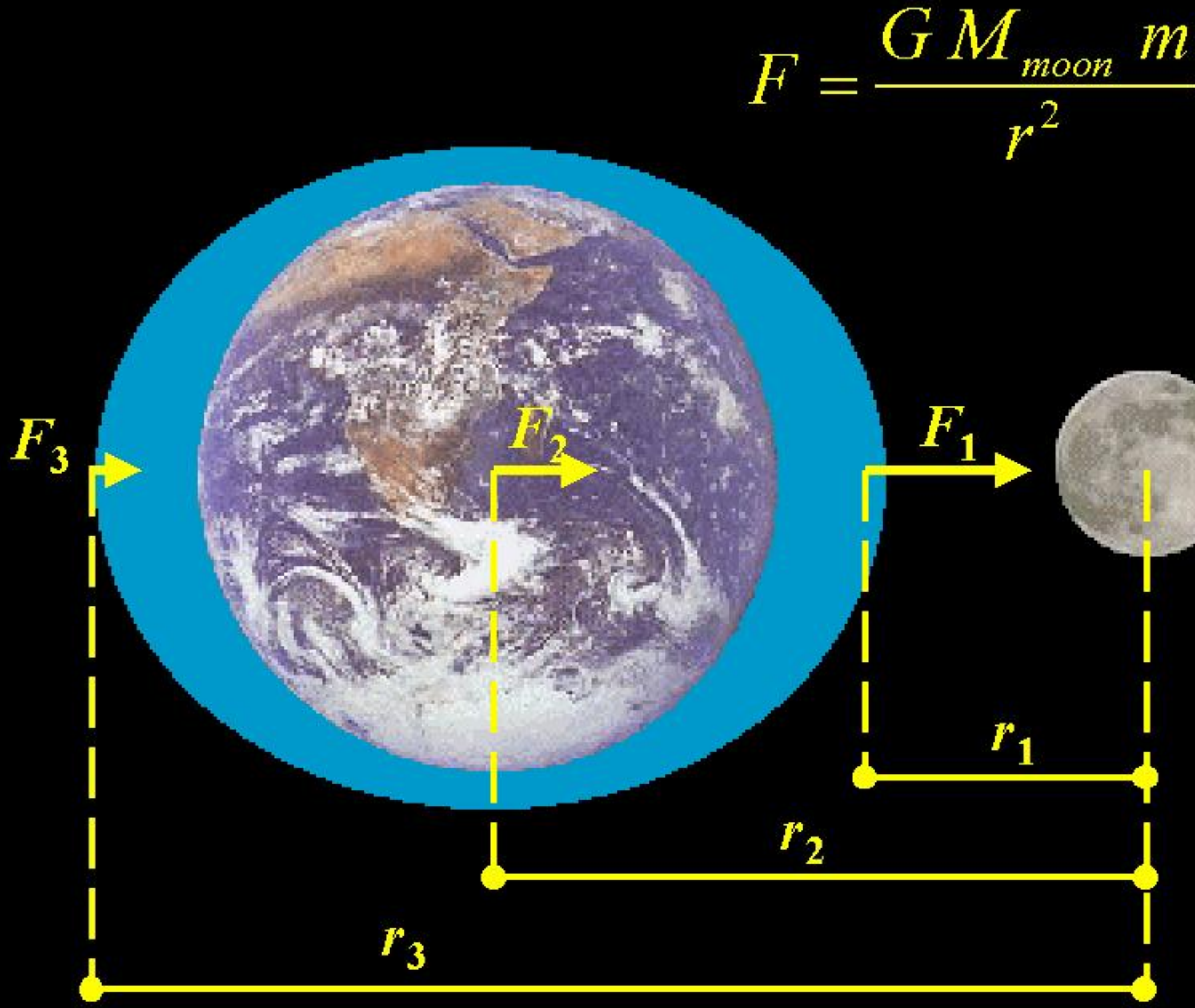
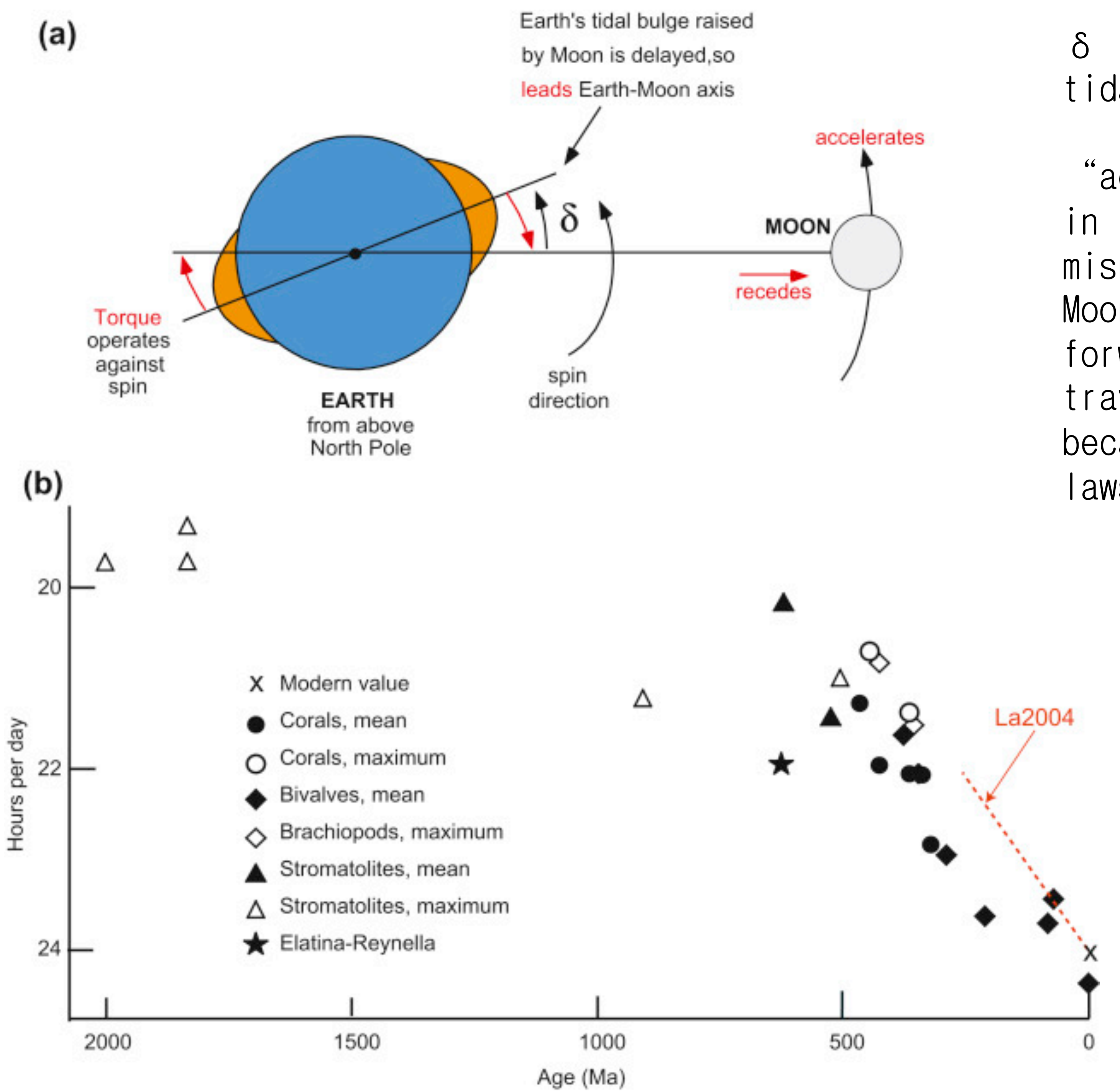
Lecture L05 - ASTC25

Physics of Tides
Tides in the solar system
Roche limit of tidal disruption

Please also read Lissauer & de Pater textbook, ch. 2.7
& see wikipedia article on "Volcanism on Io"



$F = \frac{G M_{moon} m}{r^2}$
F_3
F_2
F_1
r_1
r_2
r_3

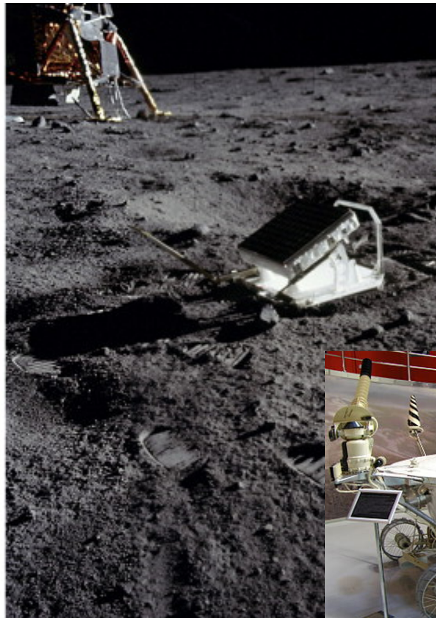


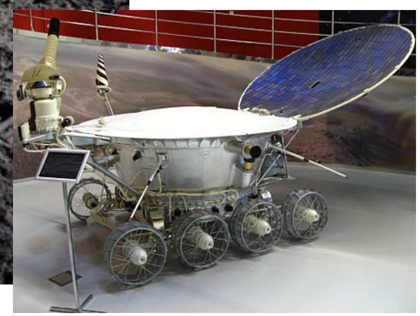
δ is called the tidal lag angle

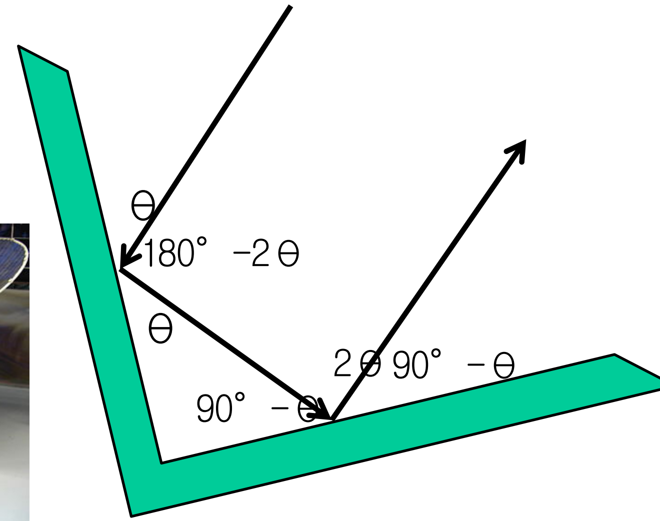
"accelerates" is in interesting misnomer: the Moon by being pulled forward actually travels slower & slowe because of Kepler's laws

As the Earth spins down, the day lengthens i.e. lasts more anc more current constant hours

In 1970s **retro-reflectors** (mirrors) were left on the surface of the Moon by American Apollo (astronautic) and Soviet Lunokhod mission (1st robotic planetary rover). We use them to track the distance to the Moon with accuracy better than 2 cm.





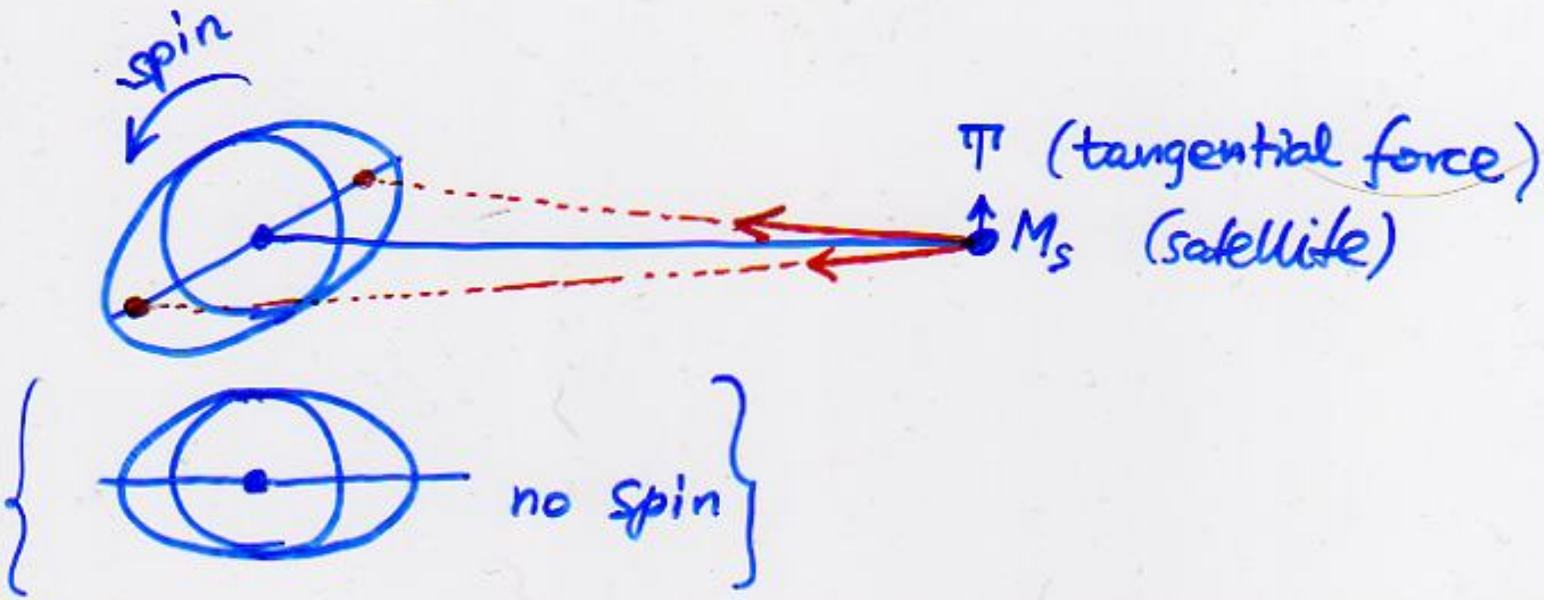


The Moon's distance currently increases 3.76 cm/yr, and on longer
time scales the semi-major axis grows as follows:

a_M = [60.142611 + 6.100887 T - 2.709407 T^2 +1.366779 T^3] R_E

where T = $(t-t_{now})$/Gyr . This is a polynomial fit to over 50+ years of data. The
validity of the formula will end in 1 to 2 Gyr, when Earth's oceans

Tides



Damped oscillator as a model equation of tidal wave

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = \frac{F}{m} \cos \omega_p t$$

braking — Free oscil. — forcing

x = **Deformation (vertical, relative to sphere)**

ω_0 = **Natural frequency of oscillations = (spring constant/m)$^{1/2}$**

ω_p = **Frequency of forcing = spin frequency = angular speed of planet**

x = deformation w.r.t. sphere
ω_0 = natural frequency of oscillation
ω_p = spin frequency of the planet

Damped harmonic oscillator solution:

$$x(t) = \underbrace{A e^{-\gamma t/2} \cos(\omega t + B)}_{\text{free oscillations}} + \underbrace{\frac{F/m}{\sqrt{\omega^4 + \gamma^2 \omega_p^2}} \cos(\omega_p t - \delta)}_{\text{tide}}$$

where $tg\,\delta = \tan\delta = \gamma\omega_p/\omega^2$; $\omega^2 = \omega_0^2 - \omega_p^2$
and A, B = const.

tide amplitude $\cong \frac{F}{m\,\omega_0^2}$, since $\omega_p^2 \ll \omega_0^2$
tidal lag angle $\delta \simeq \frac{\gamma\omega_p}{\omega_0^2}$

We can now compute $E_0 = \underbrace{\left(\frac{F}{m\omega_0^2}\right) \cdot m \frac{\omega_0^2}{2}}_{\text{oscillator energy}}$

$$Q = \frac{E_0}{\left(\frac{2\pi}{\omega_p}\right)^{-1} \oint |\dot{E}| dt} \cdot \omega_p$$

how many timescales $= \omega_p^{-1}$ before tides damped?

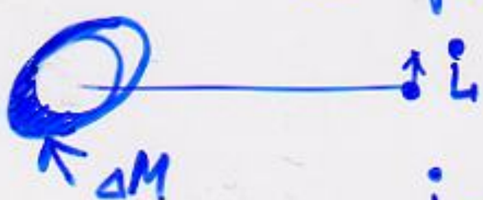
$$|\dot{E}| = m \cdot \gamma \cdot \dot{x}^2 \qquad \text{(force} \times \text{velocity)}$$

Oscillator goodness parameter

$$Q \cong \frac{1}{\delta}$$

$$\frac{F}{m} = \Delta f = \text{force stretching the planet (per unit mass)}$$

$$\Delta f = \frac{GM_s}{(r-r_p)^2} - \frac{GM_s}{r^2} \simeq \frac{2GM_s r_p}{r^3}$$



$$\Delta M \sim \mathcal{O}(1) \cdot M_s \cdot \left(\frac{r_p}{r}\right)^3$$

$$\dot{L} = \frac{\delta}{2} r_p GM_s (\Delta M) \left[\frac{1}{(r-r_p)^3} - \frac{1}{(r+r_p)^3}\right]$$

$$\boxed{\dot{L} \sim \frac{GM_s^2 r_p^5}{Q\, r^6} = \text{tidal torque}}$$

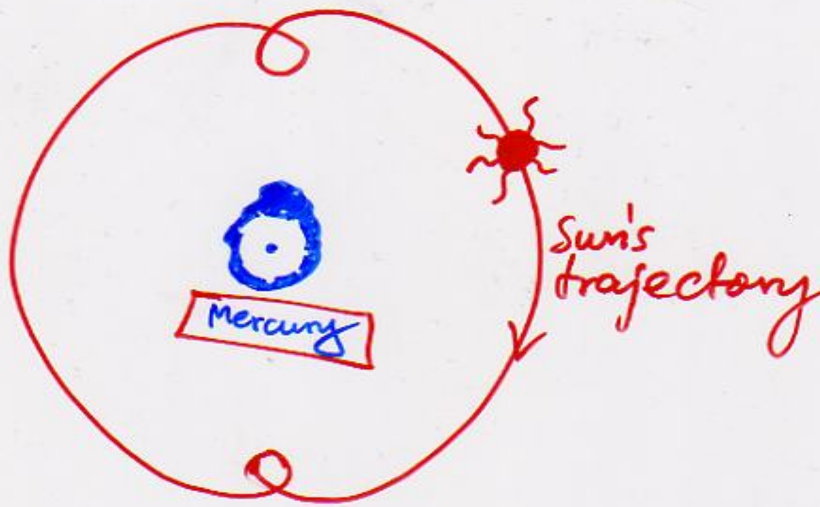
acts on the planet, and on the satellite (signs opposite)

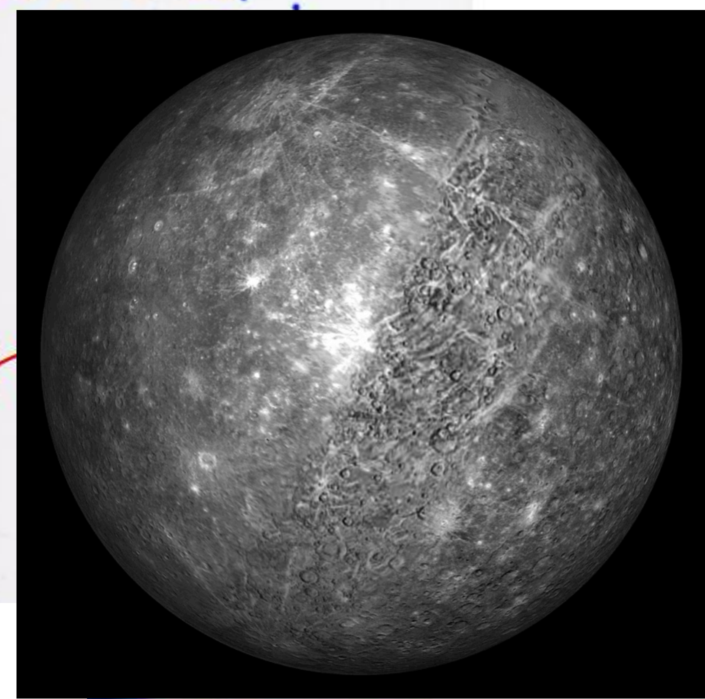
Consequences: $\omega_p \searrow$ drops | $r \nearrow$ grows

⊕ + ☾
$0.0016 \frac{s}{cent.}$
$4 \frac{cm}{yr}$

Final state: $\omega_p = 0$, $\omega_p : n = 1:1$,
planet in 1:1 corotation with M_s
e.g. Mercury was thought
& Moon was observed to corotate

However, Mercury is in $\omega_p : n = 3:2$!





Net torque = 0, stable resonance

(By coincidence, probably!) Venus rotates backwards ≈5 times between the approaches to Earth

($Q = 1400 \Rightarrow$ that would be a physical reason...)
but $Q \ll 1400$. E.g. {$Q \sim 15$ for Earth; $Q \sim 100$ without oceans}

Q: Why does Venus rotate backwards?

A: 1) Giant impact; or

2) Massive atmosphere, atmospheric oscillations couple with insolation, which bloats the atmosphere

(There is such an effect, cancelling ~10% of the gravitational tide, on Earth)

Mars : Phobos, Deimos — tidally locked spin like the Moon–Earth

Phobos very close, circles Mars $3\times$ faster than Mars spins $\Rightarrow$

$\Rightarrow \; r \searrow$

this is a measurable effect : $Q_{Mars} \sim 100$,

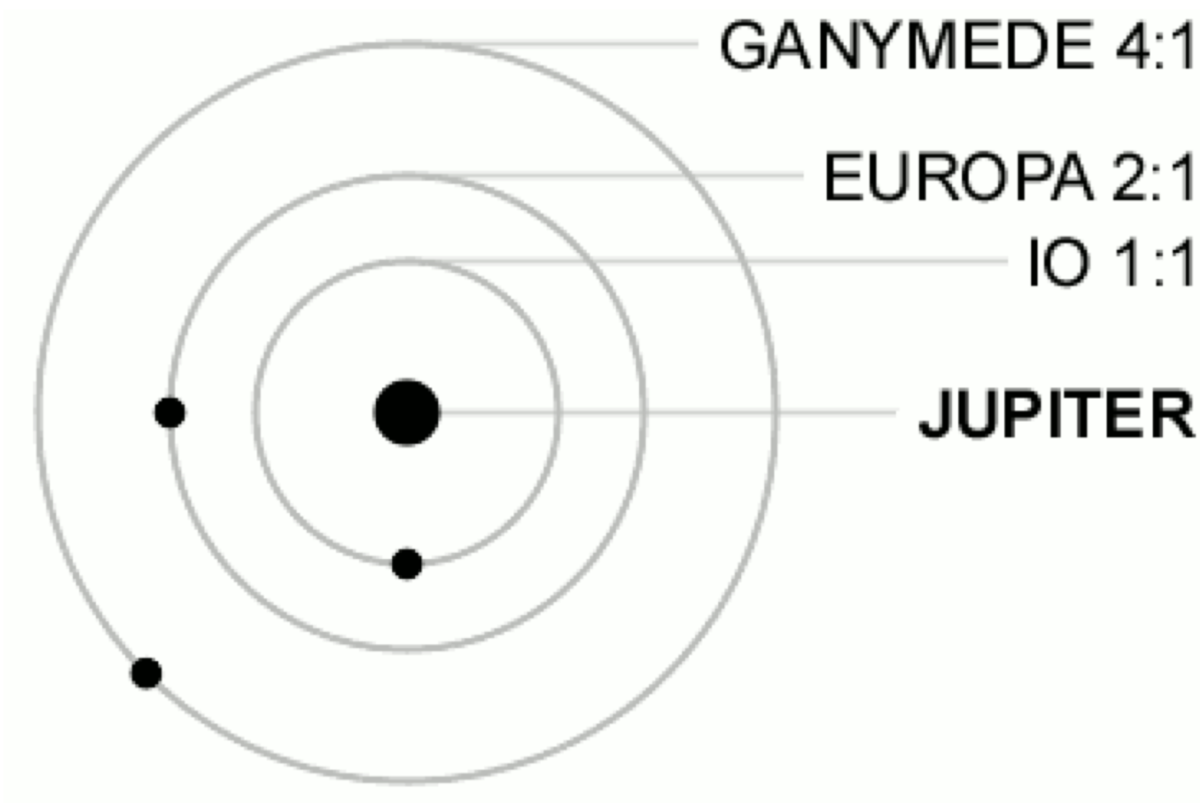
Jupiter

All 4 Galilean satellites tidally locked in spin, as well.

Q inside Jupiter must be very large & δ very small or else these satellites would already have been pushed away!

$Q \sim 6 \cdot 10^4$

Laplace resonance of Galilean satellites (except Callisto!) – the period ratios:



Physical radi

2634 km

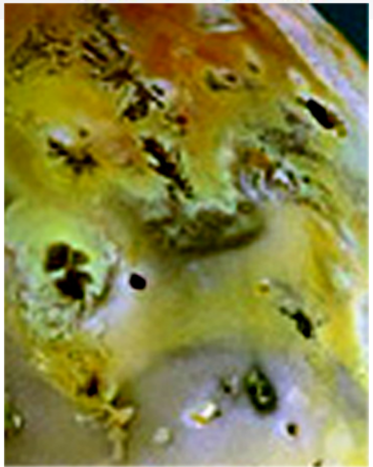
1560 km

1821 km

Io – tidally heated ⇒ the most active volcanoes in the Solar System!

Io – Europa 1:2 orbital resonance
forces eccentricity $e_{Io} \approx 0.0041$
⇒ Io has nutation (oscillation) w.r.t. mean motion, Jupiter on Io's sky moves a little.
Io flexes, releases heat of friction, which melts the interior
(predicted, then observed by Voyagers).

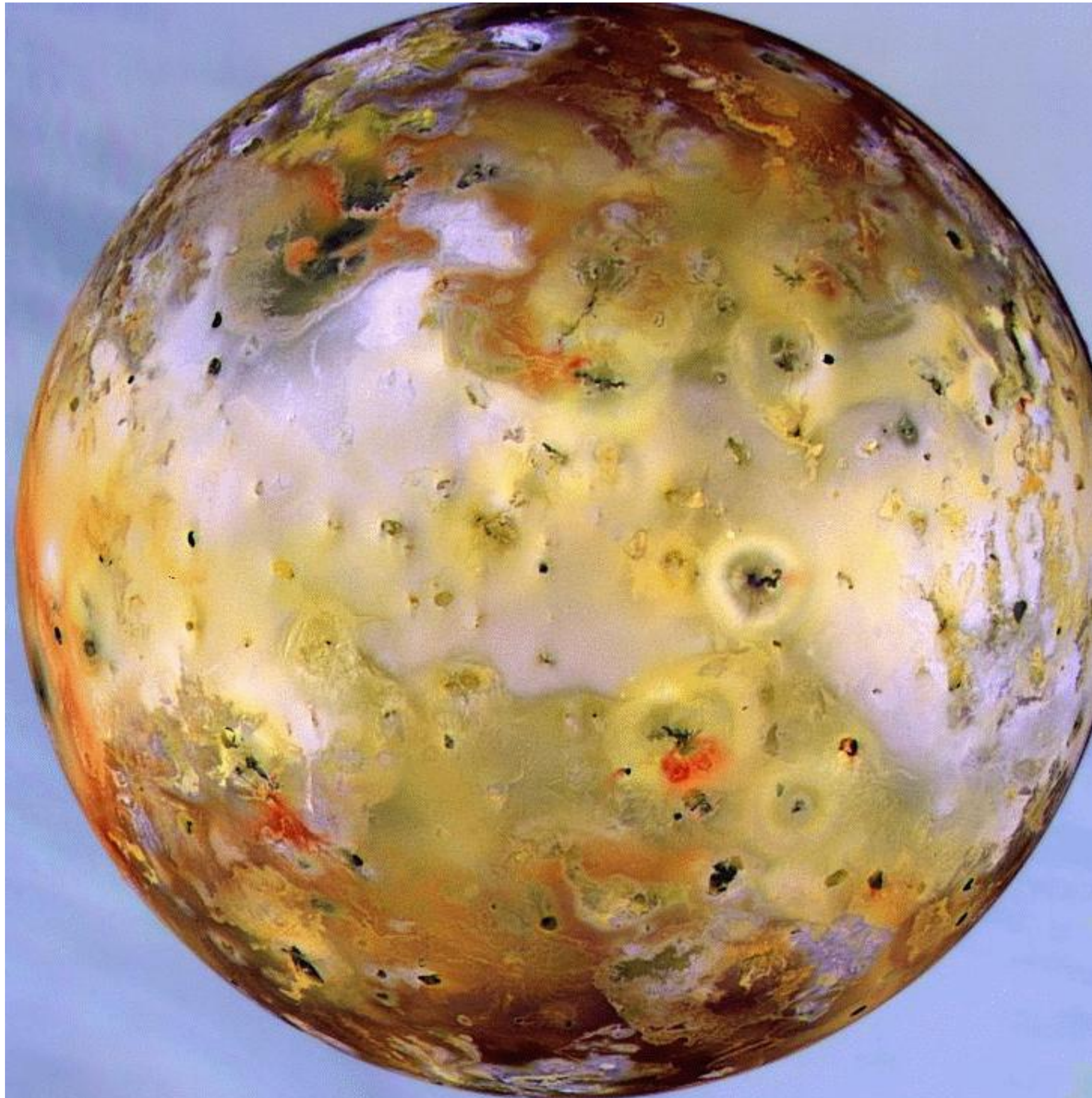




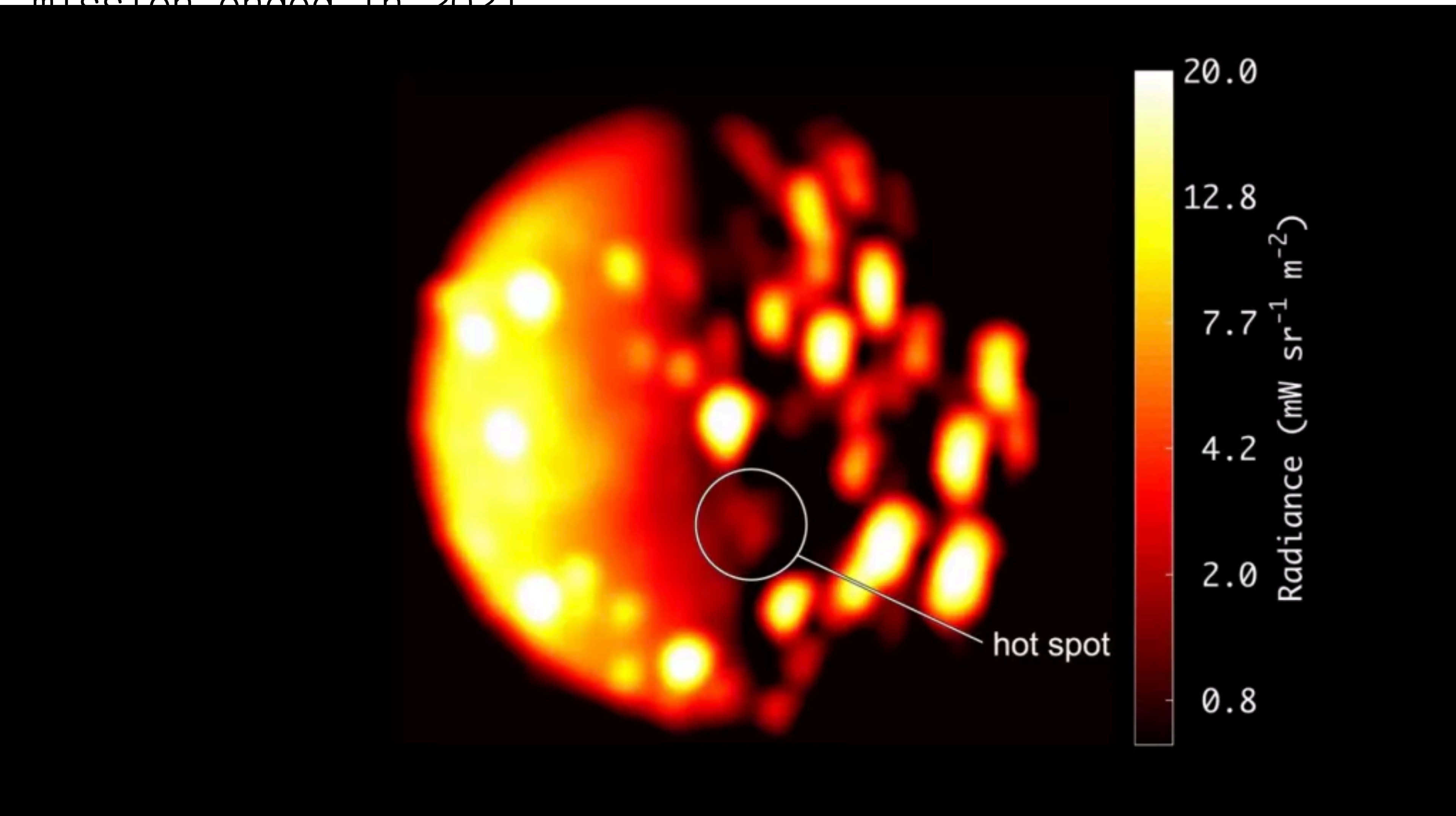
Io is tidally locked to Jupiter (same side faces it). Because of e > 0 there is nutation.

Tidal flexing and heating of Io causes the most active volcanism in our system. It has 400 active volcanoes & produces 10^{14} W of heat.

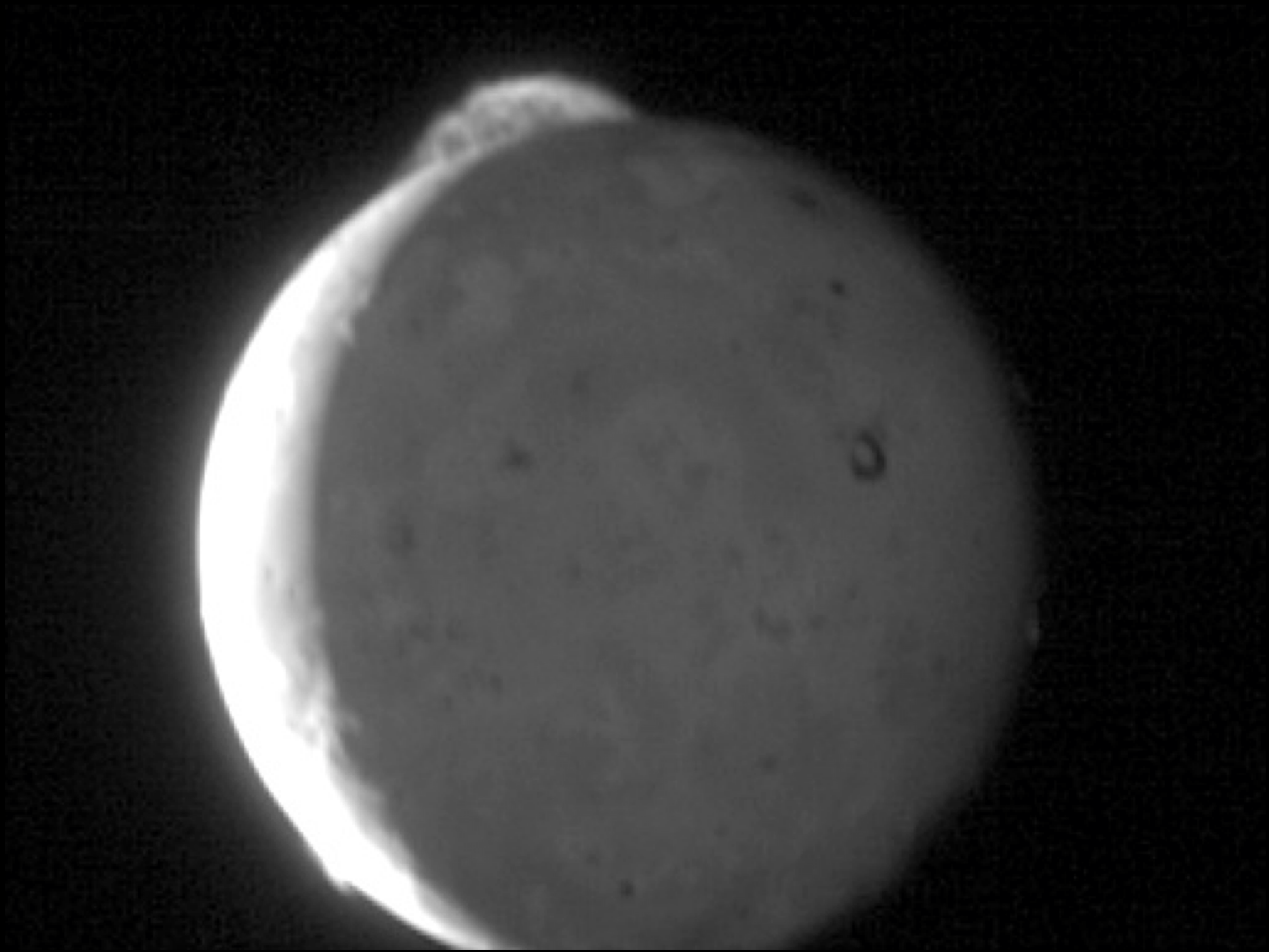
Also the moons Europa & Enceladus are tidally heated which allows underground water oceans on them.



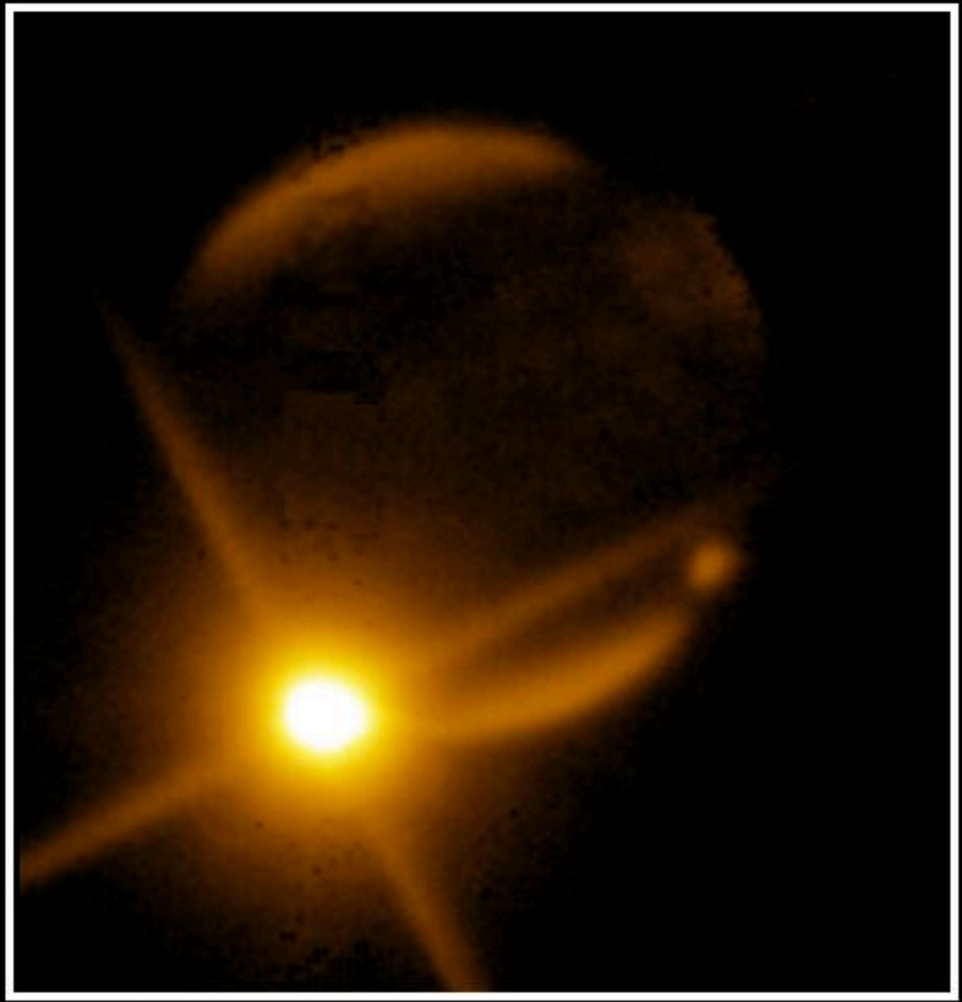
Juno probe that left Earth in 2011 and since 2016 orbits Jupiter in an elliptic orbit took this snapshot of IR hear map of Io. Multiple volcanoes are visible. Juno’s main mission ended in 2021



Plume from Tvashtar volcano on Io reaches 300 km height



spacecraft: New Horizons (NASA/JPL)

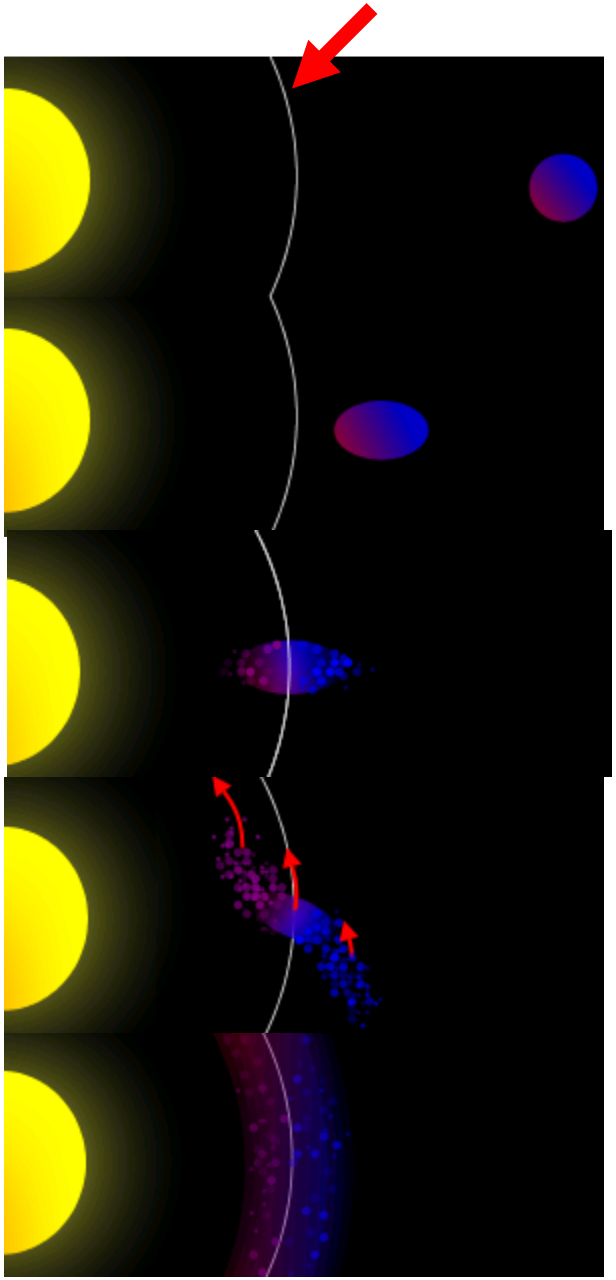


1994

Impact of Fragment G of Comet Shoemaker-Levy on Jupiter
The fireball is seen 12 minutes after impact at 2.34 microns.
The impact A site is seen on the opposite limb of the planet.

Image at 2.34 microns with CASPIR by Peter McGregor
ANU 2.3m telescope at Siding Spring

Roche limit: tidal forces destroy bodies which come too close to a source of gravitation, through differential acceleration (tide)



Consider an orbiting mass of fluid held together by gravity. Far from the Roche limit the mass is practically spherical.
The body is deformed by tidal forces

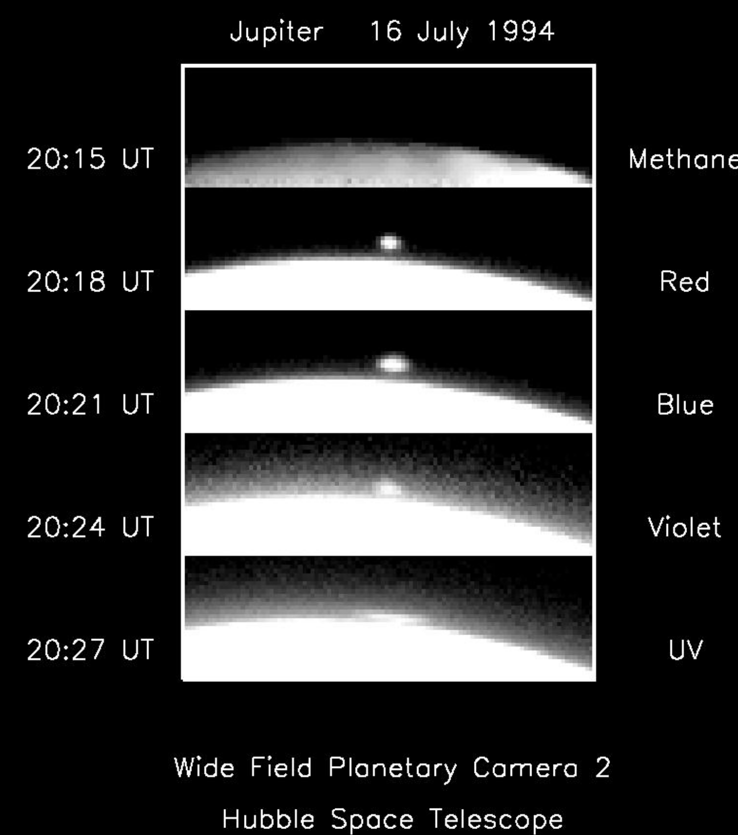
Within the Roche limit the mass' own gravity (self-gravity) can no longer withstand the tidal forces, and the body disintegrates.

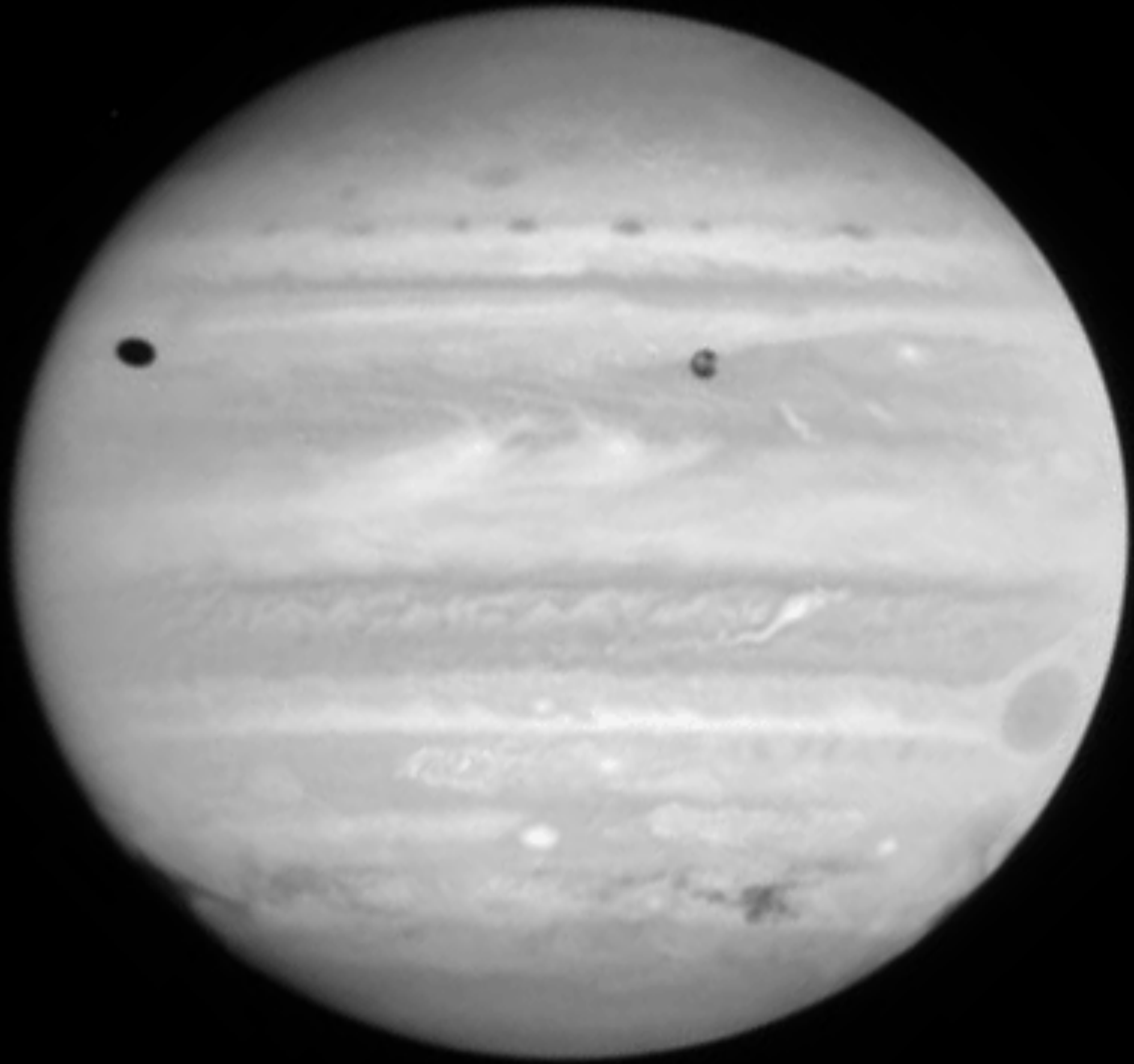
Particles closer to the primary orbit move more quickly than particles farther away, as represented by the red arrows.

The varying orbital speed of the material eventually causes it to form a ring.

http://www.answers.com/topic/roche-limit#wp-Fluid_satellites

1992 Jupiter disrupted tidally comet Shoemaker-Levy 9 (SL9)





Also see https://www.youtube.com/watch?v=7zNuT4dbdjU

First step in the computation is the estimation of the differential torque

tidal "force" = differential acceler.

Édouard Roche

(1820–1883)

$$f_{1,2} = -\frac{GM}{r_{1,2}^2}$$

$$\Delta f = f_2 - f_1 = -GM\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)$$

$$\Delta r = r_2 - r_1$$

$$\Delta f = -\frac{GM}{r_1^2}\left(\frac{r_1^2}{(r_1+\Delta r)^2} - 1\right)$$

$$\frac{1}{\left[1+\left(\frac{\Delta r}{r_1}\right)\right]^2} \approx 1 - 2\frac{\Delta r}{r_1} \qquad \left(\frac{\Delta r}{r_1} \ll 1\right)$$

$$\Delta f \approx + \frac{2GM}{r_1^2} \cdot \frac{\Delta r}{r_1} \qquad (\ll 1)$$

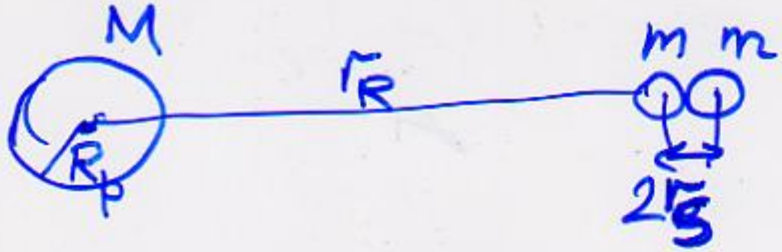
Roche limit:

Fluid limit:
tides destroy a body bound by <u>self-gravity</u> within that minimum distance r_R

Solid/rocky limit:
the same but with additional binding (electromagnetic) forces of <u>material strength</u>

Q: which limit is larger?

Roche limit calculation



tidal "force"

$$\Delta f = \frac{2GM}{r_R^2}\cdot\frac{2r_s}{r_R}$$

$$\parallel$$

gravity

$$f_{gr} = \frac{Gm}{(2r_s)^2}$$

$$4\frac{M r_s}{r_R^3} = \frac{m}{4r_s^2}$$

$$16\frac{M}{m} = \frac{r_R^3}{r_s^3} \qquad \frac{M}{m} = \left(\frac{R_p^3\rho_p}{r_s^3\rho_s}\right)$$

$$16\frac{\rho_p}{\rho_s} = \frac{r_R^3}{R_p^3} \Rightarrow r_R = R_p\left(\frac{\rho_p}{\rho_s}\right)^{\frac{1}{3}}$$

2.55

$$\hookrightarrow 4\frac{M r_s}{r_R^3} = \frac{m}{4 r_s^2}$$

$$16\frac{M}{m} = \frac{r_R^3}{r_s^3} \qquad \frac{M}{m} = \left(\frac{R_p^3 \rho_p}{r_s^3 \rho_s}\right)$$

$$16\frac{\rho_p}{\rho_s} = \frac{r_R^3}{R_p^3} \Rightarrow r_R = 2.55\, R_p \left(\frac{\rho_p}{\rho_s}\right)^{\frac{1}{3}}$$

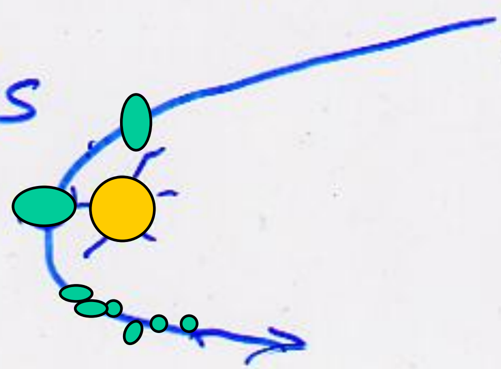
Roche limit

[pron.: *Rosh*]

Applications:

(1) satellite systems

(2) comets



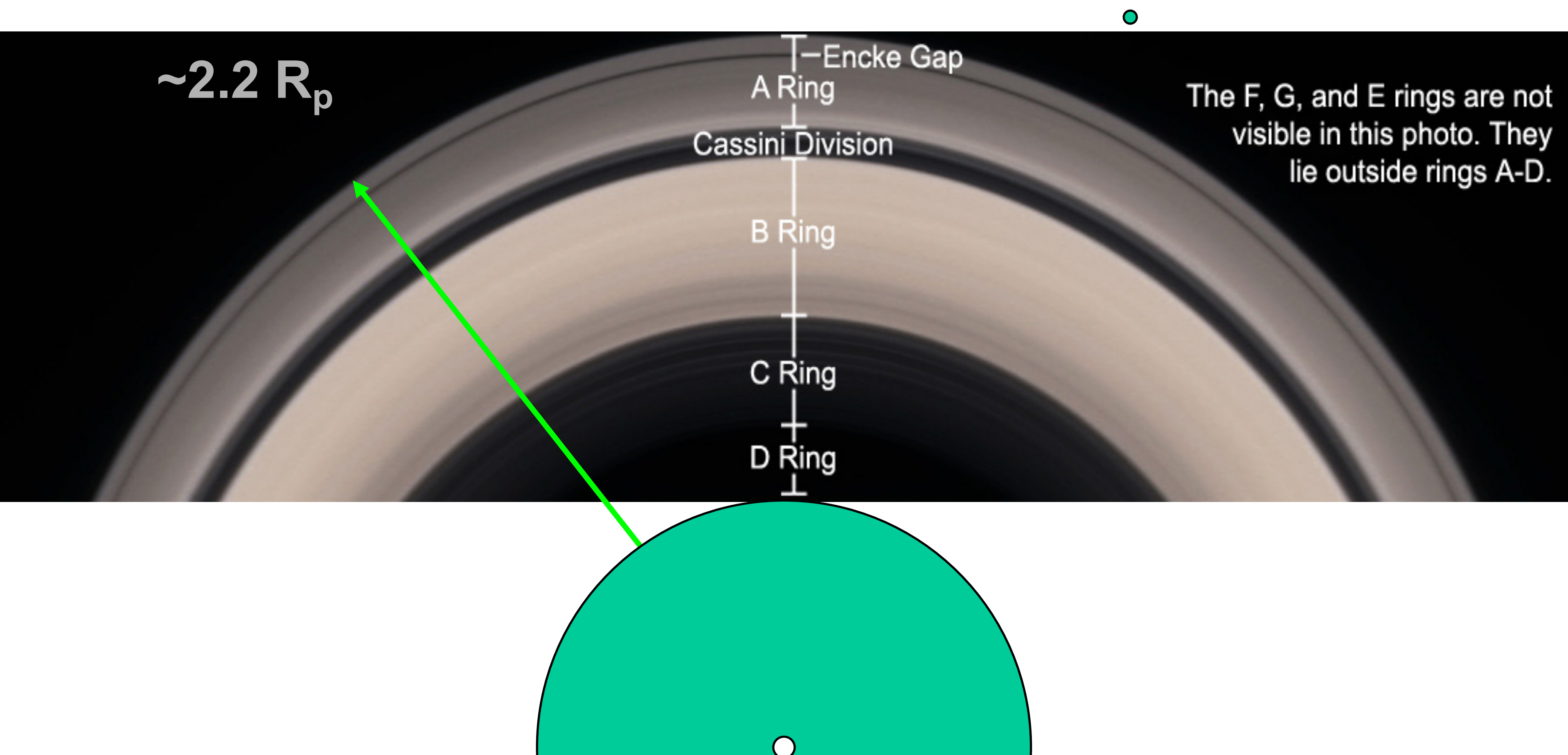
Comet Shoemaker-Levi SL-9 (1994) disintegrated after approach to Jupiter. So do the sun-grazing comets

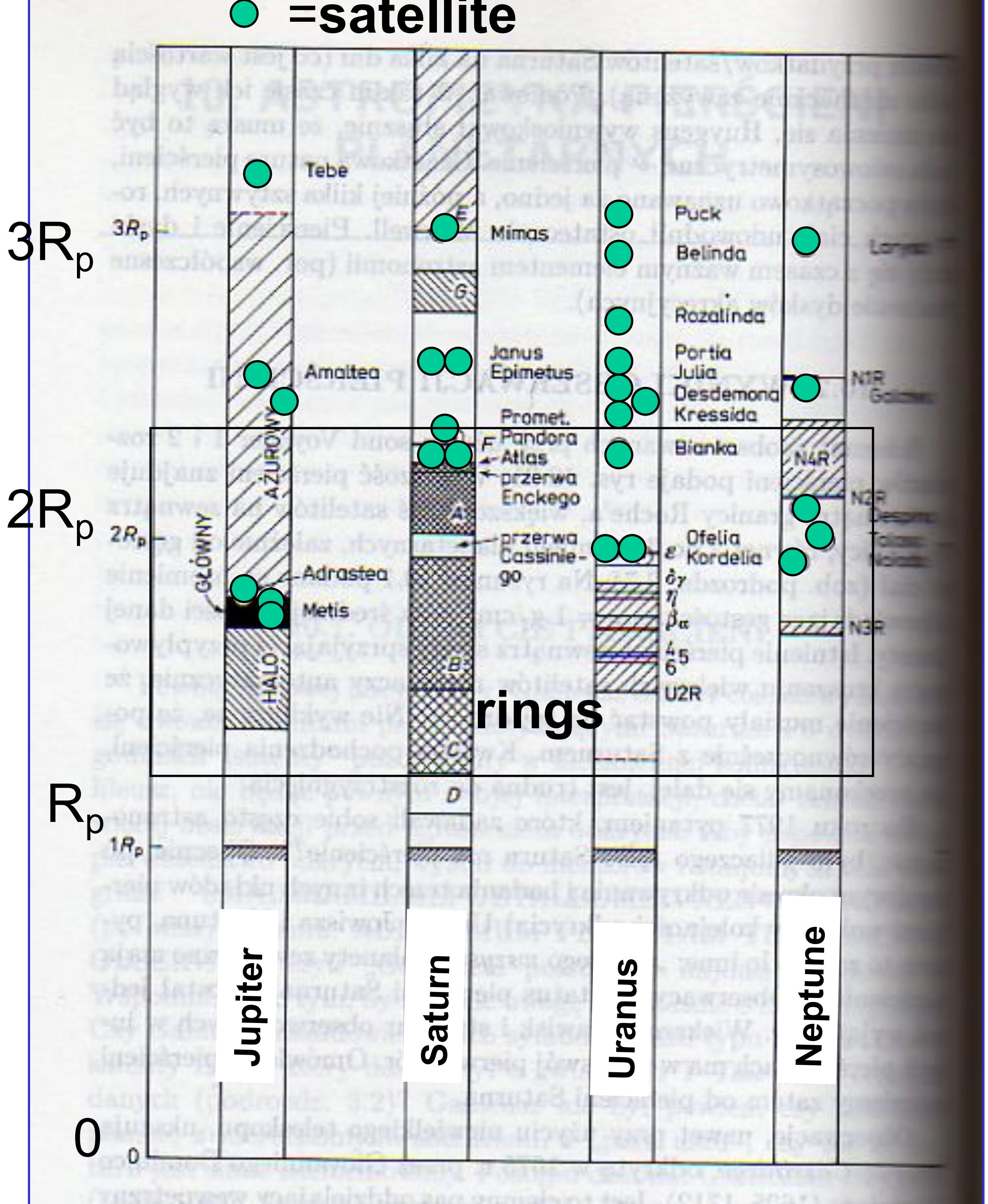
Roche limit for fluid satellite (1849): $d \approx 2.44 R \left(\frac{\rho_M}{\rho_m} \right)^{1/3}$

Roche limit for rocky satellite: $d \approx 1.4 R \left(\frac{\rho_M}{\rho_m} \right)^{1/3}$

(R_p = planet's radius, ρ_M = its density)

➡ **satellites generally are found outside the rings**





Roche limit is somewhere between 1.4 and 2 R_p: satellites are destroyed if found closer to their planets.

Ring systems drawn to scale of their planets, but not to relative scale.

Lecture L06 - ASTC25

Elements of Celestial Mechanics

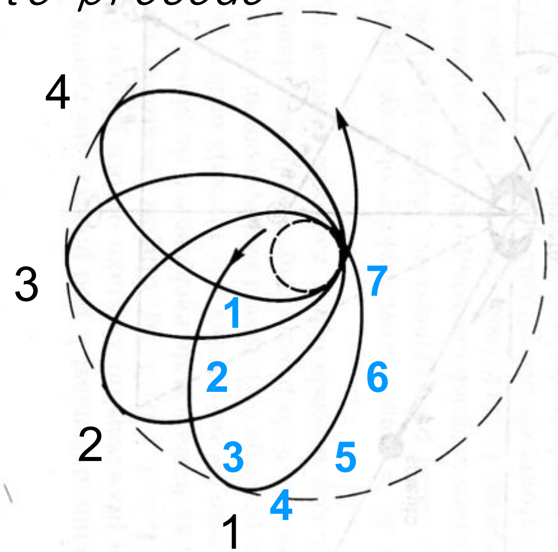
Precession of orbits and rotational axes

Lunisolar precession of Earth's spin axis

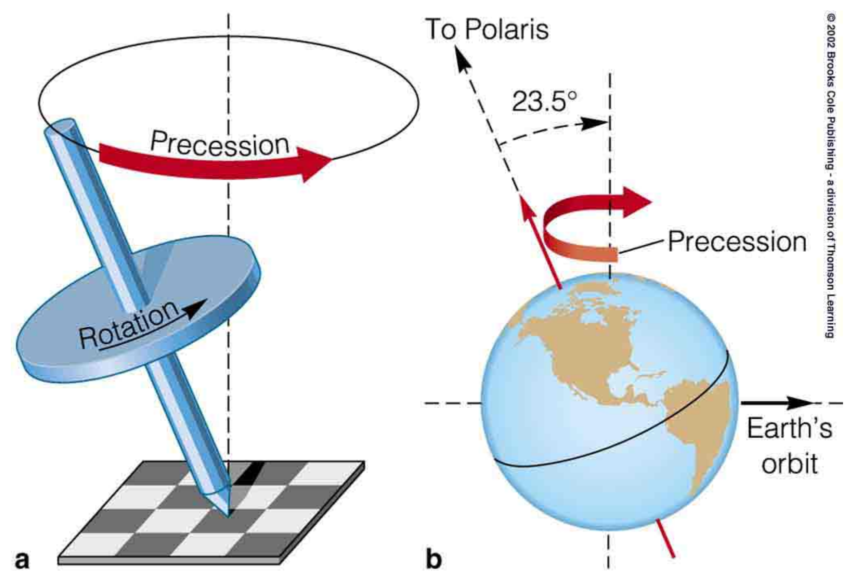
Two kinds of Precession

orbit precession,

Praecedere [latin] = to precede



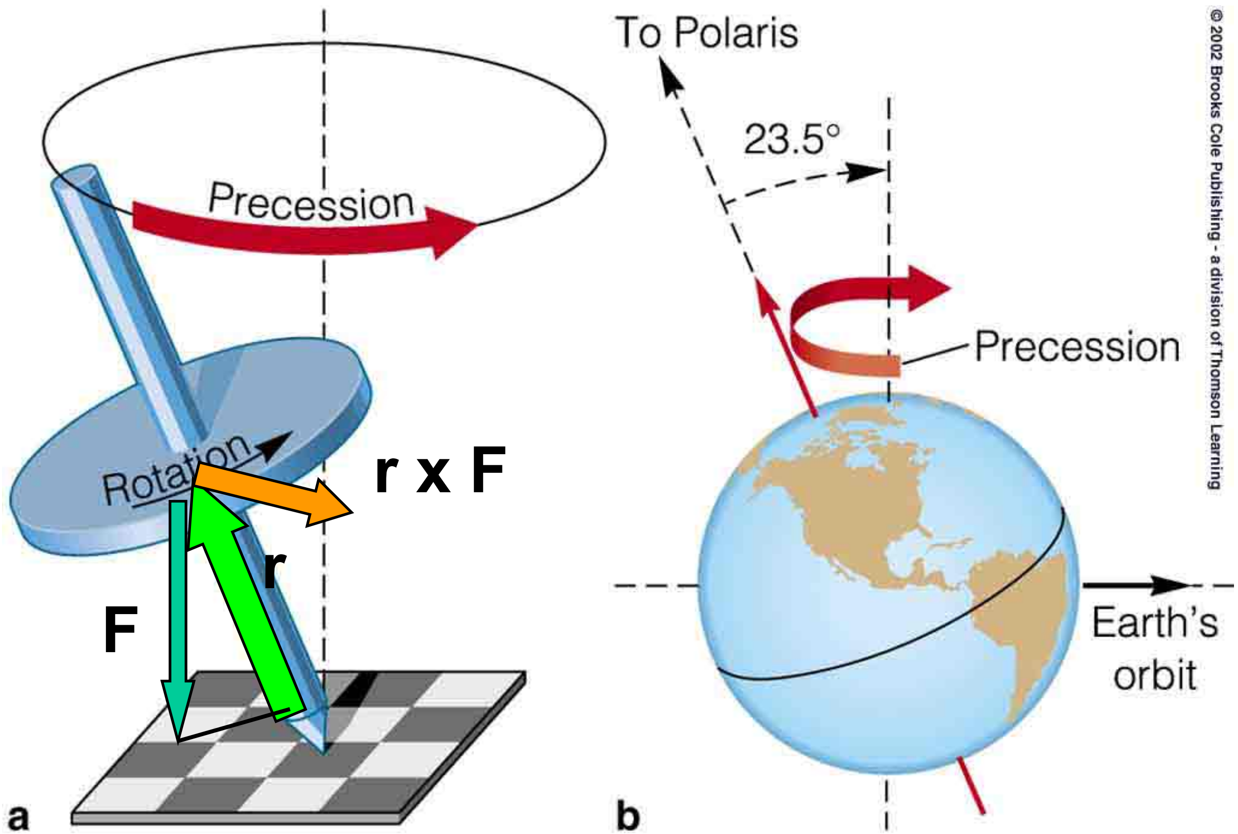
spin axis precession

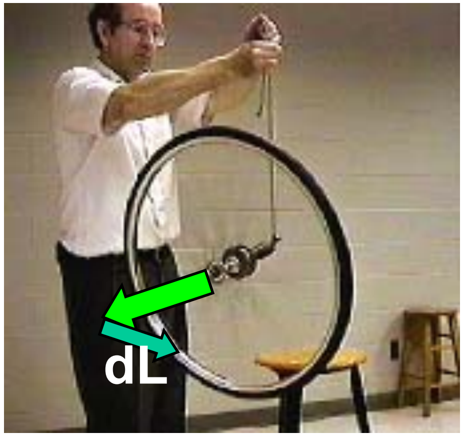


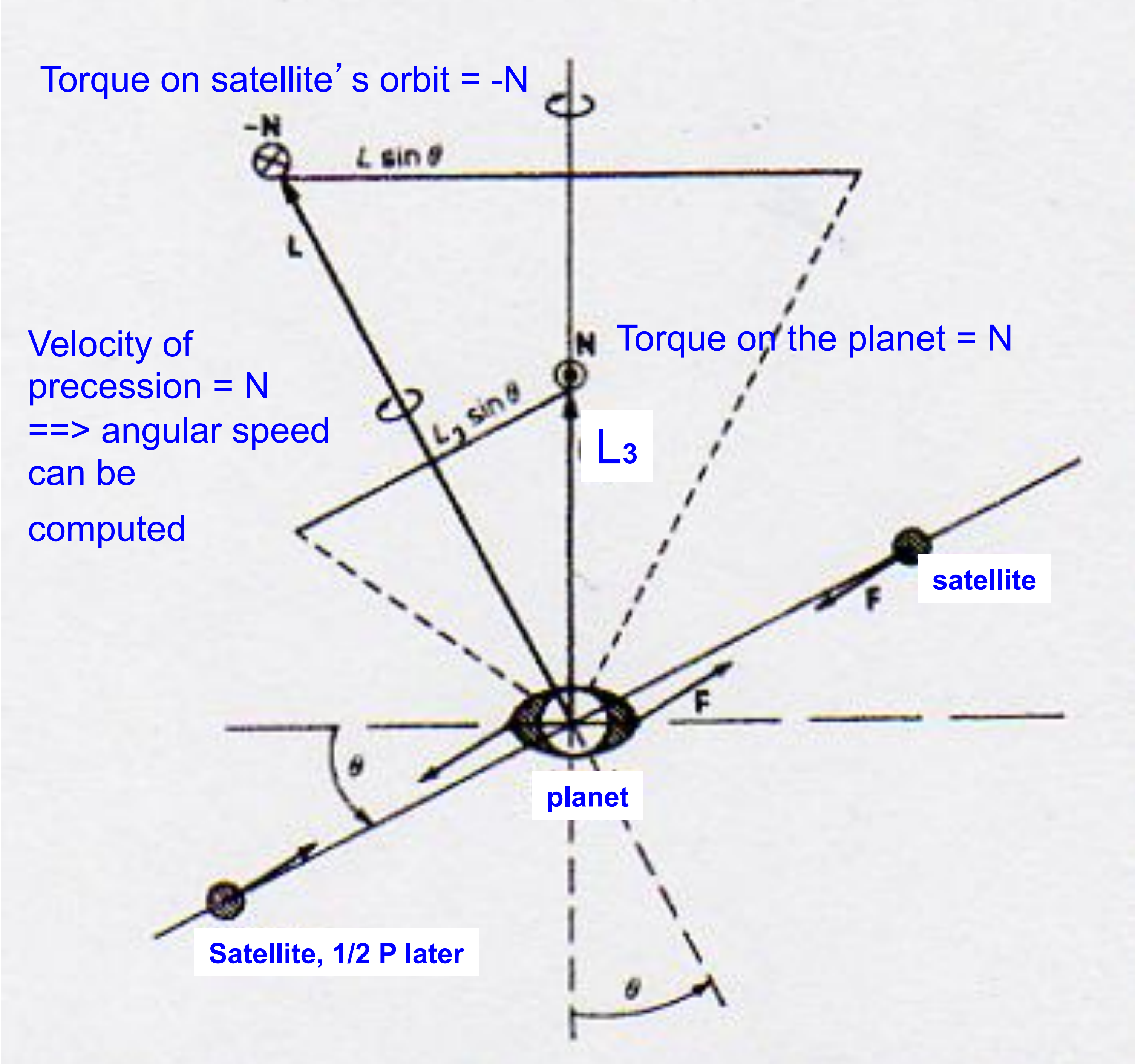
Exceptions (closed orbits):
two force laws only: $F \sim r^{-2}$, and $F \sim r$
(Bertrand's theorem)

Angular momentum vector shifted sideways by torque, value preserved ==> precession. $\overrightarrow{dL} = (\overrightarrow{torque})\,dt$

$$\overrightarrow{dL} = (\vec{r} \times \vec{F})\,dt$$






Torque on satellite’s orbit = -N
-N
L sin θ
L
Velocity of
precession = N
==> angular speed
can be
computed
N
Torque on the planet = N
L3 sin θ
L3
satellite
F
F
θ
planet
Satellite, 1/2 P later
θ

r = perturber distance

perturber mass

Obliquity (inclination)

$$N = -\frac{3GM_K(I_3 - I_1)}{2r^3}\sin\theta\cos\theta\,,$$

$I_{1,2,3}$ = three moments of inertia of the Earth, satisfying:

$$\frac{I_3 - I_1}{I_3} = 0{,}00327\,,$$

(from Earth flattenig)

Earth axis' precession (angular speed):

$$\omega_p = \frac{N}{\omega_3 I_3 \sin\theta}\,.$$

L_3 = $\omega_3 I_3$

$$\omega_p = -\frac{3GM_K}{2\omega_3 r^3}\frac{I_3 - I_1}{I_3}\cos\theta\,.$$

The corresponding period of precession of Earth axis = 33000 yr (from the Moon), and 81000 (from the sun)

Combined luni-solar precession has period 26000 yr = 51"/yr
(Note: 1/33000 + 1/81000 = 1/26000, angular speeds, not periods, add up).
Precession affects right ascention of objects in catalogues and maps, which therefore must state the "epoch" of coordinates.