Points in the square brackets give the relative weight with which the problems count toward the final score. If you need any physical constants or stellar data such as solar mass, radius, and luminosity, you may find them in our textbook or on the web.

1   [40p.] Tidal force near a star - dangerous or not?

Tidal force is a differential force. Think of the famous problem of tides in Earth's oceans. The moon is pulling the central parts of Earth with some acceleration we can call mean acceleration. Ocean on the side closer to the Moon gets pulled stronger (since it's closer), while the opposite can be said of the oceans on the far side. The deviation from the mean acceleration is called tidal acceleration a_{tid}, and gives rise to the tidal force that raises water level with respect to rocks that deform less willingly. Now forget the Moon – we will place ourselves near more massive objects.

Consider being close to the surface (which is at radius R from the center of the object) of four different objects:
(1) star Sirius A ($M = 2M_\odot$, $R = 1.71R_\odot$),
(2) White Dwarf, Sirius B ($M = 1M_\odot$, $R = 0.0084R_\odot$),
(3) Neutron Star PSR1257+12 ($M = 1.4M_\odot$, $R = 12$ km),
(4) Supermassive Black Hole (SMBH) in galaxy M87 ($M = 4 \cdot 10^9 M_\odot$, $R = R_{\text{Sch}}$ - we consider the SMBH's horizon radius, i.e. its Schwarzschild radius its "surface").

$R_\odot = 0.7\text{e}6$ km is the solar radius, and $M_\odot = 2\text{e}30$ kg its mass. See tutorial notes from the latest tutorial for how to compute R_{Sch} in case (4).

A. You want to fly by each of the four objects in a spaceship, to a minimum distance $r = 3R$ in each case. Assume your full height is $h \approx 2$ m, and the diameter of a spaceship is 20 m ($h = 10$ m). Compute the tidal acceleration a_{tid}. Decide which object: you or the ship is in more danger of disintegration.

Near which object will your life and/or the structural integrity of the spaceship be in peril? Assume that both you and your spaceship can survive differential acceleration up to 10g, where g is Earth's surface gravity acceleration $g = GM_E/R_E^2 = 9.81$ m/s^2.

B. A planet equal in mass and radius to Earth approaches the four objects' surfaces to within a distance such that 10 planetary radii separate the two surfaces. [Earth's radius is $R_E = 6371$ km, and its mass is $M_E = 3 \cdot 10^{-6} M_\odot$.] Will the planet's gravity manage to hold it together or will it be ripped apart by tides around some object(s)?

2   [20 p.] Energetics of falling into the black hole via a disk

A black hole has mass $M = 10^6 M_\odot$ and (Schwarzschild) radius $R_S = 2GM/c^2$. It is surrounded by a rotating disk of gas extending from the last stable orbit at $r = 3R_S$ to infinity. There is a small friction force (viscosity) acting withing the disk. The disk heats up and radiates because of this friction. Small disk elements of mass m move on very tightly wrapped spiral paths (almost circles) toward the inner edge of the disk from infinity. At

$r < 3R_S$, their orbits become unstable and the mass falls directly into the black hole without emitting further radiation.

Using Newtonian gravity potential $-GMm/r$ of mass m in the field of mass M, and accounting for both the kinetic and potential energy of m (assuming circular orbit at each time), find the energy of radiation E_{rad} released by mass m on its voyage from infinity to the black hole. What fraction of rest energy (i.e. mc^2) is E_{rad}? Compare your answer with the percentage of the rest mass in the process of fusion of hydrogen into helium theoretically releases (cf. Aston's results 1919, described in the textbook).

HINT: As an intermediate step, find the orbital speed v as a function of radius r from the balance of Newtonian gravity force GMm/r^2 and centrifugal force mv^2/r.

3   [30 p.] Contraction and spin-up of a star to a neutron star

Suppose a stellar core comprising $M = 1.9$ solar masses contracts to form a neutron star (entirely composed of neutrons), while preserving the angular momentum of each gas particle.

A. [15p] Calculate the radius of the neutron star R_{NS} and its mean density, assuming that neutrons in it are located on verices of a cubic grid and touching their neighbors. The radius of neutron equals $r_n = 0.87$ fm. Mass of neutron is close to the mass of proton or hydrogen atom, $m_n = 1.6749$e-27 kg.

B. [15p] Compute the rotation period of a neutron star, if the initial rotation period was 19 days and initial core radius was 0.27 solar radii. Are there really neutron stars that rotate so fast?