R. Hooke and I. Newton:

- The contributions of Hooke and his conflict with Newton
- Who really pushed Newton to develop & publish his greatest work “Phil. Naturalis Principia Mathematica”?
- Proof of constancy of areal speed
- How do we know what’s in the Galactic Center
- Cometary orbits such as Halley’s comet
- What did Newton do most of the time
- The Scientific Method

In L13 we will deal with Newton’s physics after Newton, part I: Light and its spectrum

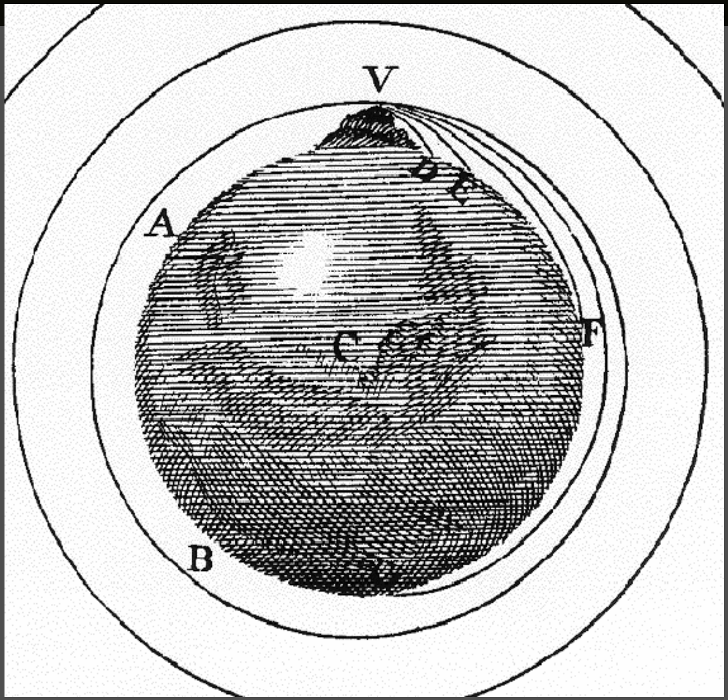
Gravity and Orbs

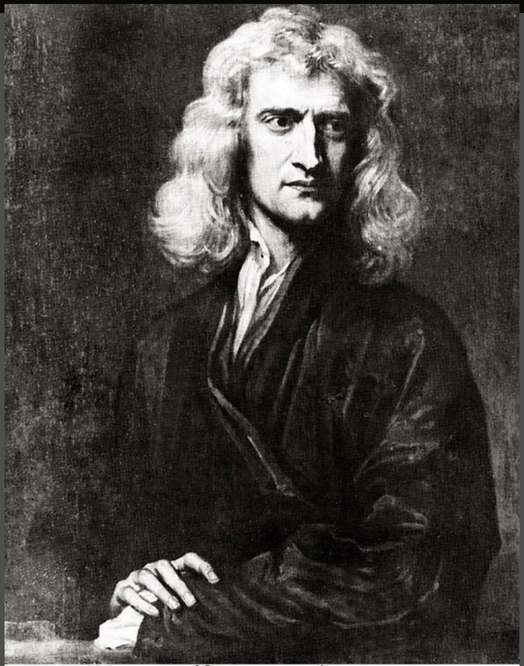
The problem of the place of Earth and humanity in the universe was resolved by the Copernican Revolution.

However, the problem of planetary motion was only partly solved by Kepler’s laws. What gave rise to these laws was unknown.

Although Galileo made extraordinary progress in observations and formulated what is now known as the first law of Newton’s dynamics, he was not able to relate his discoveries about motion to the heavens. That final step was taken in the second half of the 17th century by Isaac Newton and his contemporaries.

Isaac Newton (1643-1727)





Newton portrait

The publication of Isaac Newton’s work in his book *Philosophiae Naturalis Principia Mathematica* in 1687 placed the fields of physics and astronomy on a new firm base.

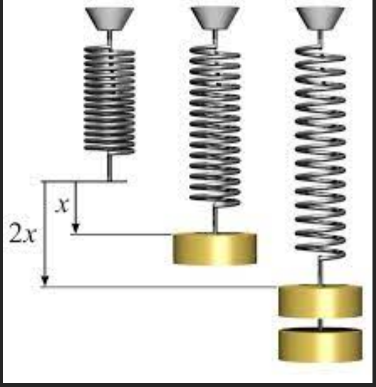
- What made Newton write and publish *Principia* were 2 things: a helpful friend, astronomer Edmond Halley, and rivalry with older colleague & arch-enemy Robert Hooke.

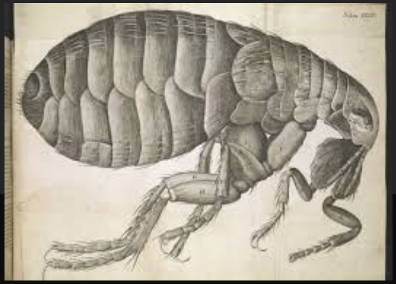
Isaac Newton and Robert Hooke

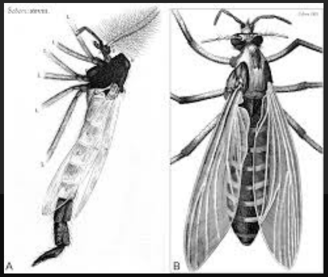
- Rivalry was a norm for Newton. He was slow to publish most of his discoveries, which later led to conflict with those who made them independently. Most famously, G. W. Leibniz discovered & published calculus, and R. Hooke formulated and published the idea of **universal gravitation** ***before*** Newton, in 1670 (printed in 1674).

Robert Hooke (1635-1703):

- Physicist and inventor. Together with Christopher Wren surveyed London and helped rebuild dozens of churches burned in the great fire of 1666.
- Discovered Hooke's law of ideal spring.
- First identified and named cells in plants using his improved design of microscope.







Robert Hooke

- Hooke made first detailed drawings of fossils

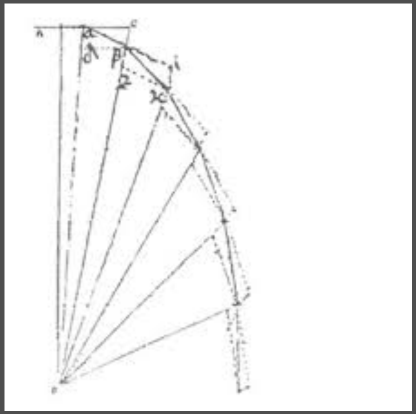
- He discovered the Great Red Spot on Jupiter by perfecting the telescope.

- Hooke was a secretary of Royal Soc. in London, and the first physicist paid for performing experiments every week (checking results for presentation to the Royal Society meetings). He was older than Newton, and when after his death Newton became the Secretary, he removed (destroyed) all Hooke's instruments & all paintings showing Hooke..

Due to this we are now uncertain about the authenticity of Hooke's portraits.

London's coffee-houses, C. Wren, E. Halley and R. Hooke & an idea hanging in the air: Universal Gravity

- Hooke suspected that universal gravitation acts between all massive bodies, whose strength follows the inverse square law of the form $F = const / r^2$. He constructed funnels for balls, which simulated the motion in a gravity field, knew that a pendulum or a funnel simulates force $F = const * r$.
- Hooke also knew how the orbits follow from "compounding" the inertial straight-line motion and gravity, which bends the trajectory by providing radial increments to velocity.



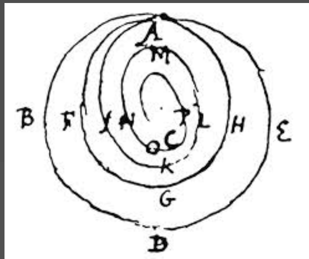
And he told Newton about all this!

- For those interested in history of science:

M. Nauenberg "Robert Hooke's Seminal Contributions to Orbital Dynamics"

https://planets.utsc.utoronto.ca/~pawel/ASTB03/hooks-contrib.pdf

At that time, some of Newton's ideas on that problem were apparently incorrect (see below)



Isaac Newton

- Newton was a quiet child from a farming family in Woolsthorpe in Lincolnshire, England.
- His progress at school was impressive & he seemed so inept at agriculture that his parents decided that Isaac should become a priest, not a farmer.
  - His uncle financed his education at Trinity College, Cambridge – where he studied mathematics and physics
  - In 1665, bubonic plague epidemic swept through England, and the colleges were closed. Obligatory quarantine was introduced and enforced. During 1665 and 1666, Newton spent 1.5 years back home in the village of Woolsthorpe.
  - Yet he didn't sit out the pandemic in the basement, binge-watch Netflix series. He was thinking and studying..

Isaac Newton

Newton claimed that it was during the 'miraculous year' of 1665 (*annus mirabilis*) that he made most of his scientific discoveries.

Today historians agree that this timing of discoveries was distorted by Newton to make priority claims.

- The anecdote of an apple is apocryphal. It was told by Newton himself but sounds suspicious.
- Young Newton studied optics, and started quarreling with Hooke about the nature of light: according to him light is made of particles emitted in bursts; according to Robert Hooke and Dutch physicist Christian Huygens light is strictly a wave phenomenon.
- Newton developed laws of motion, perhaps *started* thinking about gravity (although there is not a slightest written trace of it before 1679), and partly worked out differential calculus

Isaac Newton

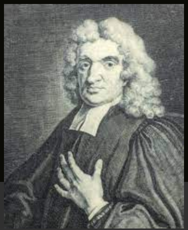
- By 1670 his talents in math were recognized by Isaac Barrow, professor of mathematics. Upon retirement, Barrow recommended the 27-yr old Newton as his replacement (Lucasian Professorship in Mathematics)

- Newton started accusing Leibniz of having stolen his ideas about calculus. Newton’s attacks were fairly vicious, they actually affected Leibniz’s life very negatively

- At the age of 35 (in 1678), Newton suffered a mental breakdown. He became a solitary man, distanced himself from others





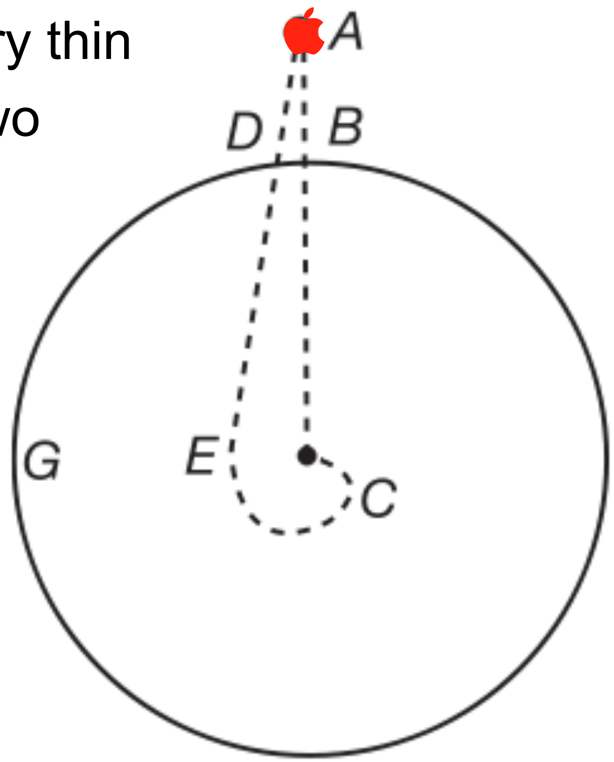


1680s

- Astronomer Edmond Halley, architect Christopher Wren, mathematician John Wallis, and Robert Hooke had meetings and lively discussions at coffee-houses of London after Society's brief weekly meetings.
- They were interested in physical causes of elliptic orbits discovered by Kepler and later confirmed in comets by Halley, although astronomers had problems tracing comets near the sun (Astronomer Royal John Flamsteed claimed that Halley's comet was stopped by the sun and repelled).
- Hooke had an exchange of letters 1679-1680 about it with Newton, which probably gave Newton both a motivation to study orbits (*which he admitted later*), as well as
- crucial ideas, like compound motion, i.e. combining inertia and gravity to produce trajectory in small steps, and the idea of Universal Gravitation (*all of which Newton later denied*)

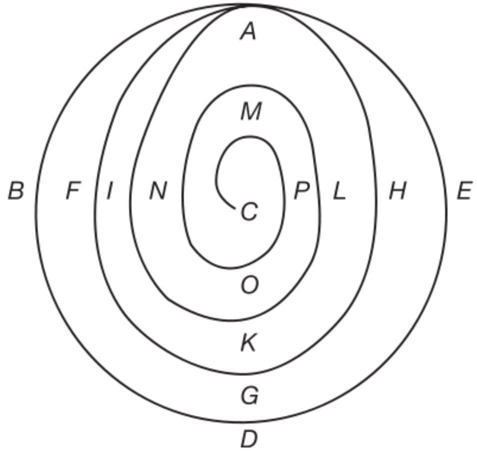
Conflict between Newton and Hooke, with gravity and orbits in the background

Instead of responding to the offer by Hooke to work together on the gravity problem, Newton suggested another problem: the proof of rotation of Earth by measuring the expected deviation of free fall from the vertical line. [You have to imagine that a very thin empty space separates two halves of the Earth, to allow this motion.]



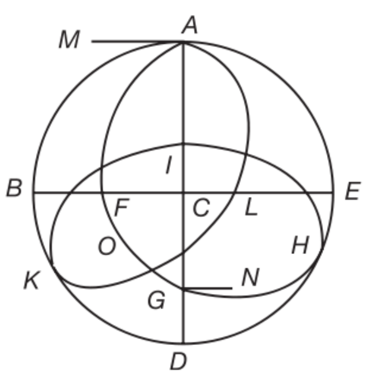
Well, this trajectory is not very good. Hooke was quick to point it out...

10. Newton's suggested path for an object dropped vertically from above the Earth's surface. The object's path is assumed to continue inside the Earth's surface as the Earth revolves around C anticlockwise (i.e. BDG).



11. Hooke in turn argued that the body described by Newton would revolve in the ellipsoid AFGHA, unless it experienced some resistance, in which case it would descend close to the centre of the Earth

Incidentally, the bottom figure inaccurately but essentially correctly sketches the path of a ball rolling in an inverted cone with opening angle 60°. Hooke did many experiments like this, perhaps Newton too!



12. Newton's response, with gravity and 'centrifugal force' alternately overpowering each other

85

Even as late as in 1680s Newton did not have a full understanding of orbits and gravity.
Neither did Hooke, but his under-standing was sometimes more correct.

This suggests that Newton did not discover Universal Gravity in 1665-1666 as he later claimed, but that he worked it out in response to original hypotheses of his contemporaries.

As they exchanged seemingly polite letters and quarreled about trajectories in person and in letters, there was intense personal dislike behind this.

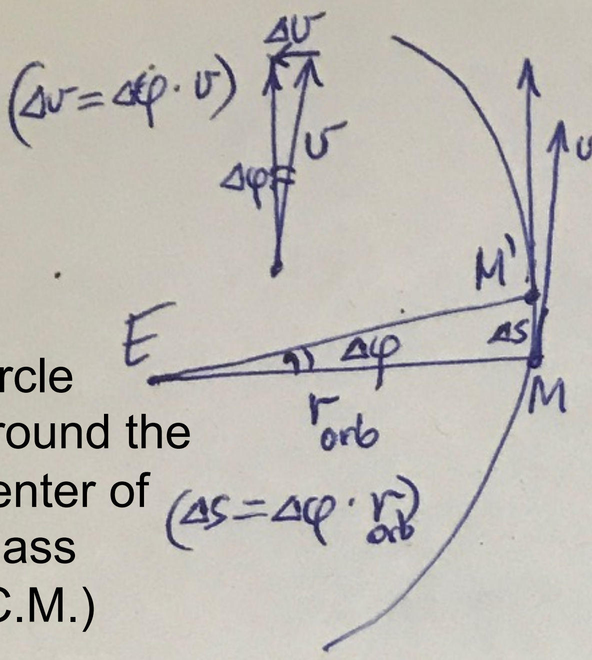
$F \sim 1/r^2$ law in times of Newton, verified for circular orbits by comparing the Moon & apple

- For demonstration, below I'll do the computation that Newton actually performed more than once, improving the approach and quality of input data. (You won't be asked to reproduce the math or details of this proof.)
- We know the gravity's acceleration that applies to apples:
  $g = 9.81$ m/s^2
- We can assume that the Moon is in a circular orbit, and that it is subject to the *force/acceleration* of the same (Earth's) gravity as apple, which somehow (perhaps as $1/d^2$) falls with distance d from the center of the Earth.

- Let us use modern astronomical data (Newton knew the approx. distance to the Moon but surprisingly imprecisely the size of Earth, so he achieved a disappointingly low accuracy of the result ~10%

$1/r^2$ law: The apple and the Moon

First, let's find the centripetal acceleration in a circular orbit

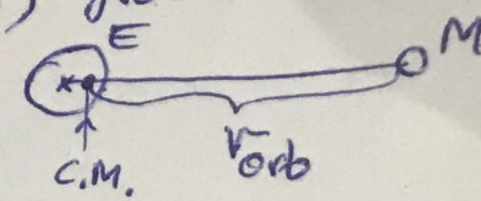


circle around the center of mass (C.M.)

In time Δt the Moon moves from M to M', dist. $\cong \Delta\varphi \cdot r_{orb} \equiv \Delta S$

$\Delta\varphi$ – some tiny angle

r_{orb} – orbital distance of the Moon (Moon's center) from the center of mass:



$r_{orb} = 379712$ km

$d_{EM} = 384400$ km = dist. Earth – Moon

$r_{orb} = \frac{81}{82} \cdot d_{EM}$ (mass ratio ~1:81)

Acceleration of Moon (centripetal):

$$a_r = \frac{\Delta v}{\Delta t} = \frac{\Delta\varphi \cdot v}{(\Delta S / v)} = \frac{v^2}{r_{orb}}$$

one way to compute Moon's acceleration

only for those interested in details

$r_{orb} = 379\cancel{432}$ km

$d_{EM} = 384400$ km = dist. Earth – Moon

$r_{orb} = \frac{81}{82} \cdot d_{EM}$ (mass ratio ~1:81)

another way to compute Moon's acceleration from the putative formula

Gravitational acceleration : $a_{gr} = g\left(\frac{R}{d_{EM}}\right)^{\alpha}$

Let us check if $\alpha = 2$

where $R = 6371$ km is Earth's radius ; $g = 9.81 \frac{m}{s^2}$

$$a_r = a_{gr} \Rightarrow \frac{v^2}{r_{orb}} = g\left(\frac{R}{d_{EM}}\right)^{\alpha}$$

What's v? Moons velocity is $\frac{2\pi r_{orb}}{P} = v$

$P = 27.3216$ days = siderial ~~per~~ month

Force balance reads:

$$\left(\frac{2\pi}{P}\right)^2 r_{orb} = g\left(\frac{R}{d_{EM}}\right)^{\alpha} \Rightarrow \alpha = \frac{\ln\left\{\left(\frac{2\pi}{P}\right)^2\left(\frac{r_{orb}}{g}\right)\right\}}{\ln(R/d_{EM})}$$

Substituting values, $\boxed{\alpha = 2.0004}$ Success!

$\therefore$ same force moves apple & Moon, ~ Inverse-squared distance

only for those interested in details

Isaac Newton – the story of *Principia*

- In 1684 Wren offered a reward for showing that

  $1/r^2$ force → elliptical orbit (1st Kepler's law)

Robert Hooke said he has a demonstration of this but said that he *can't find it* !

He said he'll provide it *next week*… but hasn't done it next week, or in the weeks after that.

(In fact, Hooke did have a demonstration explained to his friends in letters in 1685, but it was based on mechanical analogs, i.e. mechanical experiments with funnels and balls, not on formal mathematical physics.)

Isaac Newton – the story of *Principia*

Halley (1656-1742)

Edmond (or Edmund) Halley traveled in 1684 to Cambridge, to ask Newton the same question. Newton also claimed that he had a proof of elliptical orbits following from an inverse-square law of gravity.

> "Sir Isaac looked among his papers but *could not find it*, but he promised him to renew it and then to send it him…"

- Halley waited. Just as he started doubting in both Hooke's and Newton's claims… Newton made good on his promise! He wrote a brief account *De motu corporum in gyrum* (1684). After much expansion that took 18 months, the world received Newton's

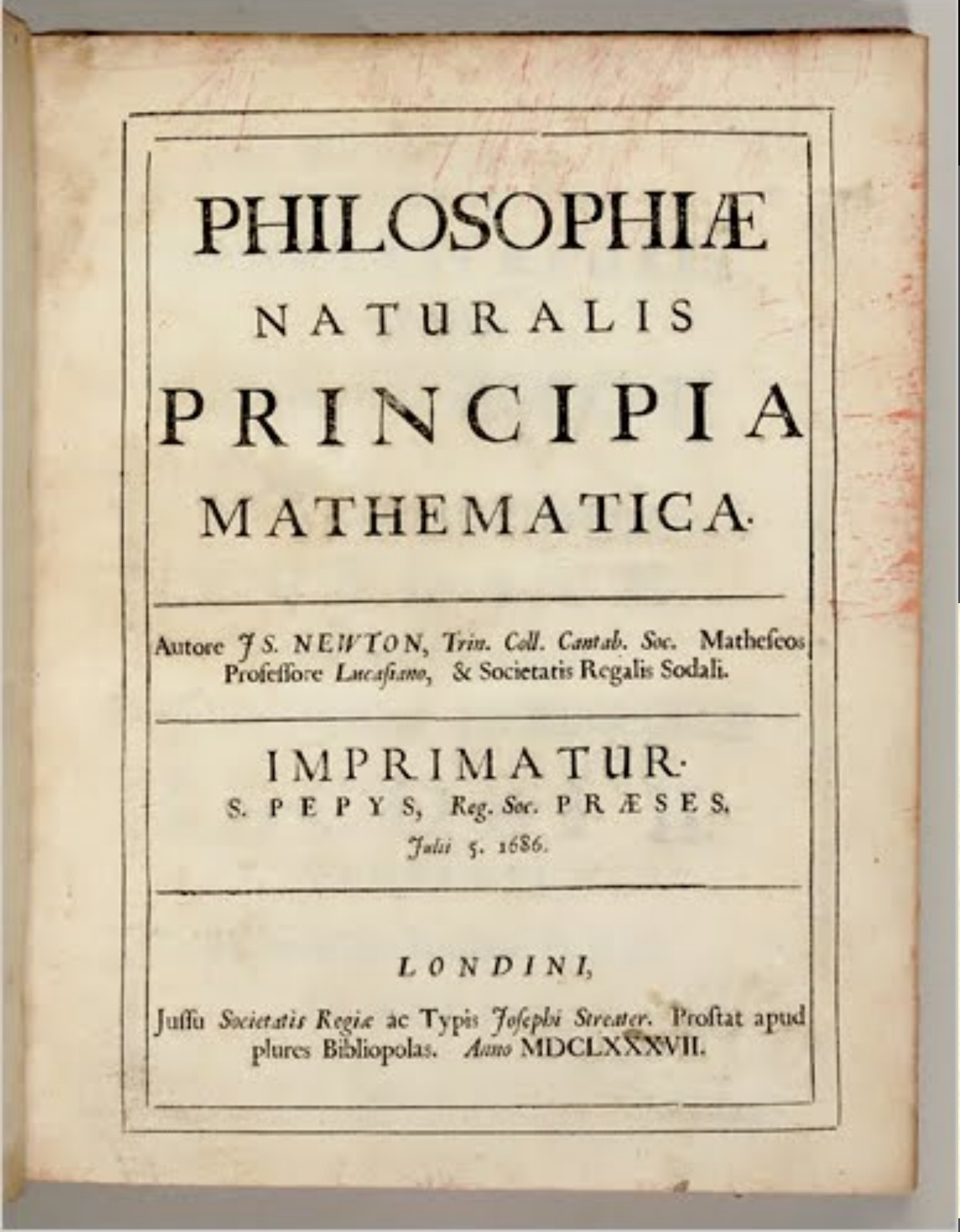
Philosophiae Naturalis Principia Mathematica (1687) means:

Mathematical Principles of Natural Philosophy

known in short as *Principia*.

- Royal Society was low on cash, the author was stingy, so the publication was edited and paid for by Edmond Halley himself.
- Newton even called it "your book" in private correspondence with Halley

MDCLXXXVII = 1687
:-/


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

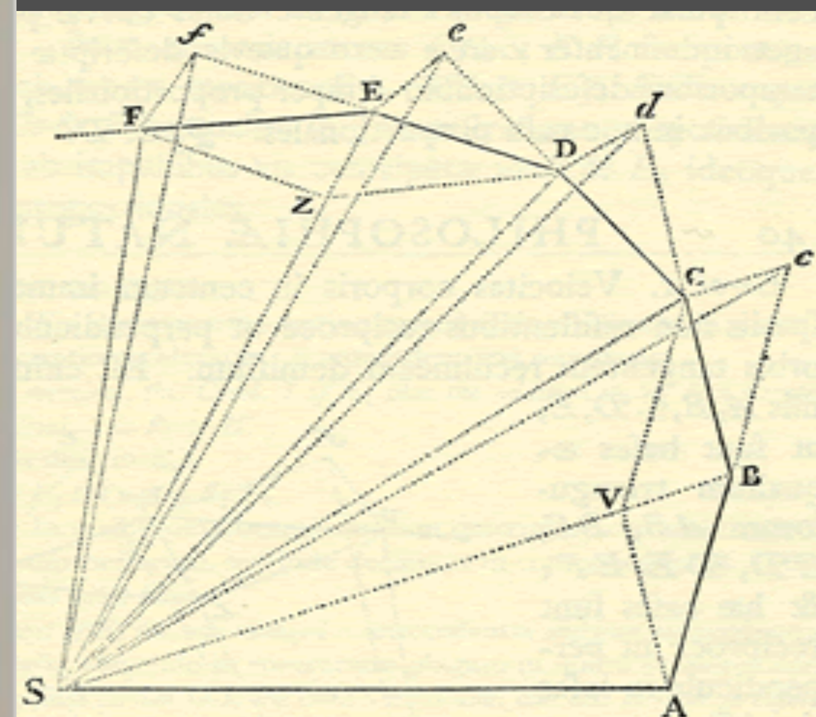
Autore J S. NEWTON, *Trin. Coll. Cantab. Soc.* Matheſeos Profeſſore *Lucaſiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

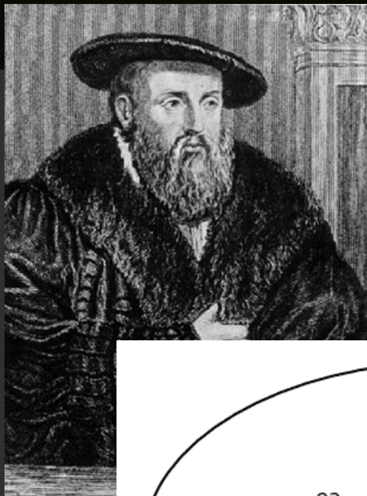
LONDINI,

Juſſu *Societatis Regiæ* ac Typis *Joſephi Streater.* Proſtat apud plures Bibliopolas. *Anno* MDCLXXXVII.

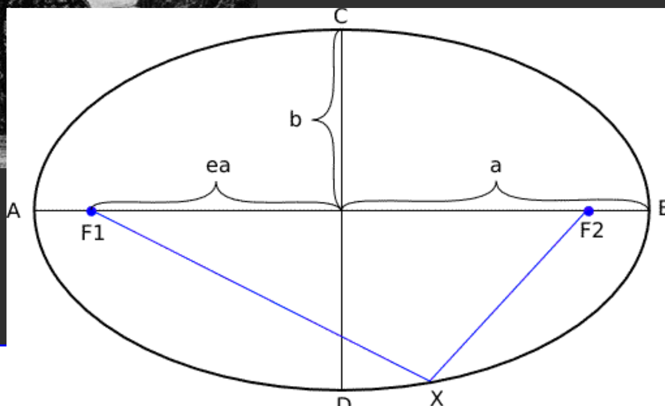
Newton's Principia today is considered the most important book in physical sciences

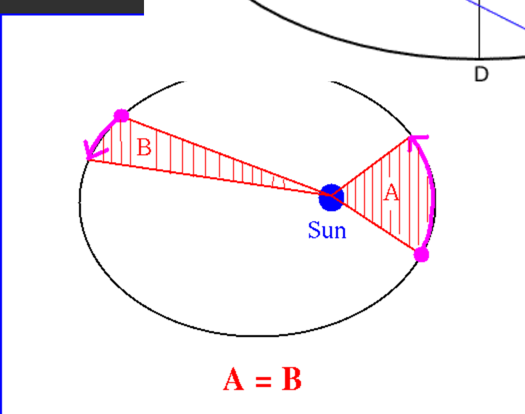


Implication needed to be shown both ways



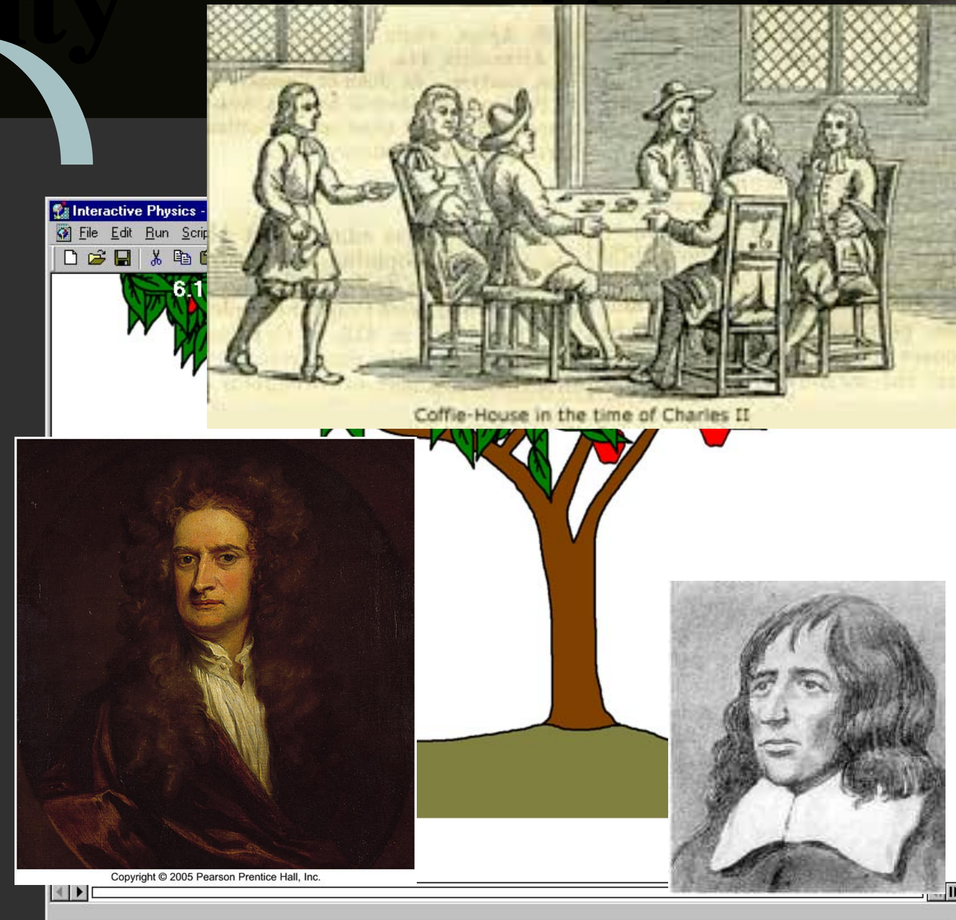
J. Kepler (1571-1630)





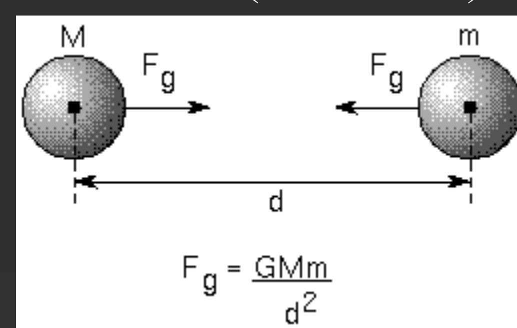
III

$$\frac{P^2}{a^3} = \text{constant}$$



I. Newton (1643-1727)

R. Hook (1635-1703)



Isaac Newton's dynamics

- Newton was able to formulate three universal laws of motion that made it possible to predict very accurately how a body would move if the forces acting on it were known.

- Newton's first law of motion *[actually formulated earlier by Hooke on the basis of Galileo's idea!]* states that an object remains at rest or moves at constant velocity unless a net force acts to change its speed or direction.

  - Example: When a car is at rest or travelling at a constant speed *and* direction, the forces exerted by the wheels to drive it forward are balanced by the airdrag and other drag forces in such a way that the net (total) force is zero. Any non-zero force accelerates the car. For instance, when the car turns, there is a net force doing this (pavement traction force that bends the trajectory).

2nd law of dynamics, found by Galileo (in case of uniform acceleration) was reformulated by Newton

- If the mass m of the object does not change, then the vector of acceleration a (rate of change of velocity – its direction and/or value, i.e. speed) is proportional to the force F that is exerted, m being the proportionality constant.

$F = m\,a$ or $a = F/m$

- An example of acceleration: All falling objects on Earth have a constant acceleration $a = g$ toward the center of the Earth. Why? It's because mass m and weight F of an object (force of gravity) are proportional.
- For example: split an apple in half and each part of it will have ½ the mass and ½ weight of the full apple. F/m is the same for all bodies, and equals $g=9.81\ m/s^2$.

Isaac Newton

The acceleration due to Earth's gravity, g, is 9.81 meters per second per second, $g = 9.81\ m/s^2$

It is the same for all objects. Their speed when dropped from rest will initially increase ~10 m/s every second, until the air drag balances the weight.

g varies slightly with the distance from the center of Earth. In a cruising airliner (10 km above ocean) you lose 0.3% of weight: $\Delta W / W = -1 / 319$, which is 0.25kg if your mass is 80 kg.

- Watch Galileo's experiment performed on the Moon

(so no air drag!) free-fall acceleration is about 7 times smaller than on Earth but, like on Earth, is the same for all kinds of bodies of various masses and shapes.

http://www.youtube.com/watch?v=GdHlWp9k_sY&feature=fvsr

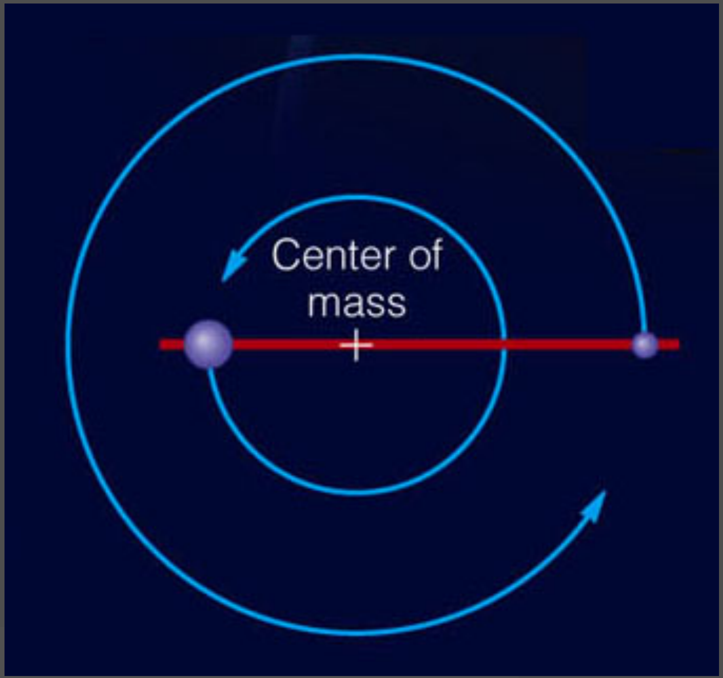
Orbital Motion

- Objects orbiting each other actually revolve around their mutual center of mass (C.M.)

Newton assumed that there is a center of the universe in our solar system, which is at rest (but never coincides with sun's center)

"Hypoth. IV. Centrum Systematis Mundani quiescere"

"Hypoth. IV. Center of the System of the World rests [is quiescent]"



- *Today we can detect and measure the mass of unseen, faint planets by looking at their much brighter stars, tugged by their planets. Their C.M rests.*

The Universal Theory of Gravitation

- Hooke, Newton and others realized that some force must pull the Moon toward Earth's center.
  - If there were no such force altering the Moon's motion, it would continue moving in a straight line and leave Earth forever.
- Hooke's and Newton's insight was to recognize that the force that holds the Moon in orbit is the same as the force that makes apples fall from trees.
- Remember Kepler? It took some time for scientists to realize that the force moving planets in their orbs is not a magnetic-type force producing whirls & pushing bodies ALONG their orbits (an idea proposed by R. Descartes and liked by Kepler), but that the force is directed always toward the sun or other central body.
- Below you will see how elegantly Newton tied this fact to Kepler's 2nd law

The Universal Theory of Gravitation

Gravitational force of attraction between two objects depends on the product of the masses of the two objects.

- for example, doubling one of the masses (m or M) would double the gravitational force F, and doubling both masses would quadruple the force.

$F = G M m / r^2$ strength of force exerted by mass M on m

where G is a gravitational constant (measured in experiment).

- Gravity is universal. So, for instance:
  - Your mass affects Neptune, the galaxy M31, and every atom in the universe.
  - Their masses, in turn, affect you – although not much, because they are so far away.
  - This is very important and is expressed as a symmetric product (M*m):

If the Earth pulls the Moon, the Moon equally pulls the Earth, which we witness as tidal waves in the oceans/seas and much smaller, in rocks.

Orbital Motion

Newton's physics makes it possible to both understand why and *how* the Moon orbits Earth, the planets orbit the Sun, and why Kepler's laws work even in the furthest galaxies.

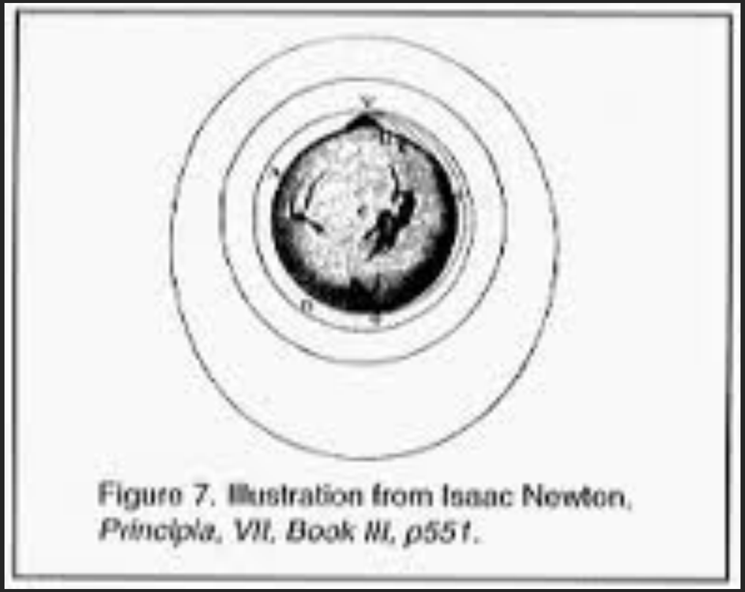
- It explains why the pendulum swings, why galaxies have spiral waves, and why they sometimes interact and merge.
- No convincing indication so far that the mM/r^2 law is NOT valid somewhere in the universe.

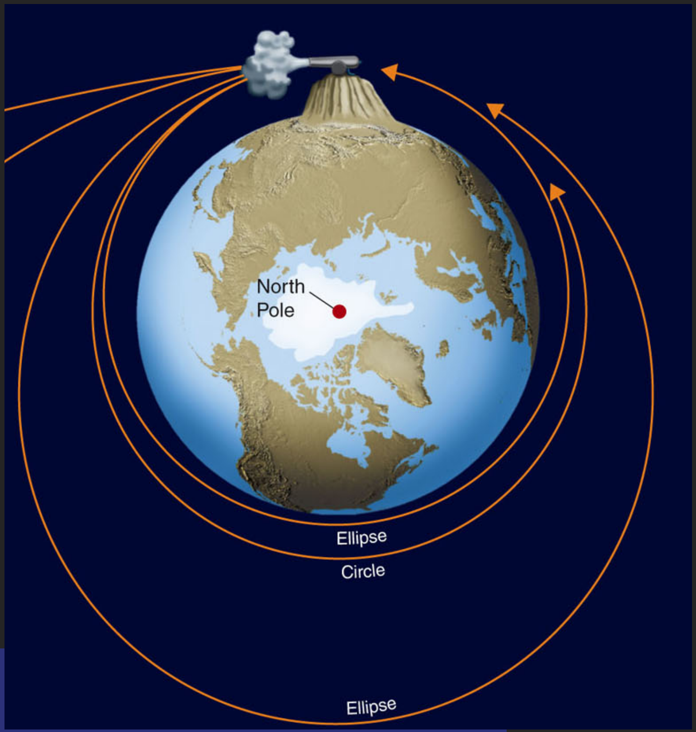
Orbital Motion around a point mass

Orbiting a planet is possible if you give an object enough speed

This is a type of illustration often found in books; it was first drawn by Newton.

An object in a stable orbit continuously falls toward the center of the planet but misses it because of its horizontal velocity and the curvature of the surface of the planet.





Orbital Motion: Newton's vs. Kepler's laws

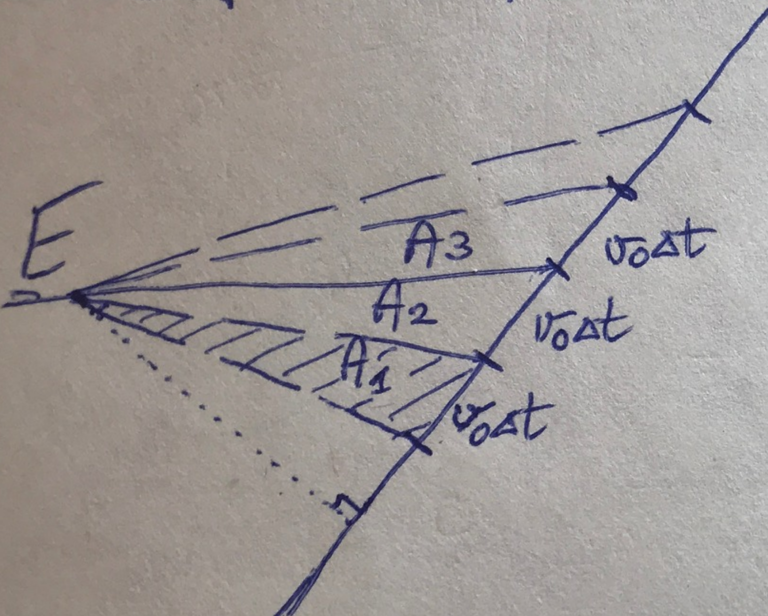
- Newton's laws of motion and theory of universal gravitation enabled Newton to explain mathematically all Kepler's laws of planetary motion. More precisely:
- Kepler's 1st law (of ellipses) is a direct consequence of the inverse-square law of gravitation. Any other type of force produces different trajectories (with a single exception of ideal spring force, which also produces elliptical paths, though differently centered)

- *The Area Law (2nd Kepler's law), however, is not because of the very specific $1/r^2$ force. It holds for any force that is central, i.e. points toward the center of attraction.*

  (Newton invented a word centripetal for such forces)
- *In modern physics, we identify 2nd law with angular momentum conservation*

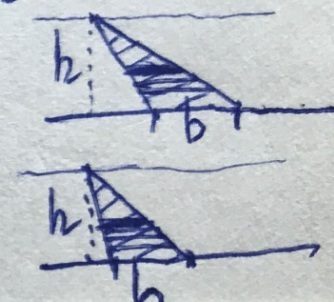
Newton's proof of areal speed constancy

$\boxed{\text{CASE 1}}$: no forces



$A_1 = A_2 = A_3 = \dots$

because $A = \frac{1}{2} h \cdot b = \text{const}$
if the top of
the triange shifts
$\parallel$ base

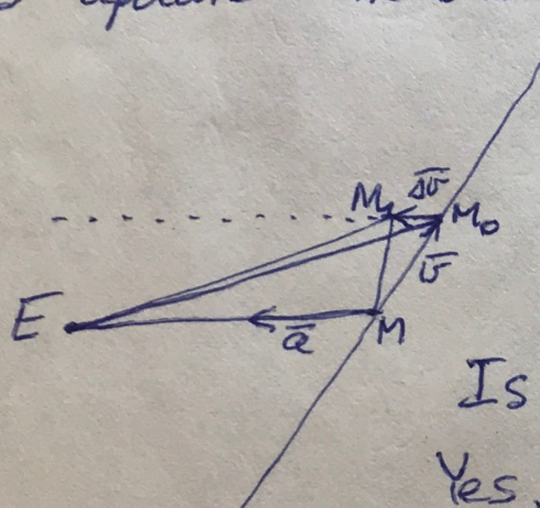


Newton's geometric proof of Kepler's 2nd law, if the force always points toward the center of attraction (is centripetal)

6

$\boxed{\text{CASE 2}}$: centripetal force

Denote acceleration as $\bar{a}$. Consider time interval Δt as update interval of velocity (later you can let $\Delta t \to 0$)



velocity gets a kick **$v \rightarrow v + \Delta v$** & the moon moves to M_1 not M_0

$$\overline{\Delta v} = \bar{a}\,\Delta t$$

toward E.

Is Area(EMM_0) = Area(EMM_1) ?

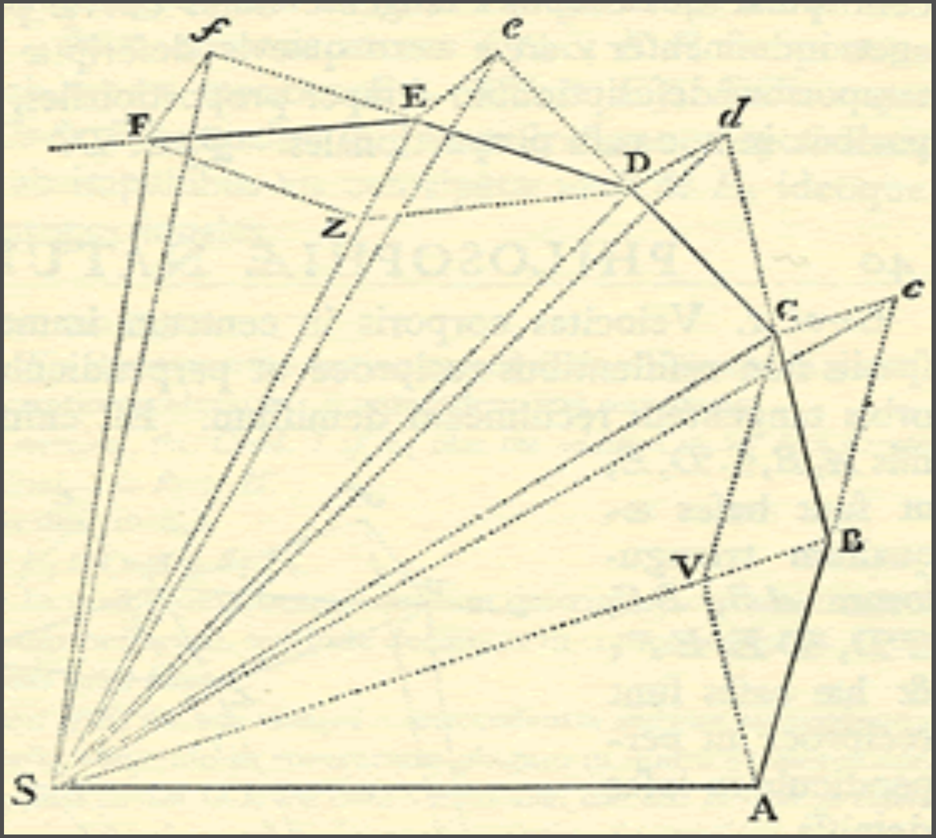
Yes, because base EM is the same and the $M_0 \rightarrow M_1$ shift is $\parallel$ base.

So the areal speed is constant.

trajectory consists of ∞ number of infinitesimal triangles.

Newton's geometric proof of Kepler's 2nd law, if the force points toward the center of attraction was beautifully simple

Philosophiae Naturalis Principia Mathematica (1687)



The proof was based on the compound motion idea of Hooke, and elementary Euclidean geometry of triangles

In today's mechanics we prefer to talk about angular momentum $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{v}$ = const., which is a vector of length = 2 * areal speed

Proof: the cross product of two vectors, $\boldsymbol{r}$ = SA and ($\boldsymbol{v}\,\Delta t$) = AB, equals twice the area "swept" in time Δt, or area SAB.

Dividing this area by Δt gives areal speed, numerically equal to the length of $\boldsymbol{L}/2$, q.e.d.

Orbital Motion – derivation of 3rd Kepler's law

- Newton was also able to combine his laws of motion with the law of gravitation to derive the 3rd Kepler's law: a relationship between a planet's orbital period (P) and the average distance from the Sun (a):

$$P^2 = (4\pi^2/G)\, a^3 / M$$

(P = period, a = distance, M = star's mass; π, G = constants)

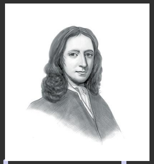
- This was earlier done qualitatively, only for *circular* orbits, by Hooke and Halley. Newton generalized the proof to *elliptic* orbits.

- Moreover, this more general form of 3rd Kepler's law with explicit mention of mass M is applicable to thousands of solar systems we have recently found, and studied with its help.

Orbital Motion – Newton’s derivation of Kepler’s laws

This is the power of Newton’s work. He was able to explain all the patterns of planetary motion observed by Kepler as three laws by using very simple and universal rules. In addition, this allows *generalization* to other planetary systems, systems of moons around a planet etc.

Gravity is also the key to understanding a huge number of other things like Earth’s ocean tides. Newton contributed to the theory of tides, choosing the geographers’ accounts somewhat selectively (those which supported the theory were preferred).

But unlike Halley,  who became the captain of the first science research ship and traveled down to Antarctica to map the magnetic field,

Newton never traveled far enough from his home to see the ocean or the tides, the correct theory of which he created.

Many of Isaac Newton’s ideas were incredibly deep and prescient.

Newton has shown the equivalence in both directions

Keplers laws ←→ $F \sim 1/r^2$ gravity p. 405 of *Principia*:

Prop. II. Theor. II.

Vires, quibus Planetæ primarii perpetuo retrahuntur à motibus rectilineis, & in Orbibus ſuis retinentur, reſpicere Solem, & eſſe reciproce ut quadrata diſtantiarum ab ipſius centro.

- *"Vires, quibus planetae priamarii perpetuo retrahuntur a motibus rectilineis, et in orbibus suis retinentur, respicere solem, et esse reciproce ut quadrata distantiarum ab ipsius centro."*
- *"Forces, which constantly retract the main planets away from rectilinear motion, and retain them in their orbits, are directed toward the sun, and are reciprocal to the squared distances from its center".*

Newton repeated analogous statements for the Earth's Moon and for the Galilean moons of Jupiter. Then he proposes (the lack of) apsidal* motion as a sensitive diagnostic tool for possible deviations from the $1/r^2$ law.

[Deviations from $1/r^2$ gravity are only possible if the central body deviates from spherical symmetry. Today we use this to remotely observe interiors of moons.]

*) Apses = Aphelion & Perihelion, the furthest & closest points of an orbit

Apsidal motion as a test of $F \sim 1/r^2$ proportion

p. 405 of *Principia*:

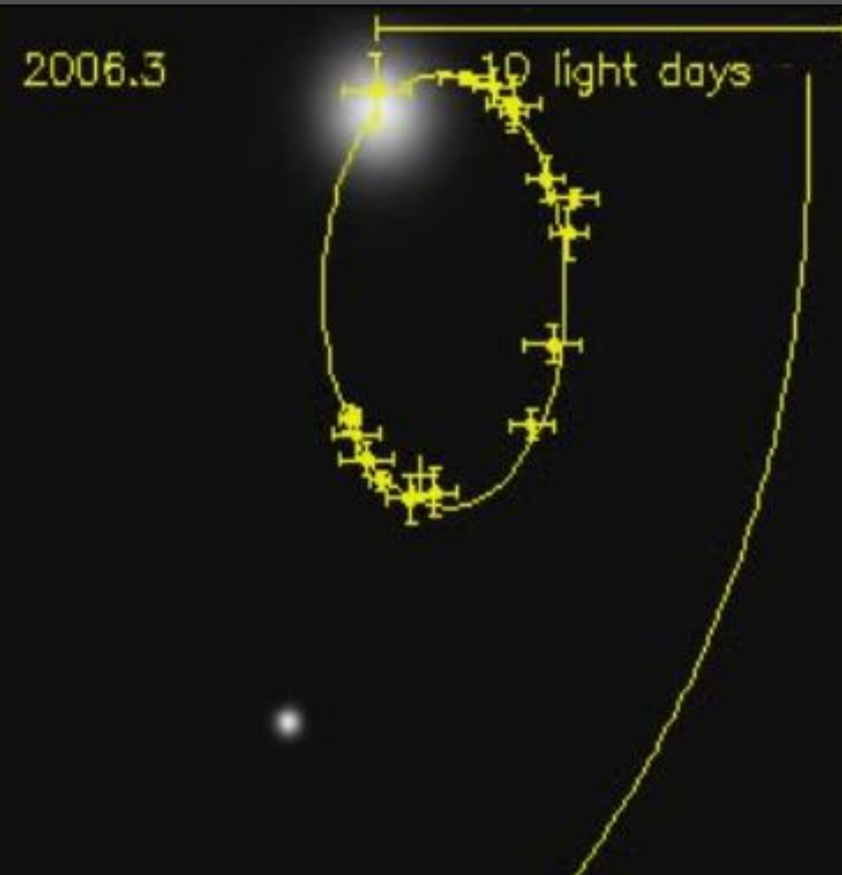
Accuratiſſimè autem demonſtratur hæc pars Propoſitionis per quietem Apheliorum. Nam aberratio quam minima à ratione duplicata (per Coràl. I. Prop. XLV. Lib. I.) motum Apſidum in ſingulis revolutionibus notabilem, in pluribus enormem efficere deberet.

“But this part of the Proportion is most accurately demonstrated by the repose of the Aphelium. For even the smallest deviation from the duplicate proportion would cause an apsidal motion that in most cases would have to be enormous.”

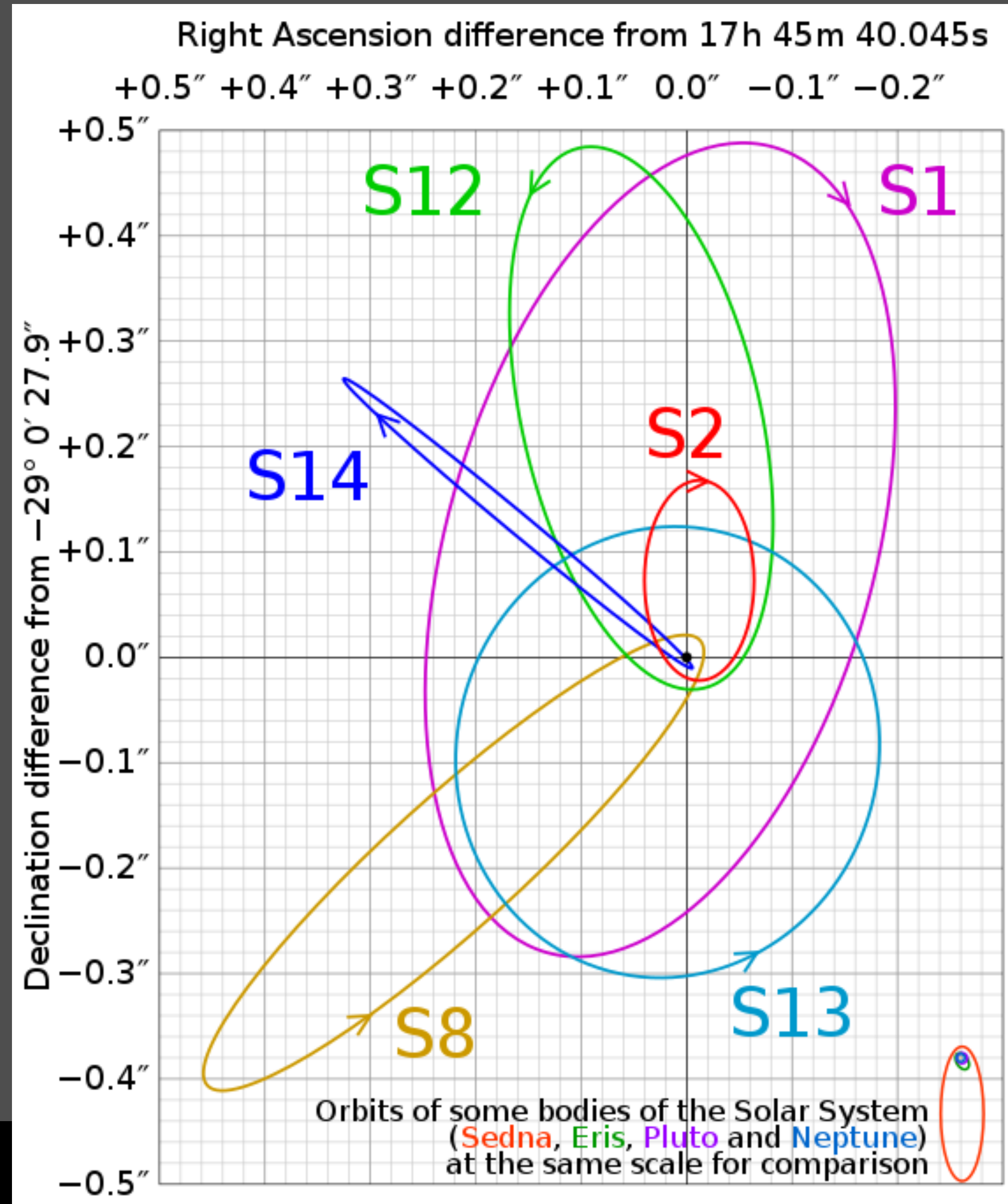
- This argument is independent of the calculation we have done in the case of an apple and the Moon.

- Newton’s ability to prove very general statements rested on his deep understanding of orbital mechanics.

Newton: F ~ $1/r^2$ → no rotation of apses in case of solar system orbits, & the S-objects (massive stars) orbiting a supermassive black hole of 4.3 million solar masses at the Galactic Center.



Observations of the orbit of object S2, P = 16.0518 yr done before 2020 did not show deviation from $1/r^2$ gravity. Later in the course we'll return to what was found in 2020.



But you should know that everybody makes mistakes.
Some ideas in the *Principia* (1st ed.) were wrong and were discussed by Halley at the meetings of the Royal Soc. of London.

Prop. XXXVII. Prob. IX.

Aquæ de vaſe dato per foramen effluentis definire motum.

Si vas impleatur aqua, & in fundo perforetur ut aqua per foramen defluat, manifeſtum eſt quod vas ſuſtinebit pondus a

od foramini perpendicu
lo aliquo occluderetur,
i perpendiculariter inc
ıdus aquæ reliquæ
:adem aquæ preſſione
us; & pondus quod
tineatur, faciet ut aqua

quæ totius effluentis is
culariter incumbentis g
quæque pondere ſuo,
e motu uniformiter ac
obſtaculum. Obſtacu
ferior defluens; & pro
ım non ſuſtinet, urgeb
ortionalem generabit.
A altitudinem aquæ
P pondus ejus, *AF* q

You can Google-translate the text into English. It's about the height to which water rises from a fountain filled to altitude A.

In reality it's A, not ½ A.

ſit $\frac{2d}{e}S$ & latitudo eadem quæ foraminis, poſſet eo tempore defluendo egredi de vaſe, hoc eſt columna $\frac{2d}{e}SF$. Quare motus $\frac{2dd}{ee}SFV$, qui fiet ducendo quantitatem aquæ effluentis in velocitatem ſuam, hoc eſt motus omnis tempore effluxus illius genitus, æquabitur motui $AF \times V$. Et ſi æquales illi motus applicentur ad FV; fiet $\frac{2dd}{ee}S$ æqualis A. Unde eſt dd ad ee ut A ad $2S$, & d ad e in dimidiata ratione $\frac{1}{2}A$ ad S. Eſt igitur velocitas quacum aqua exit e foramine, ad velocitatem quam aqua cadens, & tempore T cadendo deſcribens ſpatium S acquireret, ut altitudo aquæ foramini perpendiculariter incumbentis, ad medium proportionale inter altitudinem illam duplicatam & ſpatium illud S, quod corpus tempore T cadendo deſcriberet.

Igitur ſi motus illi ſurſum vertantur; quoniam aqua velocitate V aſcenderet ad altitudinem illam S de qua deciderat; & altitudines (uti notum eſt) ſint in duplicata ratione velocitatum: aqua effluens aſcenderet ad altitudinem $\frac{1}{2}A$. Et propterea quantitas aquæ effluentis, quo tempore corpus cadendo deſcribere

Personality of Newton

- *Newton was unable to tolerate a slightest criticism.*

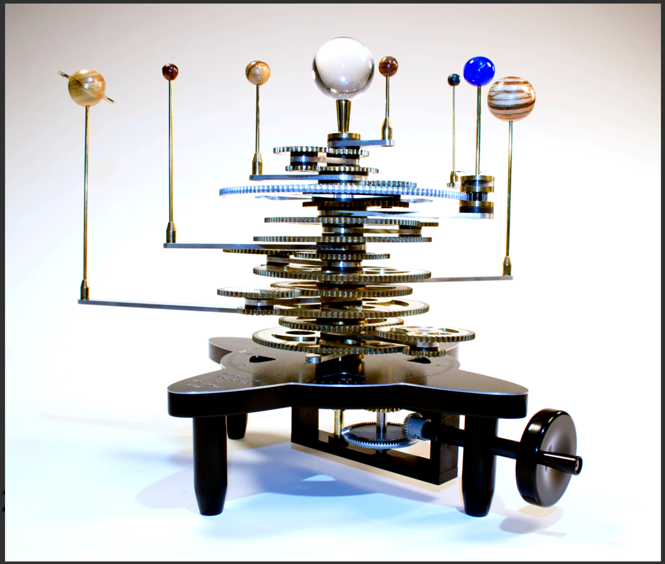
  He could be rather nasty at times to both friend and foe.

- Newton mentioned his colleagues' names in the 1st edition of *Principia – one of the few exceptions of Newton giving credit to anybody besides himself. He mentioned "Wren, Hooke and Halley" to diminish Hooke's contribution! In later editions, Newton tried to remove all mention of Hooke.*

- *He wrote a sentence about being able to see further than others by standing on the shoulders of giants – not to show humility but to take a snipe at Hooke who was known to all to be of small, crooked stature.*

A soul-less, mechanistic picture of a world is sometimes called "Newton's Universe". But this is a misconception.

Newton's universe was in fact full of spirits. The world was continuously driven by God. Newton was obsessed with precise timing of world's creation by God. He interpreted Bible and believed in prophecies, incl. Armaggedon (end of the world, which he estimated will happen in 2060, if we understand his notes correctly).
Newton also spent countless years on alchemy, trying to find Philosopher's Stone. In the words of modern historians, Newton was the "last magician", even though some of his mathematical proofs were so ahead of their time that they were only fully appreciated in the past century.



Orrery:

a mechanical model of the universe