ASTB23 (STARS & GALAXIES) PROBLEM SET #4. DUE 30 NOV. AT NOON.

Points in the square brackets give the relative weight with which the problems count toward the final score. If you need any physical constants, you may find them in the textbook(s) or on the web. Sorry for ugly typography of letter 'vee' as v which looks like Grek 'nu' (ν).

1 [26p.] Milky Way as a Logarithmic potential galaxy

Observations of the Milky Way galaxy reveal that the circular speed at the distance $R = 8.5$ kpc is 220 km/s, while it is 170 km/s at the distance of 2.2 kpc from the center.

Find two constant parameters R_0 and v_0, in units of kpc and km/s correspondingly, of the following logarithmic potential

$$\Phi(R,z) = \frac{v_0^2}{2} \ln[(R^2 + R_0^2 + z^2/0.9^2)/R_0^2]\,,$$

that provides the best fit to the observational data.

What circular speed is found at infinity (i.e. when $R \gg R_0$)? What speed is found at $R = 0$?

Notice: The circular speed is defined in the midplane of the galaxy only, i.e. at $z = 0$.

2 [24p.] Epicyclic and orbital frequency

Orbital frequency is $\Omega(R) = v_c(R)/R$, also known as angular frequency or angular speed (in radians per second) of circular orbital motion.

Epicyclic frequency is the frequency of radial oscillation of distance r around the guiding center (imaginary circular orbit, around which a real trajectory, e.g. of the sun, oscillates, between some minimum and maximum distance from the center of the galaxy). One can prove (but you do not have to!) that this frequency equals

$$\kappa^2(R) = R^{-3}\frac{d(R^4\Omega^2)}{dR}$$

in arbitrary potential, that is in arbitrary rotation curve.

Based on the formulae from the previous problem, show that the Galaxy near its center ($R \ll R_0$) has a linearly rising rotation curve $v_c(R)$, and an approximately constant velocity curve $v_c \approx const.$ at very large radii $R \gg R_0$. Compute the ratio of epicyclic to orbital frequency κ/Ω in these two asymptotic cases. (You don't have to use the precise $\Omega(R)$ curve, you may use two separte asymptotic expressions). Frequency is inversely proportional to period. Based on that, qualitatively describe the consequences of your results for the shape of the orbit of a star (like the sun) on a nearly circular orbit, in particular whether the orbit is closed and if not, then which way the it precesses in space. Make a qualitative top-view sketch of non-circular sun's trajectory.

3 [23p.] Surface brightness in a disk galaxy NGC3011

Galaxy NGC3011 has an exponential disk dominating the visible light image. Surface brightness (luminosity per unit area) is given by the exponential law

$$I(R) = I_0 \exp(-R/R_d)$$

where I_0 is the central surface brightness, R distance from the center, and $R_d = 4$ kpc the exponential radial scale of the disk (e-folding distance, distance over which density changes by factor e). The total luminosity of the galaxy equals $L = 2.5 \cdot 10^{10} L_\odot$.

A. Considering that the total luminosity is the surface brightness $I(R)$ integrated over the *area* of the whole disk from $R = 0$ to $R = \infty$ (not over the radial distance, a mistake some students make!), compute I_0 in units of $L_\odot/\text{pc}^2$, solar luminosities per square parsec.

B. What is the surface brightness of the galaxy at radius 8 kpc, $I(8kpc)$? Assuming a standard disk light-to-mass ratio $\Upsilon = 4M_\odot/L_\odot$ appropriate to many disk galaxies, convert your answer into the surface density of disk stars in units of $M_\odot/\text{pc}^2$ and compare with the knowledge of the solar neighborhood of our Milky Way (roughly 60 $M_\odot/\text{pc}^2$ of luminous matter).

4 [27p.] Cosmological constant and the accelerated expansion of the Universe

Expansion of the Universe does not have one central point. Any point in space can be taken as a point from which the rest of space is expanding. Let's pick one such a point and draw a sphere of radius R around it. In an expanding universe, $R(t)$ is an increasing function of time t, while the amount of matter M inside the sphere of radius $R(t)$ remains constant.

The expansion of the universe can be approximated by the following dynamical equation

$$\ddot{R} = -GM/R^2 + (\Lambda/3)R$$

where $M = const.$ and each dot stands for $\frac{d}{dt}$ operation. The underlying assumptions will become clearer at the end of the course when we discuss flat spacetime and cosmological constant Λ. The first term is, as you see, the Newtonian gravity term, the second was introduced by Albert Einstein as sort-of antigravity opposing the gravitational attraction (In 1999 Λ was observationally discovered to be a small positive number; its unit is s^{-2}). This means that vacuum for some as yet unclear reason has nonzero energy per unit volume and likes to expand; this energy is called dark energy.

• Derive the law of the speed of universal expansion: $\dot{R}(t)$ and make a sketch of its time dependence. Based on that also provide a rough sketch of $R(t)$. Hand-drawings will suffice.

Hint: Define the potential function $\Psi(R)$ via: $-d\Psi/dR = -GM/R^2 + (\Lambda/3)R$. • Find function $\Psi(R)$.

• Verify by taking the full time derivative of E that the dynamical equation has the following energy-like constant

$$E = \frac{1}{2}\dot{R}^2 + \Psi = const.$$

• Show that the universal expansion is initially dominated by the gravity term, but after a certain time the speed $\dot{R}$ stops decreasing and then increases, driven by the lambda-term, asymptotically leading to an exponential growth of the scale factor R.

[Hint: Cosmologists argue that $E = 0$ is the correct value of the constant. Follow their advice.]

• What is the e-folding timescale in the late stage?

[Zoom in on the text of this problem, R and $\dot{R}$ are sometimes hard to distinguish. If in doubt, as always, check the units!]