Earth (30 km/s) around the sun, we get this equality

$$M_c \approx (\sigma/30\text{km/s})^2 (r_0/1\text{AU}) M_\odot \sim 10^6 M_\odot$$

Incidentally, the quadratic dependence of the result on σ explains why we should pay a little more attention to the accuracy of estimation of that parameter, as opposed to r_0 – the relative error in σ produces twice as large relative error of mass than due to uncertainty in r_0. In our case, however, the basic source of error are the missing factors of order unity in the virial-like equation we're using. They could be specified more exactly using, for instance, King model of density distribution, but the formulation of the problem does not require such sophistication.

The rest of the problem can easily be done using lecture 9. One can assume that all stars are 1 $M_\odot$ stars. For instance, the mean distance is obtained by taking the 1/3 power of the volume of the core per one star.

However, caution must be exercised not to use the estimate of relaxation time evaluated in the lecture (10^{10} yr) because that pertains only to the cluster as a whole and we are interested in its small core. Therefore, expect a shorter relaxation time.

15 [1p.] Arm passages

If our Galaxy is a constant-speed with $V = 220$ km/s, has two main arms, and the corotation radius of spiral pattern is at $R_{CR} = 10.2$ kpc, then how often does a star in a nearly circular orbit at $R \approx 8$ kpc pass through spiral arms?

HINT: $V = \Omega R$, spiral arms form a rigidly rotating pattern (Ω_p = const).

16 [3p.] Does a Supermassive Black Hole chew stars?

We're interested in predicting whether or not a SMBH can swallow a star whole. Tidal forces around a black hole are very large; they depend on the mass M of a black hole. We'd like to know for which masses M sun-like stars (1 $M_\odot$ mass) are torn apart by tidal forces before reaching a distance equal to one Schwarzschild radius from the center.

If a star "bursts" under the tidal forces of a SMBH, but misses its "surface" and a gas cloud is flung outwards, what is the highest speed of gas you except to detect spectroscopically?

HINTS:

Assume that Newtonian dynamics is valid down to one Schwarzschild radius from the black hole. Assume that this radius is much larger than the star's radius. Compute when tidal force (difference of gravitational acceleration on the near side and center of a star) is stronger than gravity keeping the star together, neglecting any orbital/centrifugal effects - only gravitational forces.

17 Some possibly useful constants

If you don't have a calculator, you must state that in your solution and provide your calculation rounded off to 2 significant figures (numerical error less than $\sim$10% will not lower your score.) Otherwise, at least three significant figures should be carried.

$c = 2.99792 \cdot 10^8$ m/s, $= 2.99792 \cdot 10^{10}$ cm/s (speed of light)
$G = 6.67259 \cdot 10^{-11}$ m^3 kg^{-1} s^{-2} = $6.67259 \cdot 10^{-8}$ cm^3 g^{-1} s^{-2} (gravitational const)
$G = 4.302 \cdot 10^{-3}$ pc (km/s)$^2 M_\odot^{-1}$. (useful in galactic calculations)
$k = 1.3807 \cdot 10^{-23}$ J/K $= 1.3807 \cdot 10^{-16}$ erg/K (Boltzmann)
$m_H = 1.66054 \cdot 10^{-27}$ kg $= 1.66054 \cdot 10^{-24}$ g (hydrogen mass)
$M_\odot = 1.9891 \cdot 10^{30}$ kg = $1.9891 \cdot 10^{33}$ g (solar mass)
$R_\odot = 6.9598 \cdot 10^8$ m $= 6.9598 \cdot 10^{10}$ cm (solar radius)
$L_\odot = 3.8515 \cdot 10^{26}$ J/s $= 3.8515 \cdot 10^{33}$ erg/s (solar luminosity)
1 AU = $1.496 \cdot 10^{11}$ m = $1.496 \cdot 10^{13}$ cm
1 pc = 206256 AU = $3.09 \cdot 10^{18}$ cm = $3.09 \cdot 10^{16}$ m
1 yr = $3.1558 \cdot 10^7$ s
1 km/s $\simeq$ 1 pc/Myr

distance to the center of the Galaxy (old IAU recomm.) = 8.5 kpc, newer value 8 kpc (use that one!)