Accretion disks

ASTC25

Lecture 11-12

Accretion disks

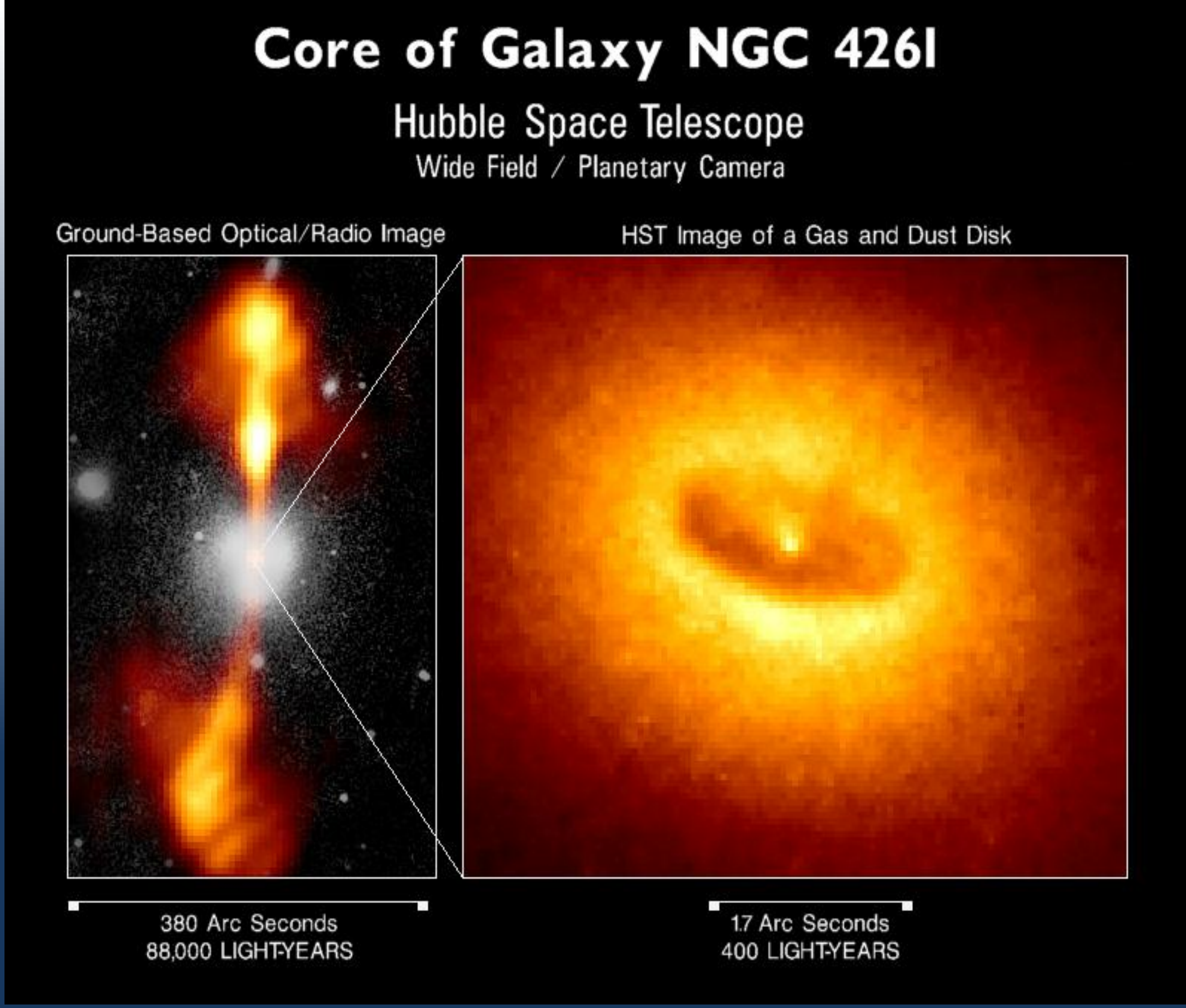
[to accrete ~ to collect, accumulate, gather;
accretion ~ inflow]

1. Huge accretion disks: QSOs, AGNs
2. Big disks in binary stars
3. Small disks – protostellar and protoplanetary disks = solar nebulae of Democritus, Kant, and Laplace
4. Tiny disks – planetary rings

1.

Accretion disk + Black Hole in the core of elliptical galaxy NGC 4261

(Hubble Space Telescope)



A disk of cold gas and dust fuels a black hole (BH). 300 light-years across, the disk is tipped by 60 deg, providing a clear view of the bright inner disk. The dark, dusty disk represents a cold outer region, which extends inwards to an ultra-hot accretion disk with a few AU from the BH. This disk feeds matter into the BH, where gravity compresses and heats the material. Hot gas rushes from the vicinity of the BH creating the radio jets. The jets are aligned perpendicular to the disk. This provides strong circumstantial evidence for the existence of BH "central engine" in NGC 4261.

Large AGN/Quasar disk luminosities

$L \sim 10^{46}$ erg/s (quasars) = 10^{12} $L_\odot$ ~ Milky Way, derive from the following fact:

Gravitational energy release in disk:
L_{dsk} = 50% * |-GM/R| (dM/dt)

R = 2 GM/c^2 (Schwarzschild radius, where v_{esc} = c in Newtonian physics)

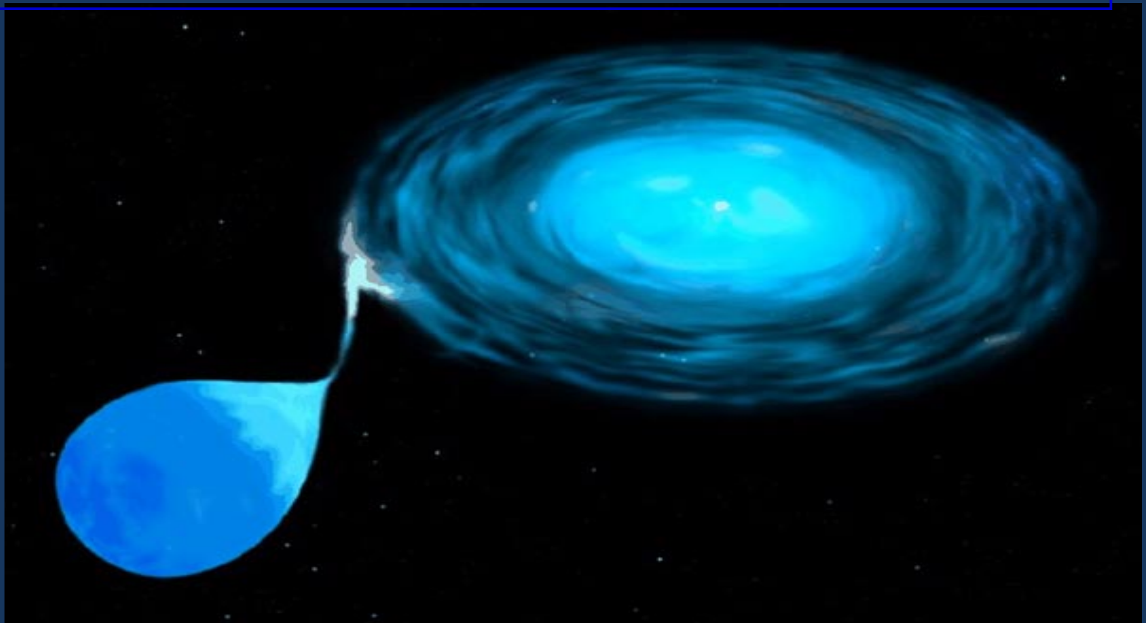
L_{dsk} ~ 25% d(Mc^2)/dt

ARTIST'S VIEW...

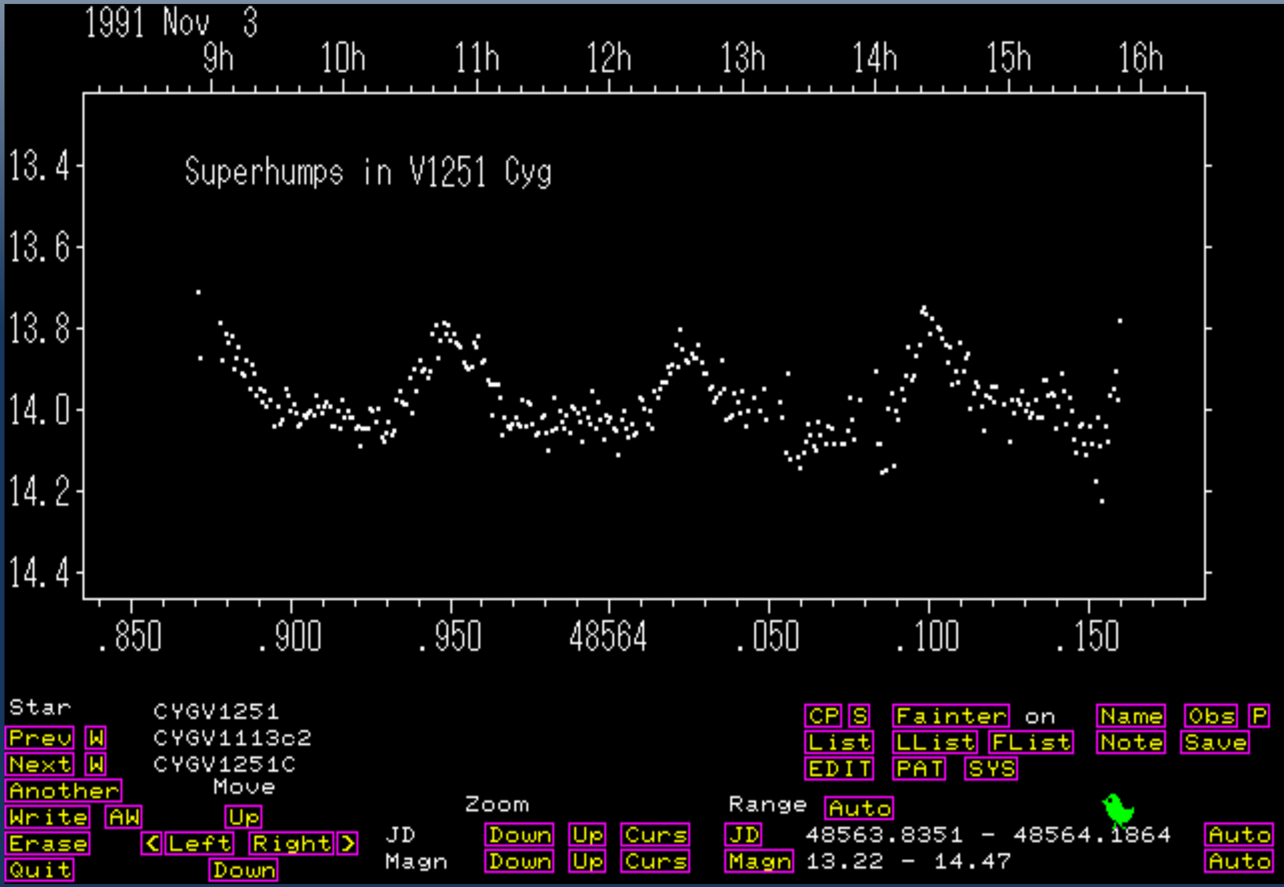
BH radius (Schwarzschild radius)
R = 2GM/c^2 = 3 km ($M/M_\odot$)
e.g.,
10 $M_\odot$ ==> 30 km (binaries)
3e6 $M_\odot$ ==> 0.06 AU (galaxies)
1e8 $M_\odot$ ==> 2 AU (AGNs)
1e9 $M_\odot$ ==> 20 AU (quasars)

2.

Accretion disks are often found in close, interacting pairs of stars, such as the cataclysmic variables (CVs). One star, originally more massive, evolves to a compact companion: a white dwarf or perhaps a neutron star (pulsar) or a black hole. The other, originally less massive, star bloats toward the end of its main-sequence life and fills the critical surface (Roche Lobe) after which it sends a stream of gas onto a compact companion, creating an accretion disk.



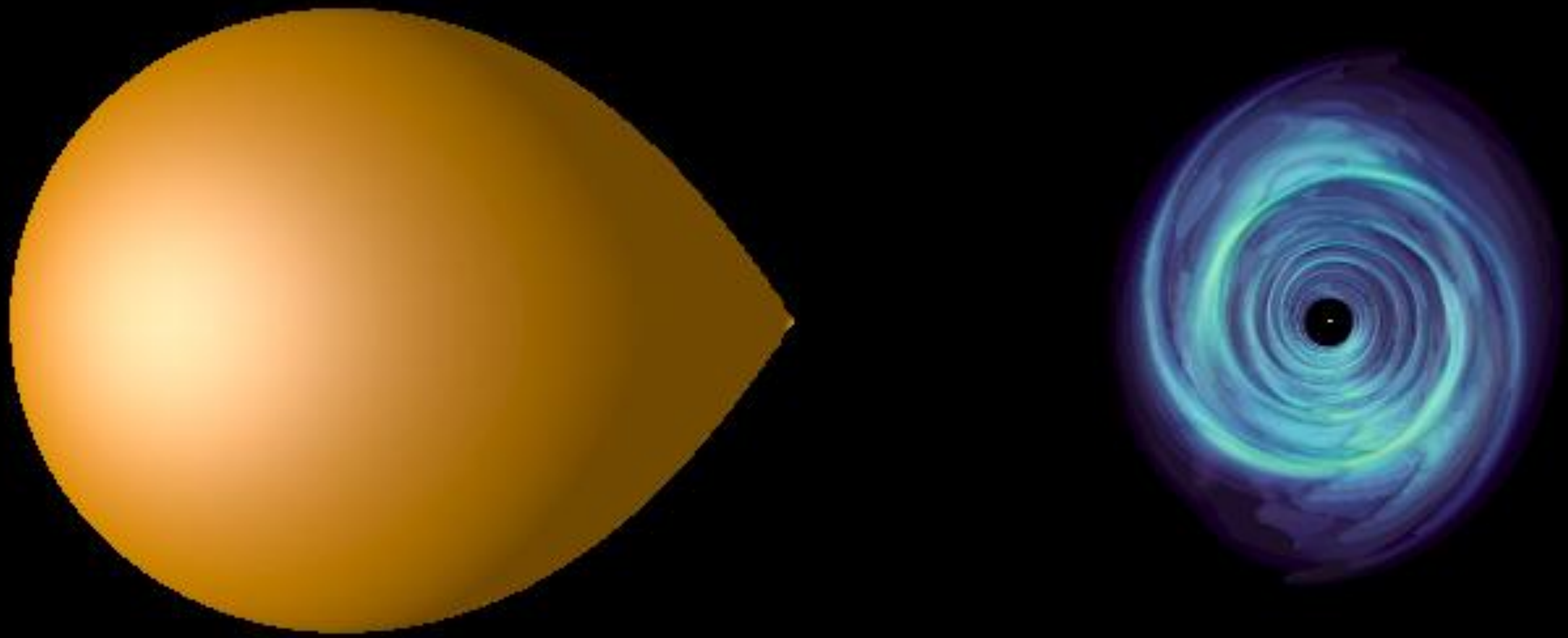
Superhumps are distortions (local maxima) of the light curve of the s-called dwarf novae systems, belonging to cataclysmic variables class. The light curve is due to a varying viewing angle of the accretion disk and companion. Superhumps are due to resonances and waves in the disk.



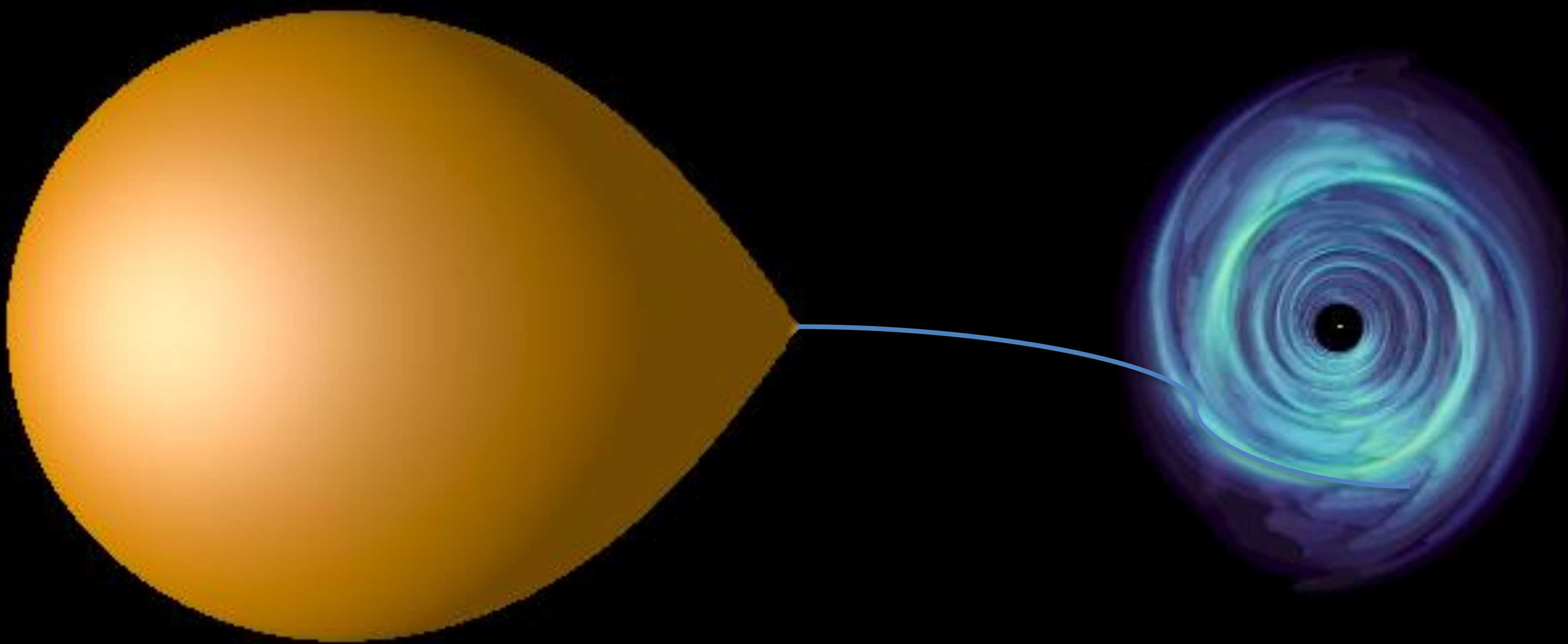
VH-1

3. Small disks in binary stars

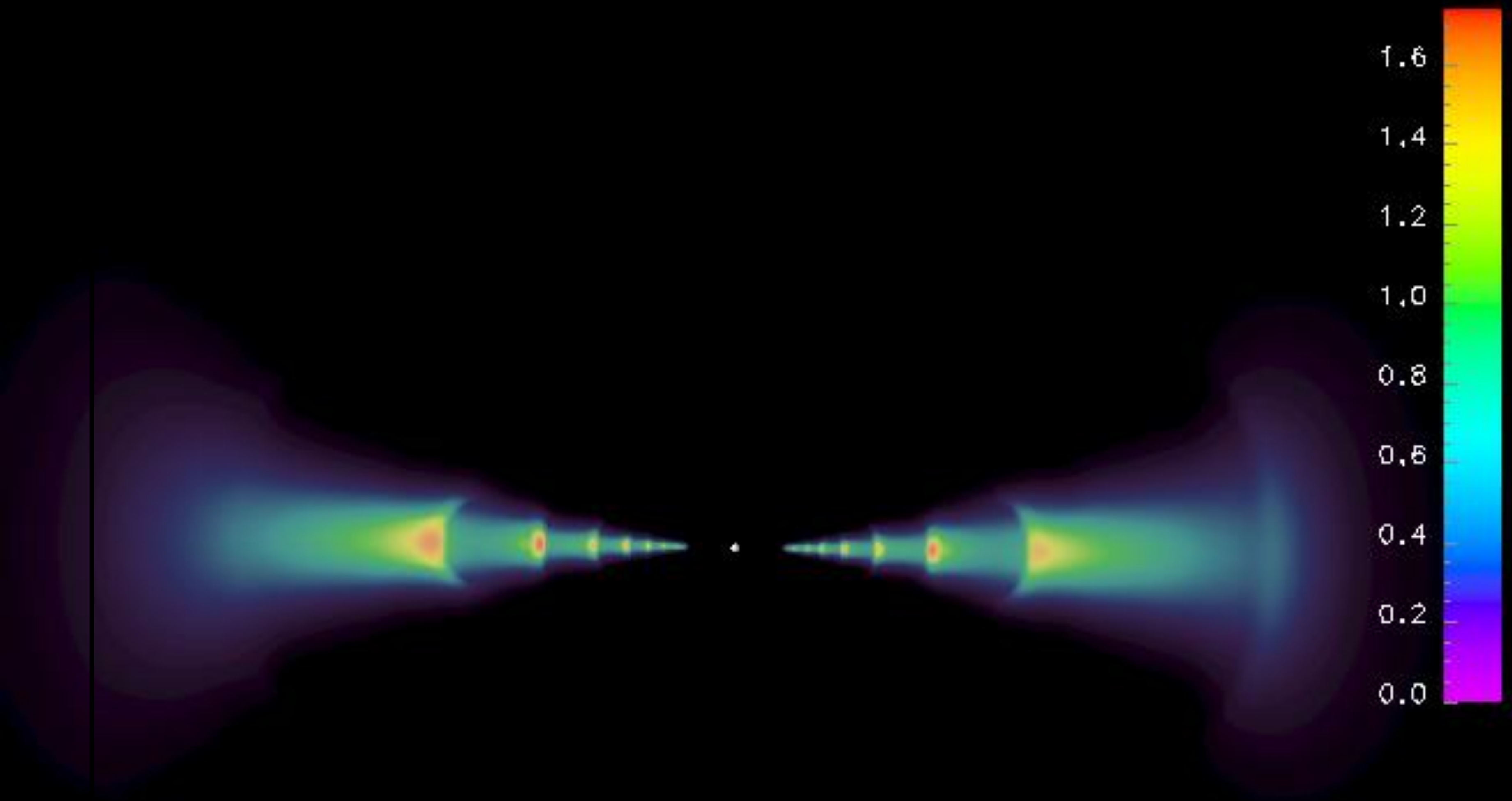
sizes up to ~ 10 AU



VH-1



Roche lobe overflow & mass transfer

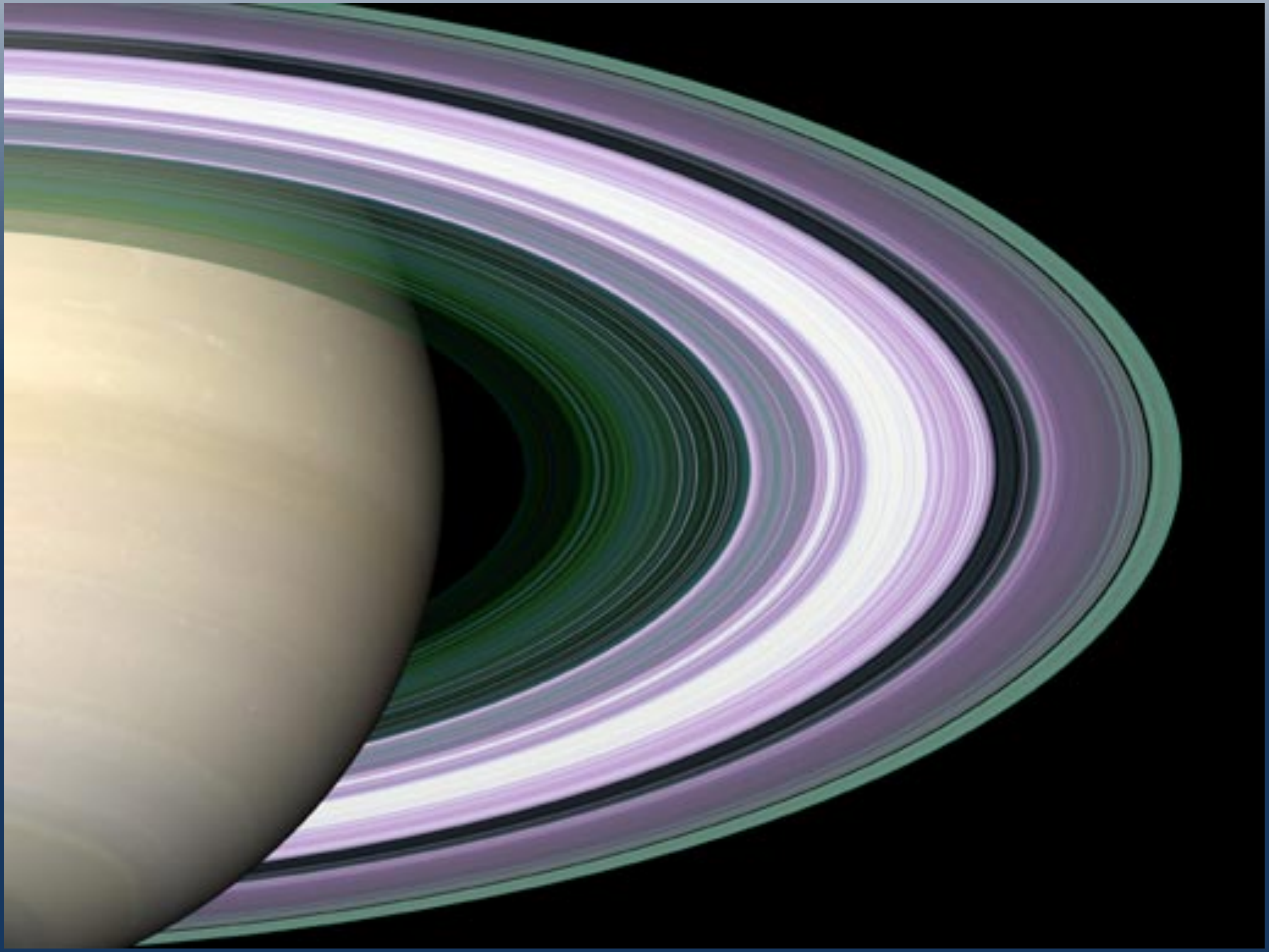


shock waves in a vertical cross-section of a disk

4. Planetary rings are also accretion disks, sort of.

They are special: their thickness is extremely small:

$z/r \sim 10$ m / 66000 km $\sim 10^{-6}$,   very slowly accreting disks.



From: Diogenes Laertius, Φιλοσοφοι βιοι (3rd cn. A.D.), IX.31

The first description of an accretion disk?

"The worlds come into being as follows: many bodies of all sorts and shapes move from the infinite into a great void; they come together there and produce a single whirl, in which, colliding with one another and revolving in all manner of ways, they begin to separate like to like."

Leucippus, ca. 460 B.C.?

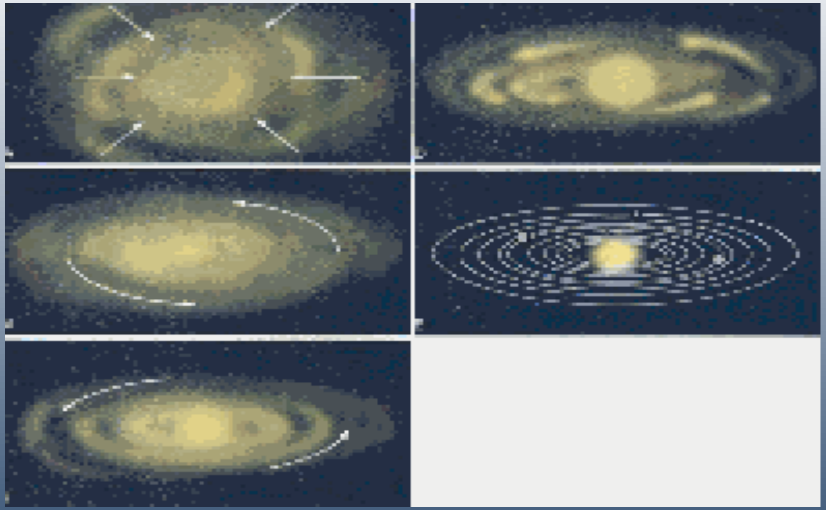
Kant-Laplace nebula ~ primitive solar nebula ~ accretion disk ~ protoplanetary disk ~ T Tauri disk

R. Descartes (1595-1650) - vortices of matter -> planets

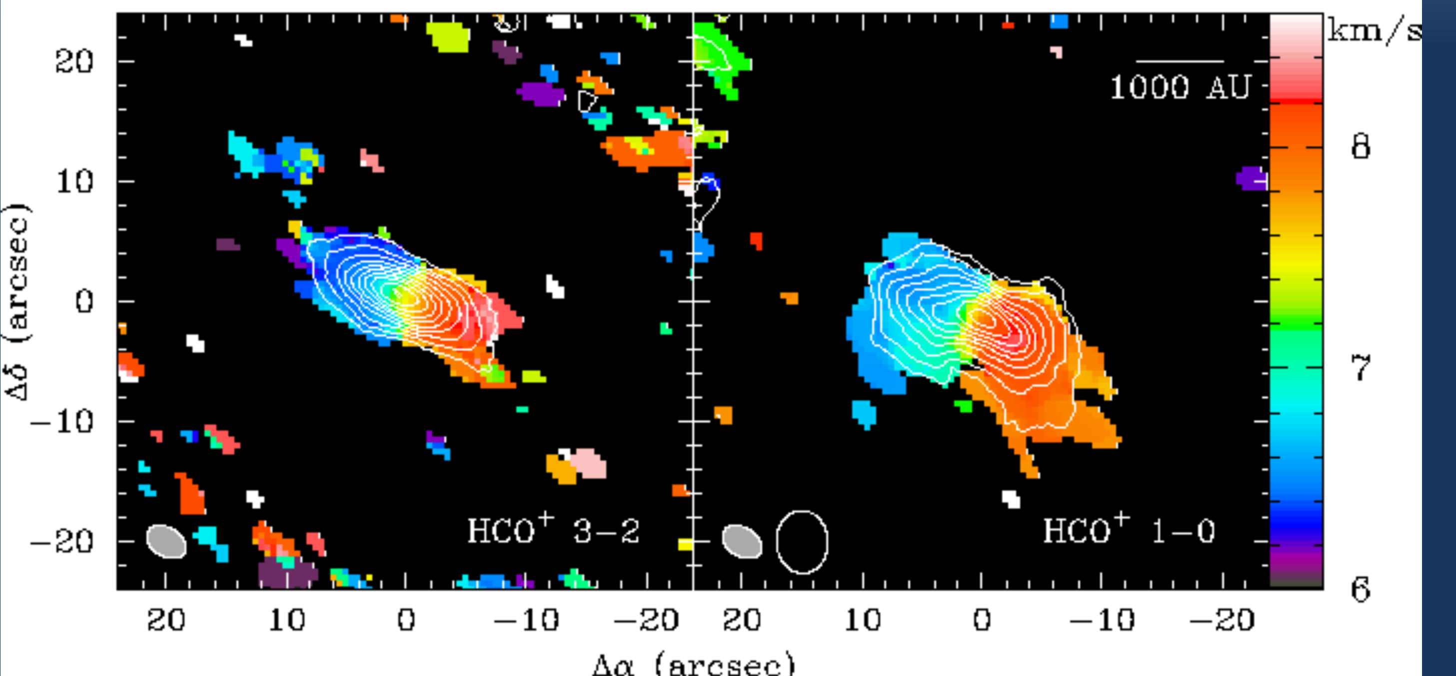
I. Kant (1755) - nebular hypothesis

(recently revived by: Cameron et al, Boss)

P.S. de Laplace (1796) - version with rings



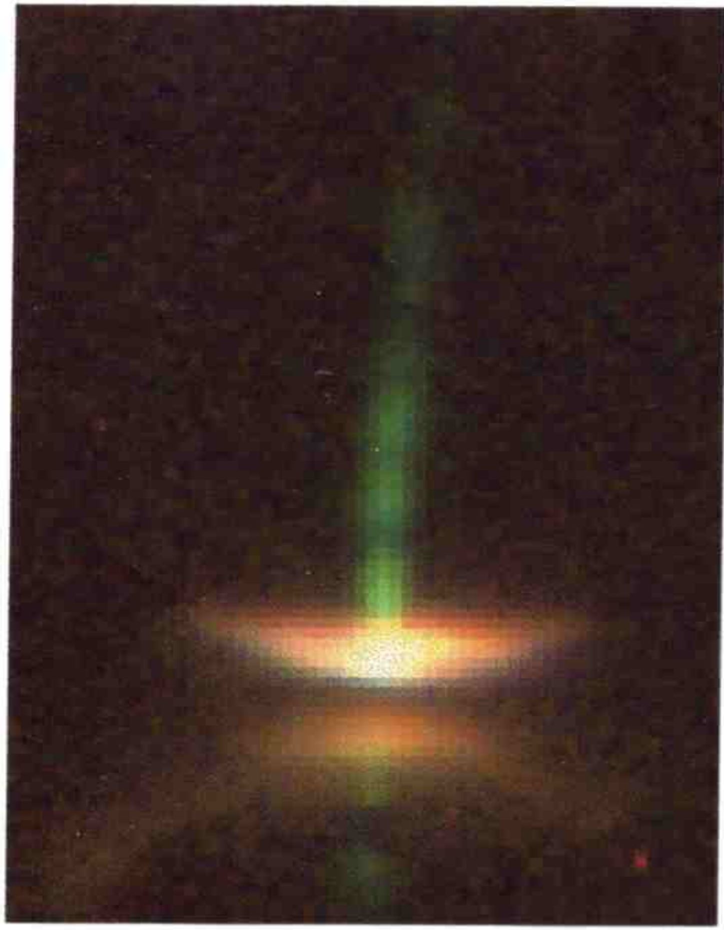
A rotating infalling disk around the young stellar object L1489 IRS imaged at 267 and 89 GHz with the Berkeley–Illinois–Maryland Array



(Hogerheijde 2000)

• Age ~1 Myr or less

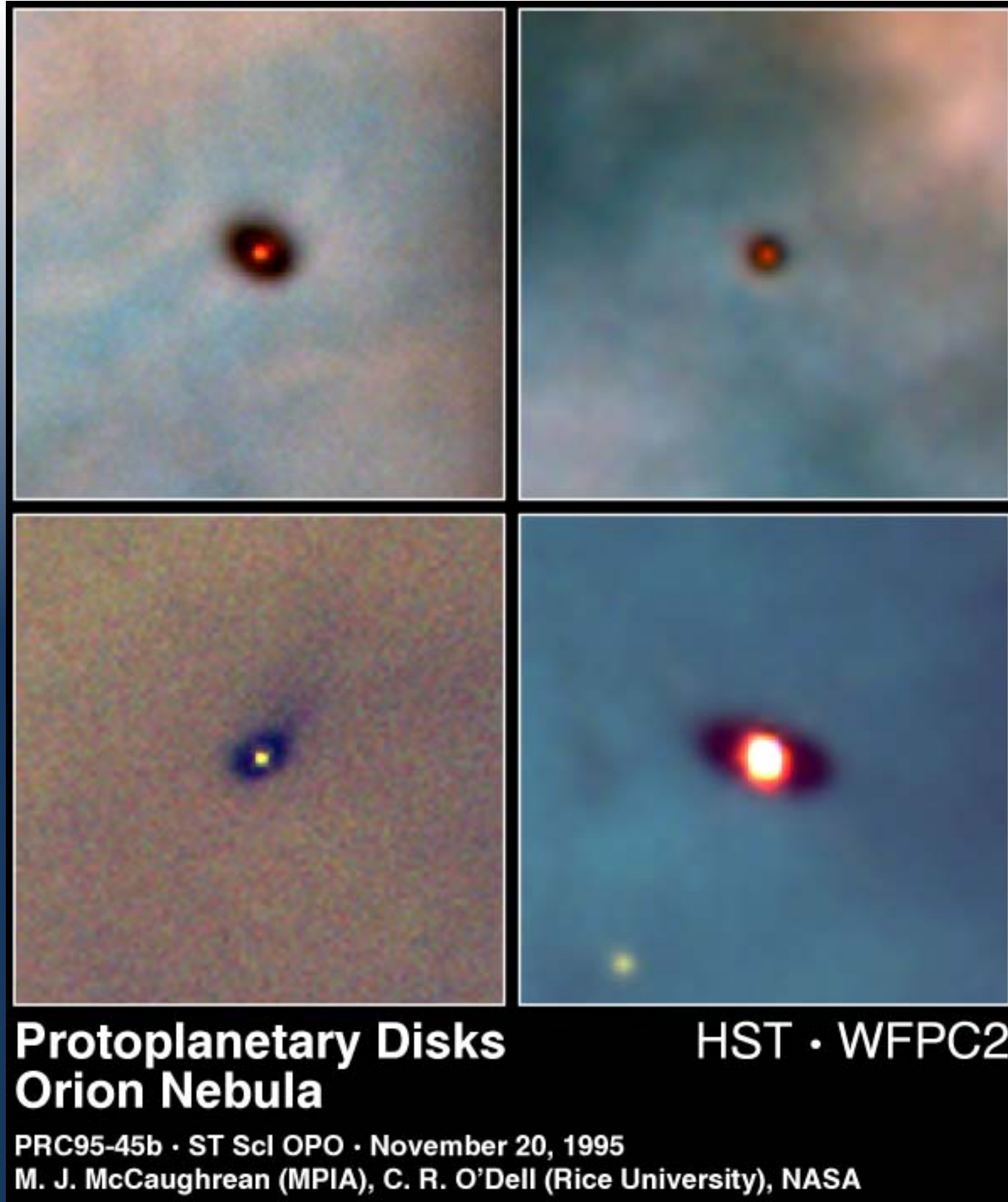
Primordial solar nebulae = protoplanetary disks =
= protostellar disks = T Tau disks = accretion disks

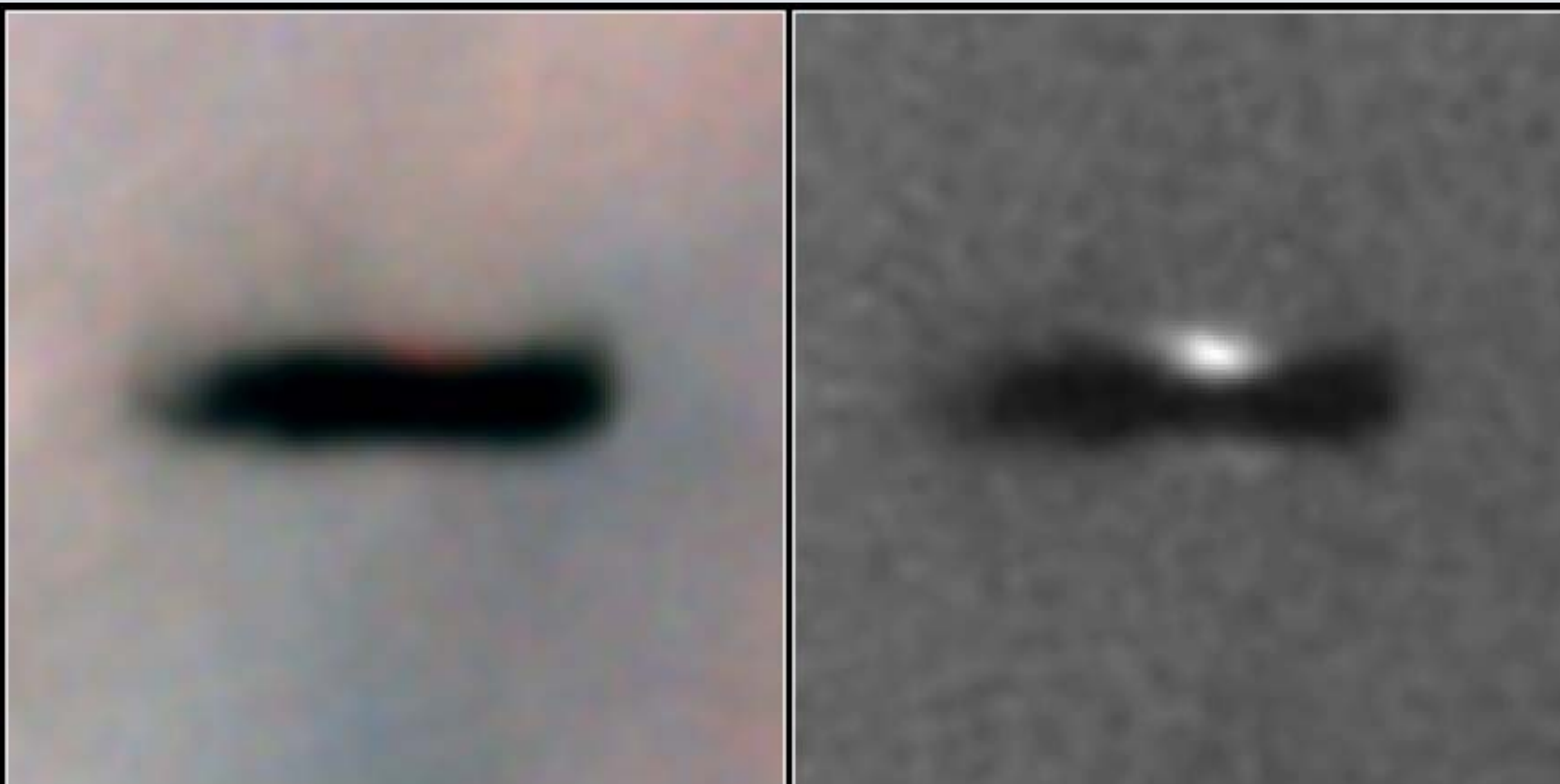


HH30 (C. Burrows, 1997)

Silhouette disk in Orion
(McCaughrean, O'Dell)

These disks are very opaque (opt. thickness $\tau \sim 10^5$ in the visible)
Dust(+ice) in such disks is frozen dynamically to gas,
Dust:gas = 1:100 by mass; gas = H + He (mostly)

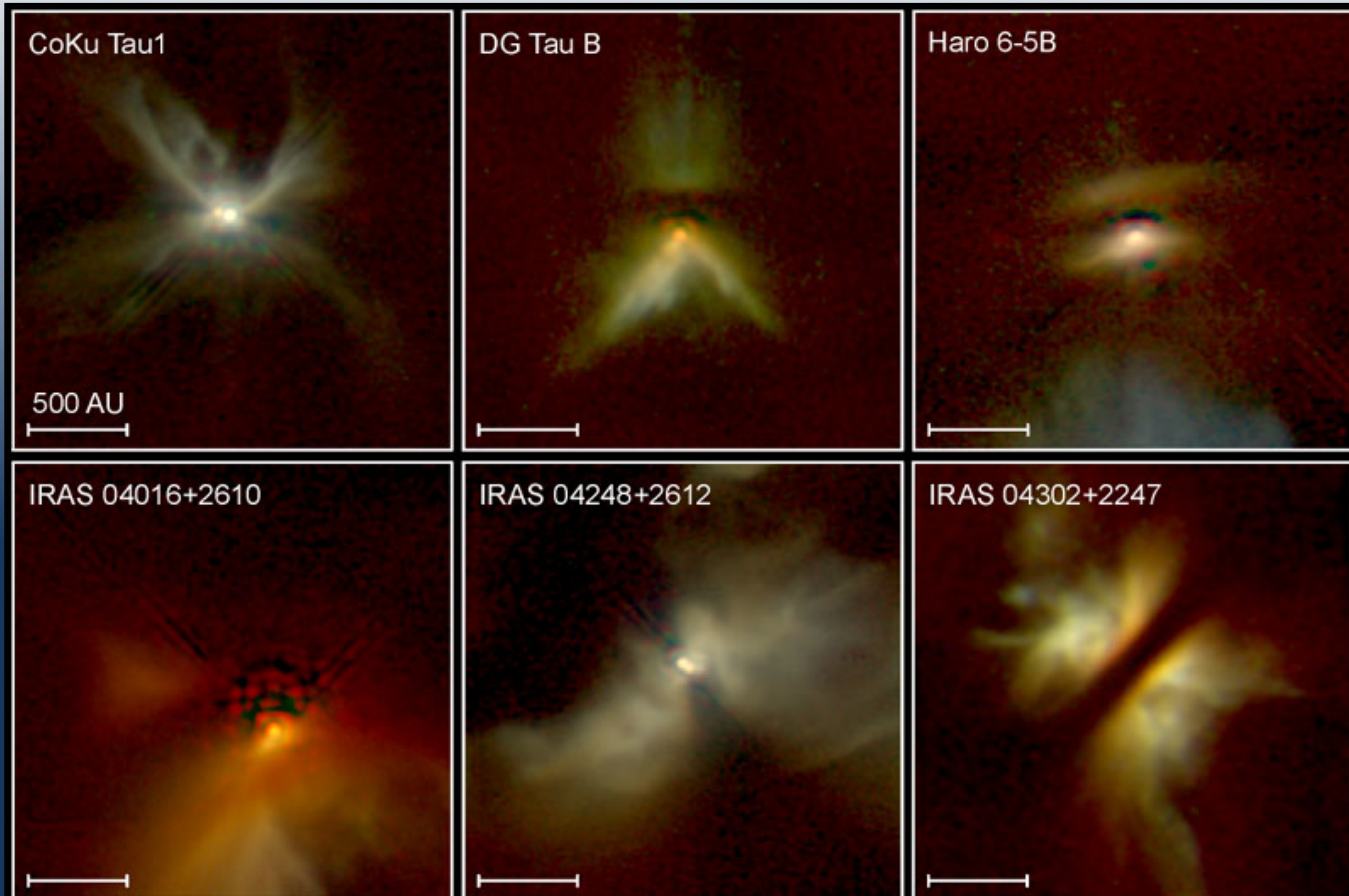

Protoplanetary Disks
Orion Nebula
HST • WFPC2
PRC95-45b • ST ScI OPO • November 20, 1995
M. J. McCaughrean (MPIA), C. R. O'Dell (Rice University), NASA



Edge-On Protoplanetary Disk
Orion Nebula

HST • WFPC2

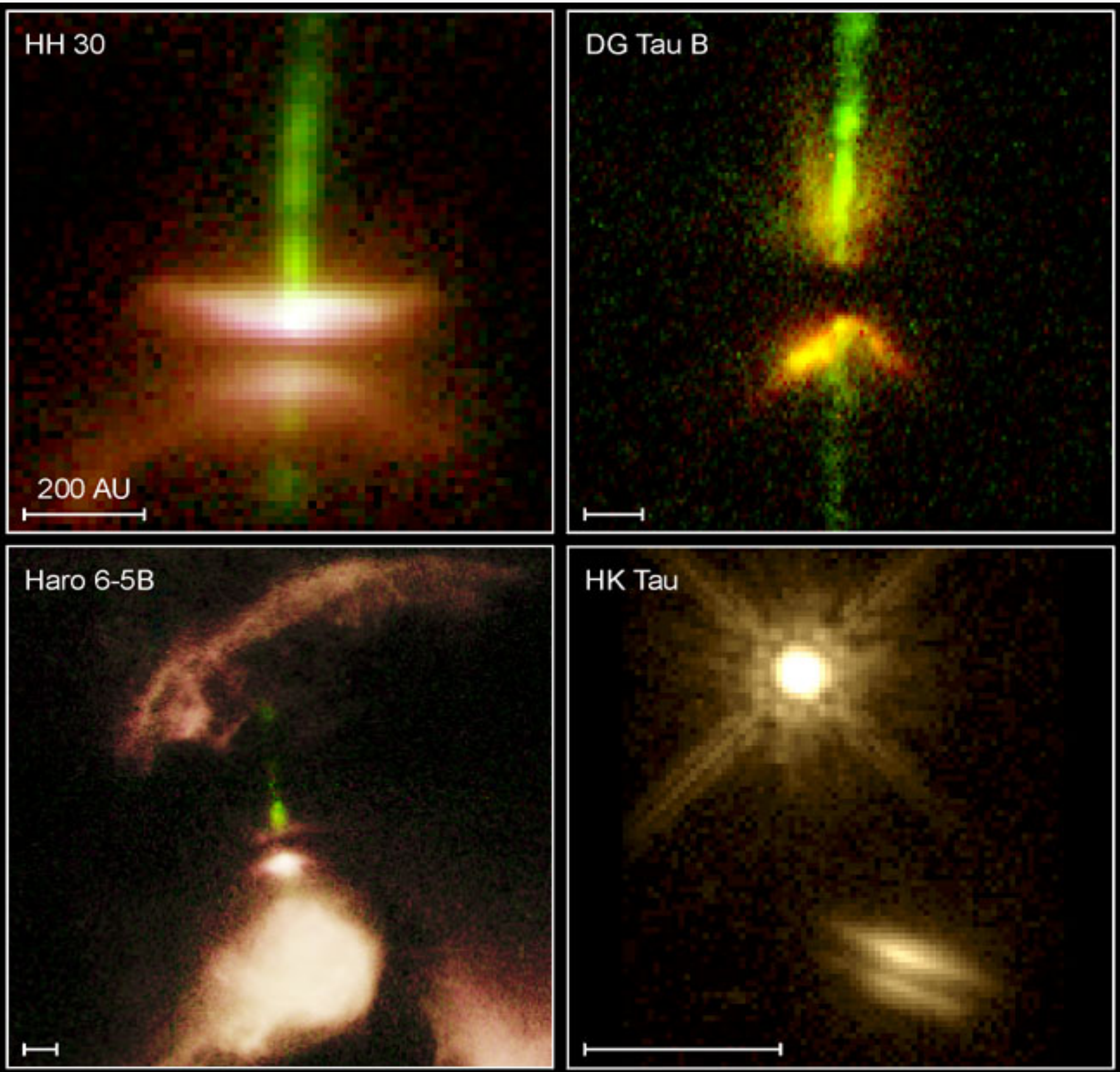
PRC95-45c • ST ScI OPO • November 20, 1995
M. J. McCaughrean (MPIA), C. R. O'Dell (Rice University), NASA



Young Stellar Disks in Infrared **HST • NICMOS**

PRC99-05a • STScI OPO

D. Padgett (IPAC/Caltech), W. Brandner (IPAC), K. Stapelfeldt (JPL) and NASA

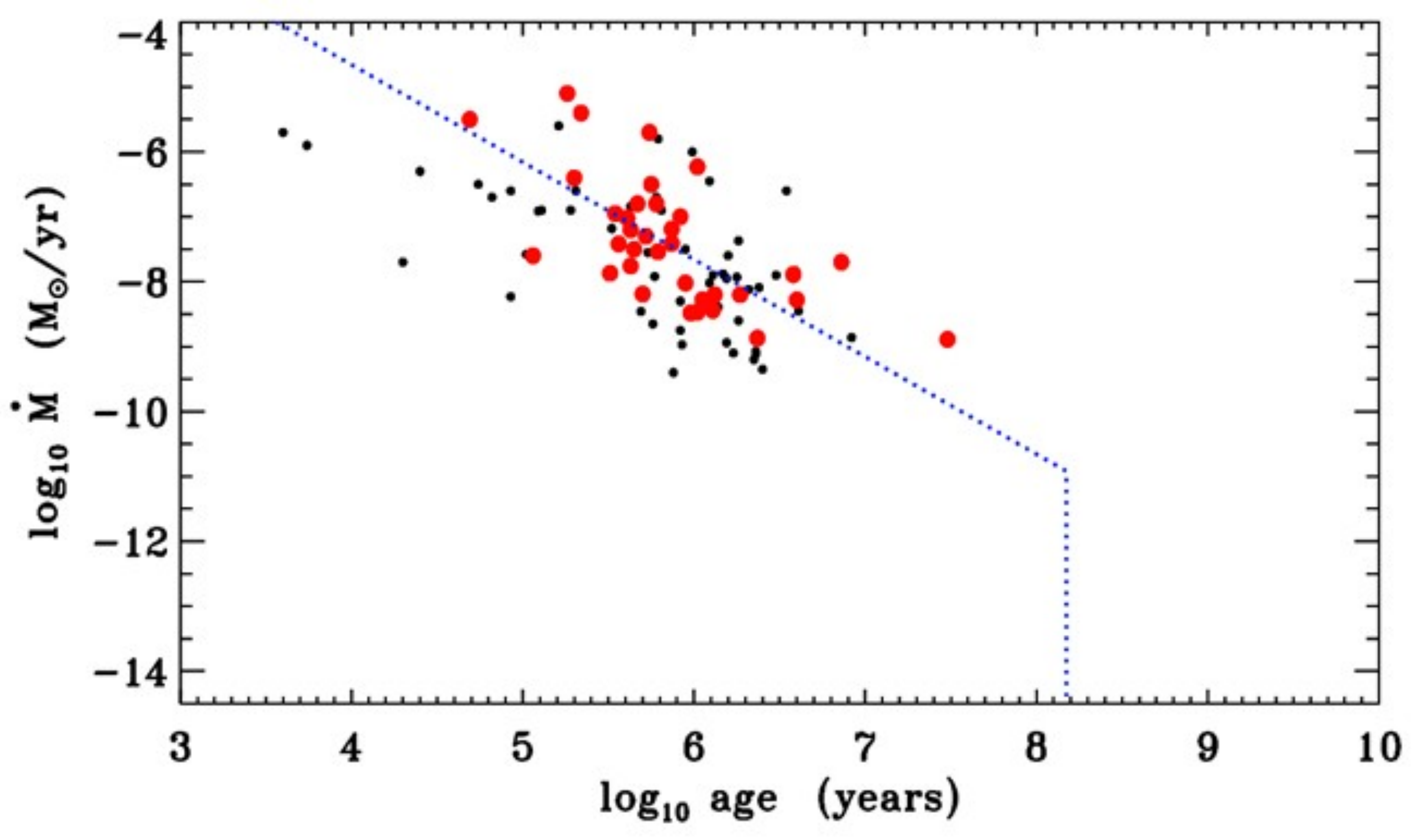


Disks around Young Stars HST • WFPC2

PRC99-05b • STScI OPO

C. Burrows and J. Krist (STScI), K. Stapelfeldt (JPL) and NASA

Accretion rate in T Tau disks decreases with time (age)



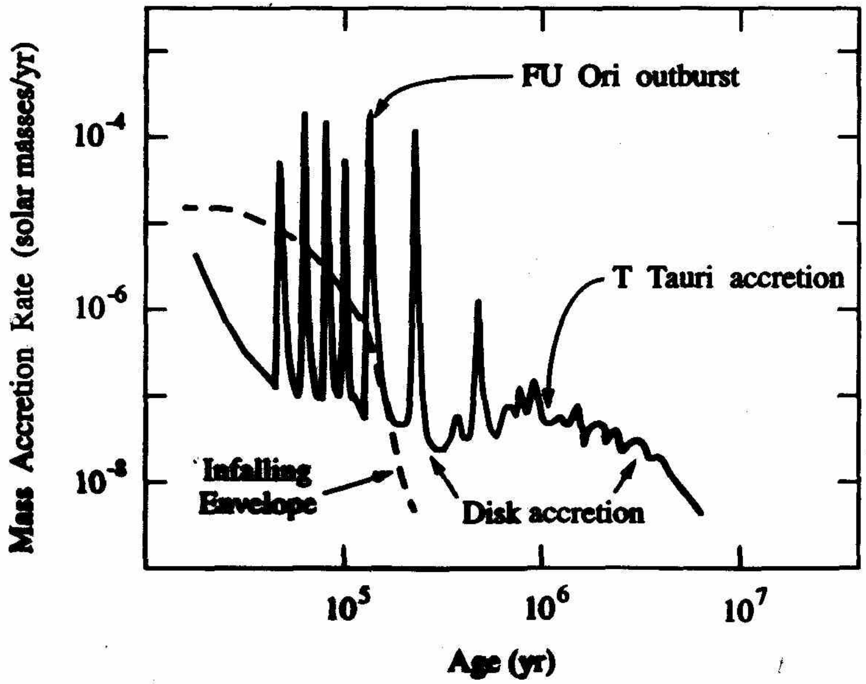
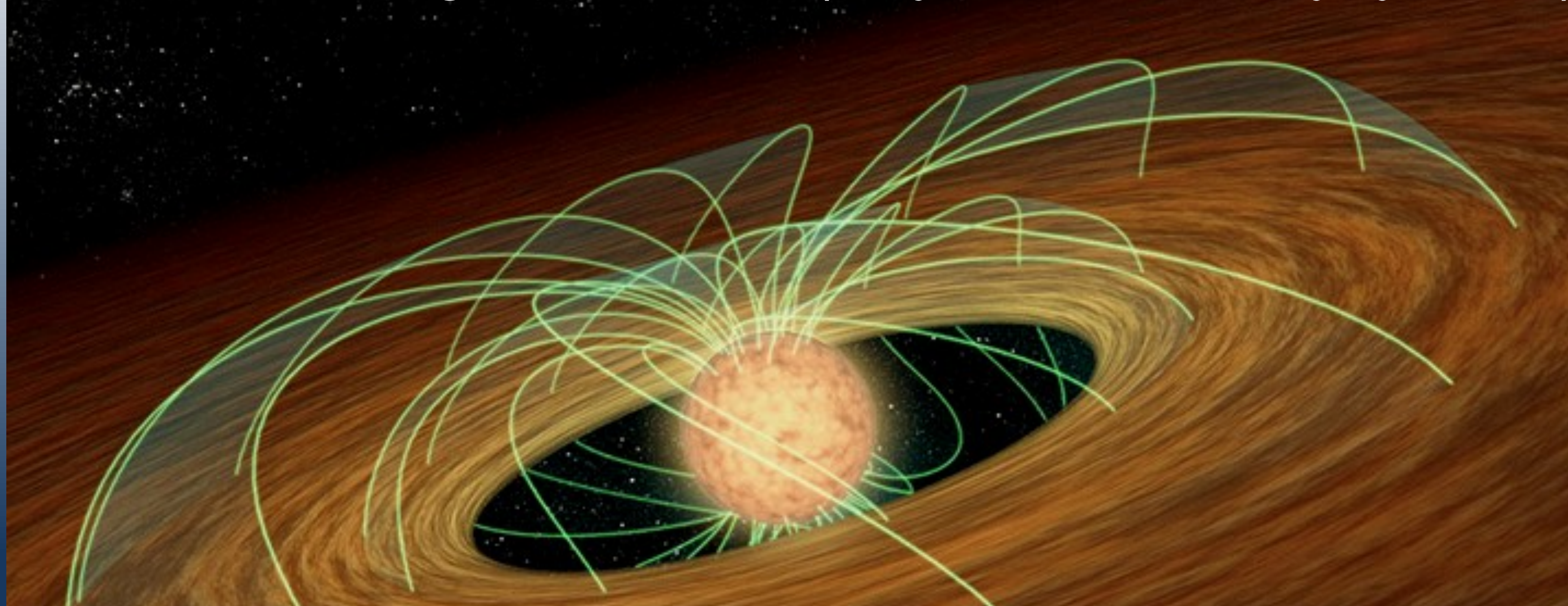
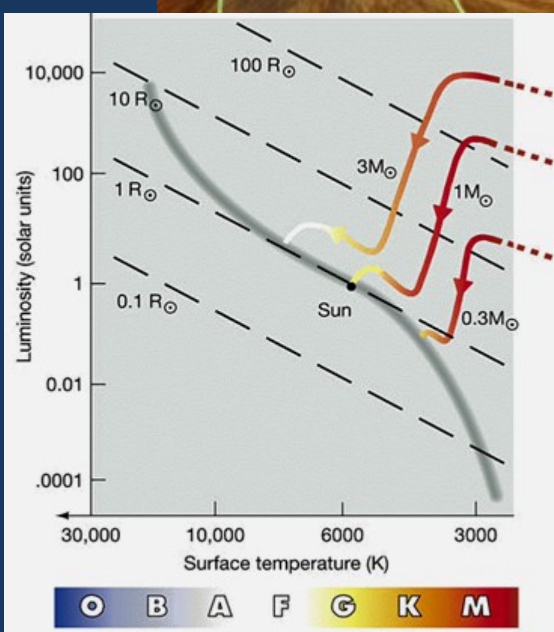


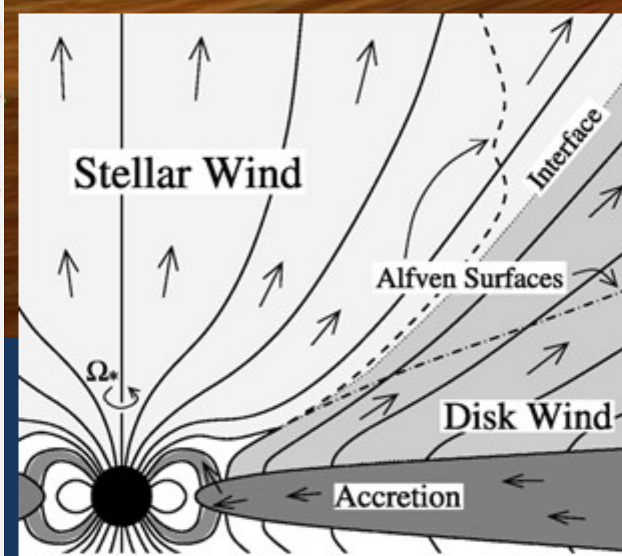
Figure 7. Sketch of disk evolution with time, summarizing the ideas presented in this chapter. The disk remains most of the time in a quiescent state, punctuated by episodes of high $\dot{M}$ as long as the envelope feeds mass to the disk. When infall ceases, the disk evolves viscously, and $\dot{M}$ slowly decreases with time.

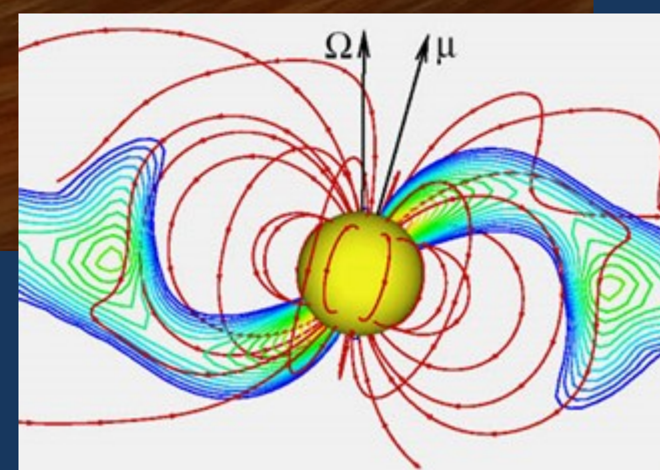
Observed dM/dt ~ 10^{-6} M_{sun}/yr for ~0.1 Myr time
➔ total amount accreted ~0.1 M_{sun}
Observed dM/dt ~ 10^{-7} M_{sun}/yr for ~1 Myr
➔ total amount accreted ~0.1 M_{sun}

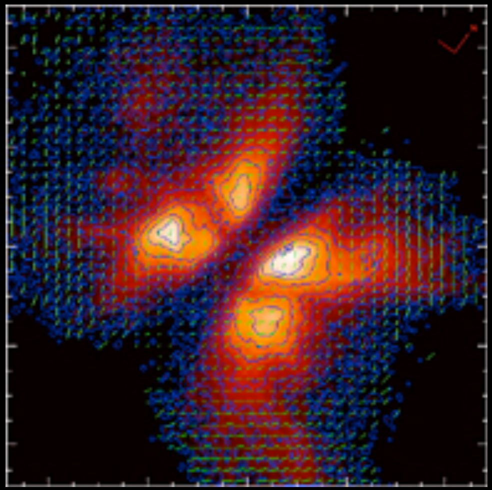
T Tau star, schematic diagram of magnetic field in the central clearing & evolution (Hayashi and Henyey track)



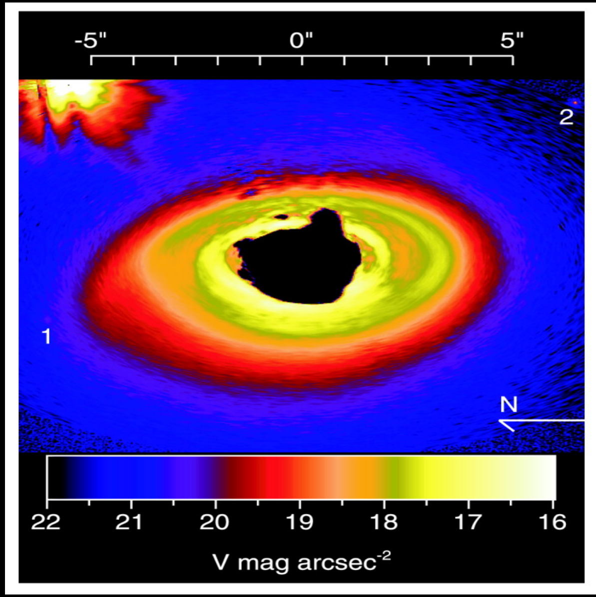






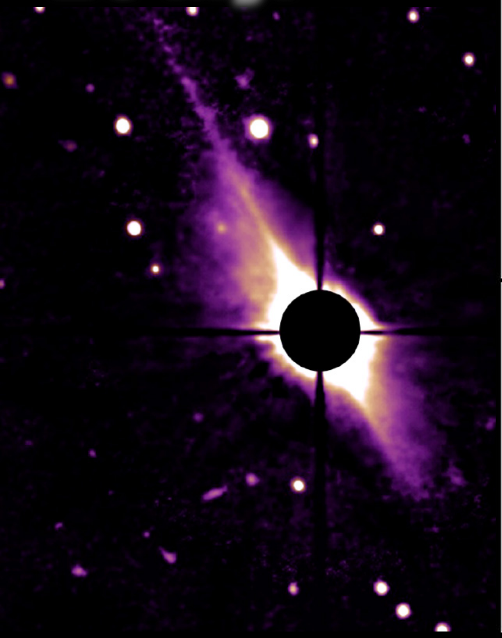


<1 Myr

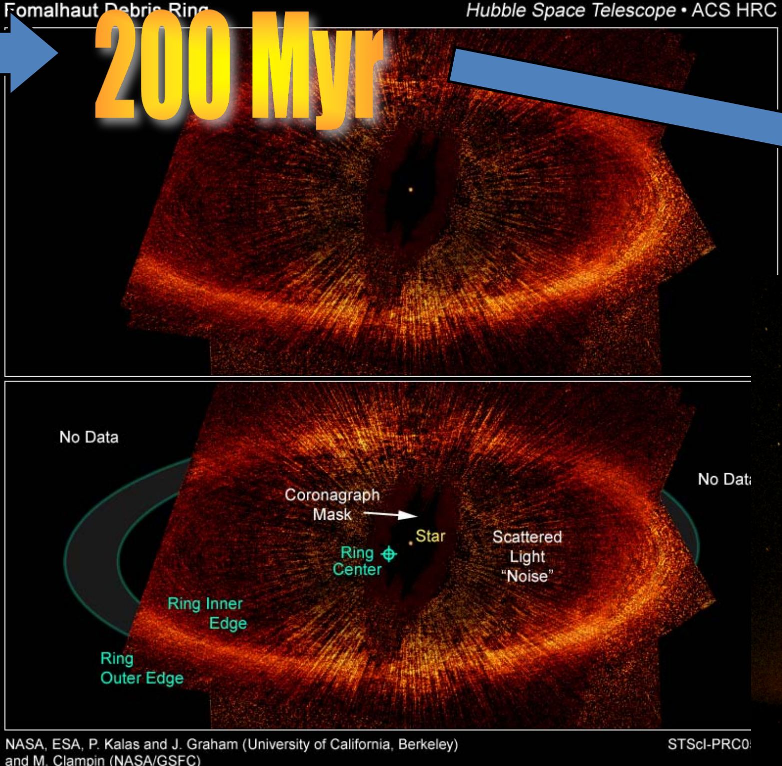


5 Myr

20 Myr



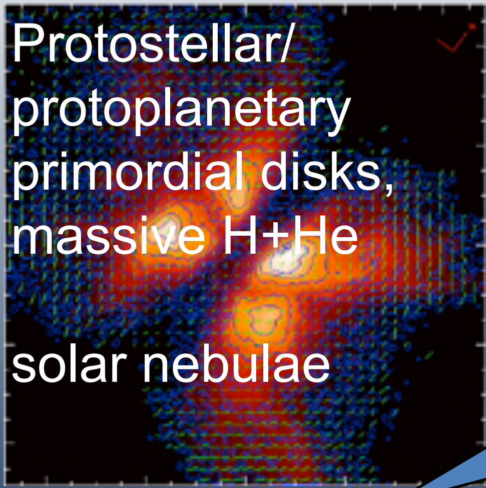
200 Myr



4567 Myr

T Tau, Classical

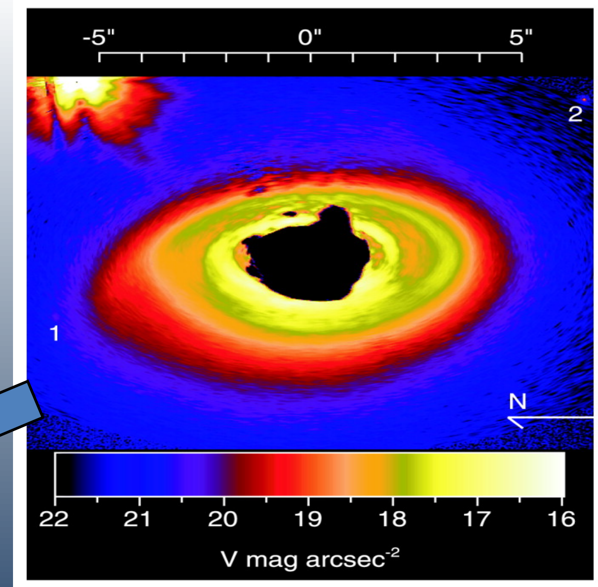
<1 Myr



Protostellar/ protoplanetary primordial disks, massive H+He

solar nebulae

5 Myr



Transition disks,

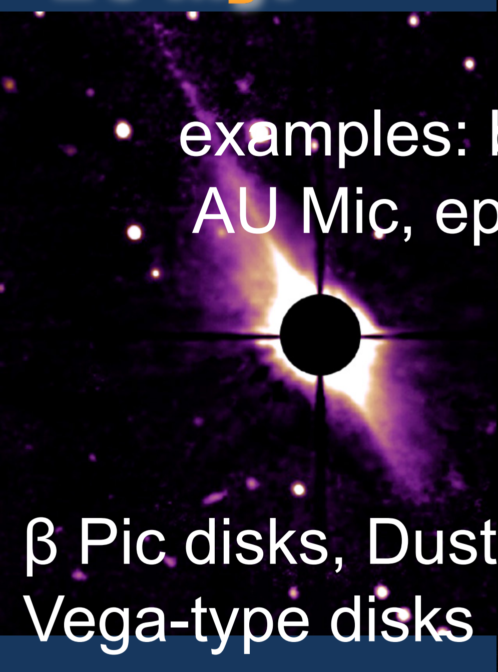
most H+He lost

20 Myr

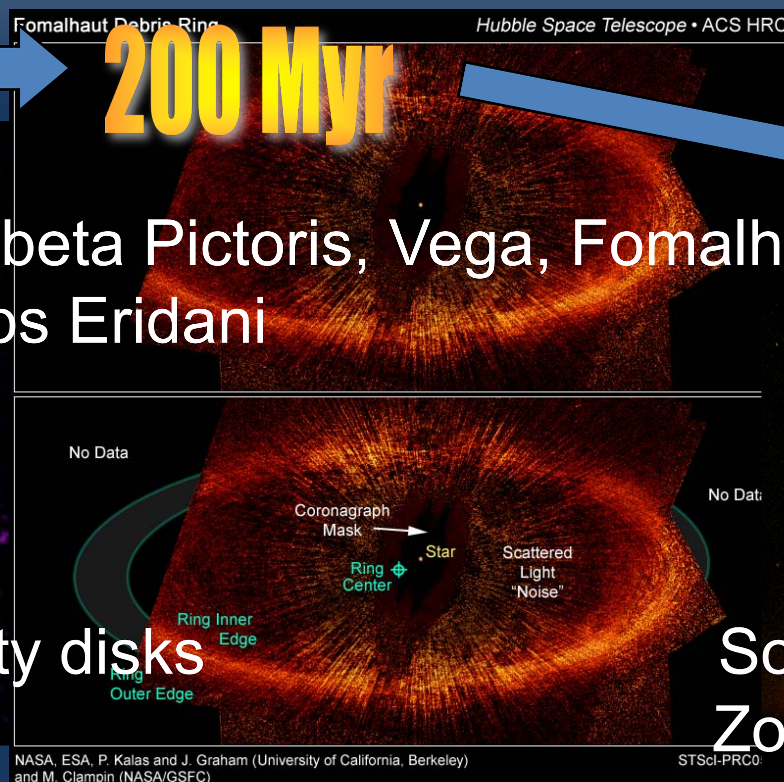
200 Myr

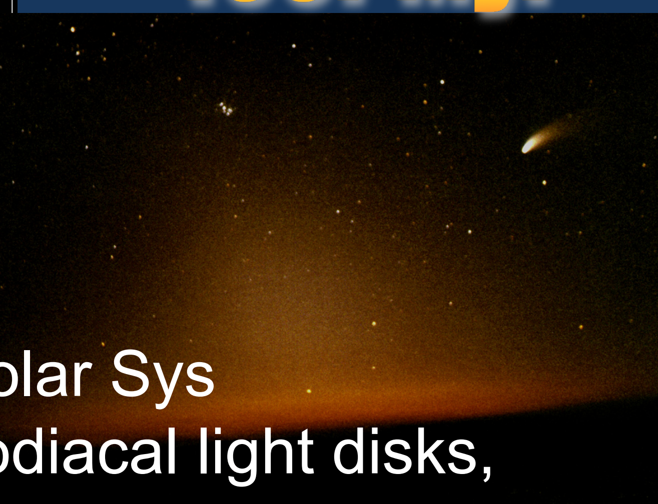
4567 Myr

- examples: beta Pictoris, Vega, Fomalhaut, AU Mic, eps Eridani



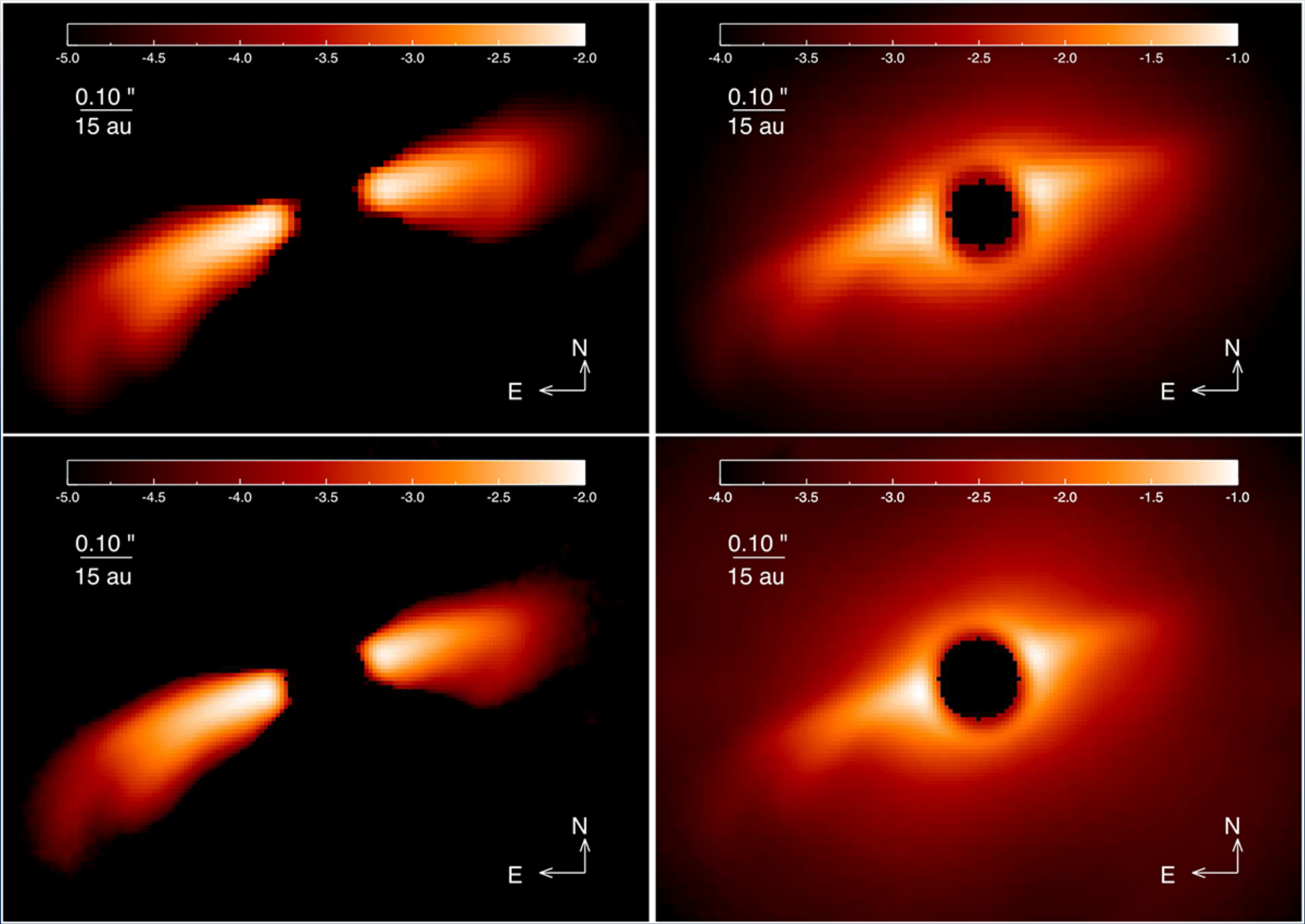
β Pic disks, Dusty disks
Vega-type disks



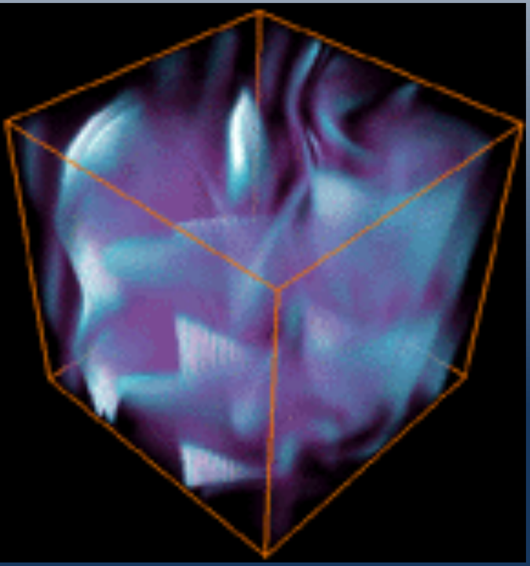


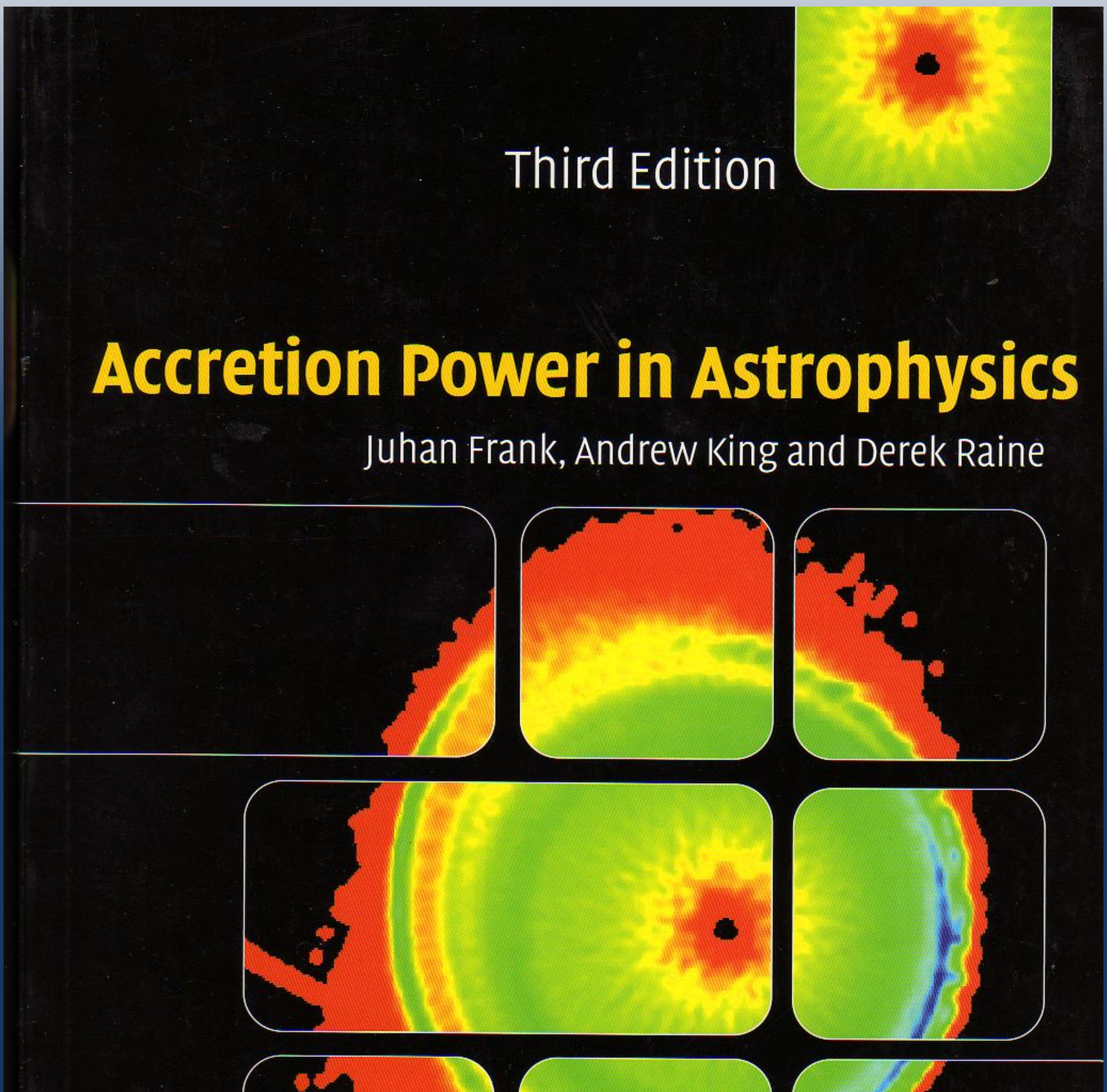
Solar Sys
Zodiacal light disks,

RY Lupi - the first scattered light image of a transition disk



Facts from of accretion disk theory





FUNDAMENTAL EQ'S OF GAS DYNAMICS

(1) $$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{v} = 0$$ continuity equation (mass eq.)

mass flux

(2) $$\frac{D\vec{v}}{Dt} = -\frac{1}{\rho}\vec{\nabla}p + \frac{1}{\rho}\vec{\nabla}\cdot\hat{\sigma} - \vec{\nabla}\Phi$$ eqs of motion (momentum eqs)

acceleration = pressure gradient + viscous forces + gravity

this Lagrangian form is written in an Eulerian frame as

$$\frac{D\vec{v}}{Dt} = \left.\frac{\partial \vec{v}}{\partial t}\right|_{\vec{x}} + \left.(\vec{v}\cdot\vec{\nabla})\vec{v}\right|_{\vec{x}}$$

Viscous forces in (x,y,z) coordinate system:

$$\hat{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \quad \text{where}$$

$$\sigma_i = +\lambda \vec{\nabla}\cdot\vec{v} + 2\mu \frac{\partial v_i}{\partial x_i}, \qquad i = 1,2,3$$

$$\tau_{ij} = \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right).$$

$\rho \nu = \mu$ = shear viscosity coefficient

λ = bulk —"— "—

e.g., for monoatomic gas $\lambda = -\frac{2}{3}\mu$

Largest shear in a thin disk is azimuthal

Other components and bulk viscosity neglected!

$\Rightarrow$ acceleration $\propto \nu$ = kinematic (shear) viscosity coefficient

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(3) $$\frac{DU}{Dt} = -\frac{1}{\rho}\vec{\nabla}\cdot\vec{F} - \frac{p}{\rho}\vec{\nabla}\cdot\vec{v} - \hat{\sigma} : \overset{>0}{\nabla\vec{v}} + \varepsilon$$

rate of gain of internal energy per unit vol. = conduction + reversible compression + irreversible viscous heating + internal sources

Energy equation in the simplest version of theory is replaced with polytropic relation

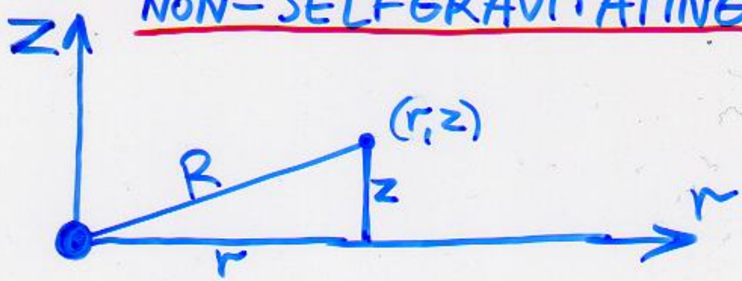
$$p = p(\rho) = K \cdot \rho^{\gamma}$$, or even

$$p = c^2 \rho \quad (\gamma = 1)$$ isothermal equation of state of gas.

(c: soundspeed)

AXISYMMMETRIC disks in cylindrical coordinates:

NON-SELFGRAVITATING



$$M_d \ll M_*$$

$$-\vec{\nabla}\Phi \cong -\frac{\vec{R}}{R} \cdot \left(\frac{GM_*}{R^2}\right)$$

$$\simeq -\frac{z}{r} \cdot \frac{GM_*}{r^2}$$

Vertical motions very slow $\Rightarrow$ negligible contribution to pressure ($\approx$ zero dynamic or ram-pressure)

STATIONARY disk : $\frac{\partial}{\partial t} \equiv 0$

Vertical balance of forces gives

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - \frac{z \cdot G M_*}{r^3} \iff \frac{1}{\rho}\frac{\partial p}{\partial z} = -\Omega_K^2 z$$

where $\Omega_K^2 = \frac{G M_*}{r^3}$ is the square of the Keplerian rotation frequency $\Omega_K = v_K \cdot r^{-1}$

Suppose the local isothermal gas state:

$$\int \frac{\partial \rho}{\rho} = -\frac{\Omega_K^2}{c^2} \int z \, \partial z$$

$$\ln \rho = -\frac{z^2}{2 z_0^2} \quad \text{where} \quad z_0 = \frac{c}{\Omega_K}, \text{ or}$$

$$\rho = \rho_0 \exp \frac{-z^2}{2 z_0^2}$$

Gaussian vert. profile

$$\boxed{\frac{z_0}{r} = \frac{c}{\Omega_K r} = \frac{c}{v_K}}$$

Geometry
(z/r, shape)

Kinematics (in stellar disks dispersion of veloc. plays a role of the soundspeed c)

Thermodynamics (c, T)

$$\frac{z_0}{r} = \frac{c}{\Omega_K r} = \frac{c}{v_K}$$

$$c^2 = \frac{kT}{\mu_{mol} m_H}$$

$$c \approx 1 \, \frac{km}{s} \left(\frac{T}{10^2 K}\right)^{1/2}$$

$$v_K = \sqrt{\frac{GM_*}{r}}$$

$$v_K \approx 30 \, \frac{km}{s} \left(\frac{M_*}{M_\odot}\right)^{1/2} \left(\frac{r}{AU}\right)^{-1/2}$$

Geometrical thinness of accretion disks is a direct consequence of their relatively low thermodynamical temperature T(r) e.g., gas heated by blackbody grains, which absorb the flux from the soure L_* in the center **(this works for optically thin conditions only!)**

$$T(r) = 280\,K \left(\frac{L_*}{L_\odot}\right)^{1/4} \left(\frac{r}{AU}\right)^{-1/2}$$

Solar System: aver. Earth temper. (280 K); 1 ($L_*/L_\odot$); 1 @ Earth (r/AU)

e.g., observations of flat-spectrum sources ($\nu F_\nu \approx$ const)

$$T(r) \approx 300\,K \left(\frac{r}{AU}\right)^{-0.5} \left(\frac{L_*}{L_\odot}\right)^{0.25}$$

Therefore

$$\frac{z_0}{r} = \frac{c}{v_K} \cong \overbrace{\frac{1}{30}\sqrt{3}}^{0.058} \left(\frac{L_*}{L_\odot}\right)^{-1/8} \left(\frac{M_*}{M_\odot}\right)^{-1/4} \left(\frac{r}{AU}\right)^{1/4}$$

slightly flaring disk shape

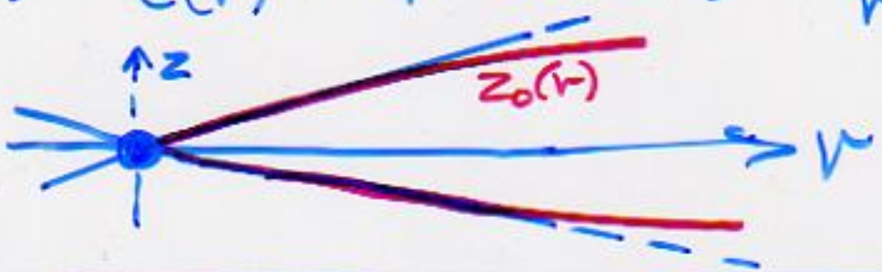
(↔ surface irradiation)

Another example of $T(r)$:

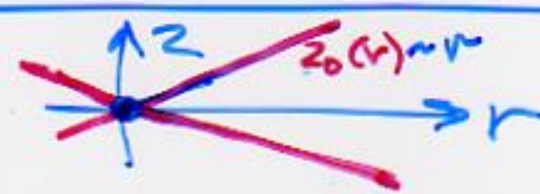
Simplified disk model of Morfill et al (PP II)

$$T(r) \sim r^{-3/2} \Rightarrow$$

$$\Rightarrow c(r) \sim r^{-3/4} \Rightarrow \frac{z_0}{r} = \frac{c}{v_K} \sim r^{-\frac{3}{4}+\frac{1}{2}} = r^{-\frac{1}{4}}$$



$$T(r) \sim r^{-1} \Rightarrow$$



wedge disk

Accretion disks are interesting objects, in a sense they are a crossover between planetary systems (in radial direction, force of gravity is mainly balanced by centrifugal force) and flat stars (in vertical direction, force of gravity is balanced by pressure gradient force).

In the rest of this lecture (L12), we outline some of the basic physics of circumstellar gas disks. You do not need to master all the details.

Quiz may test you on your understanding of some the key concepts (but not derivations about them – those are optional reading for those interested!)

There will be no computational problems involving disk (thermo)dynamics in the written part of the final exam.

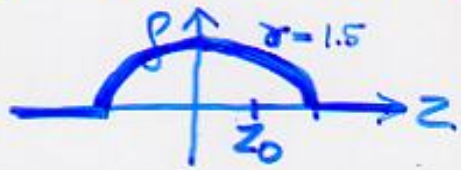
optional material!

POLYTROPIC disks

For $p = \rho^\gamma \cdot \mathrm{const}$ instead of a Gaussian vertical profile we obtain

$$\rho = \rho_0 \left[1 - (\gamma - 1) \frac{\Omega_K^2}{c^2} \frac{z^2}{2}\right]^{\frac{1}{\gamma-1}}$$

$$\rho = \rho_0 \left[1 - \frac{\gamma-1}{2} \frac{z^2}{z_0^2}\right]^{\frac{1}{\gamma-1}}, \quad z_0 = c / \Omega_K \text{ again!}$$



$\rightsquigarrow$ same shapes of disks but under c, T

we must understand the midplane values ($z = 0$)

$$T \sim \frac{p}{\rho} \sim T_0 \left[1 - \frac{\gamma-1}{2} \left(\frac{z}{z_0}\right)^2\right]$$

Better, more realistic results for the vertical disk structure must take into account radiation transport

Radiation transfer in stars, disks etc.

optional derivation

$$P_{rad} = \frac{a}{3} T^4$$ (from Planck formula integration)

= radiation pressure of photon gas, or flux of momentum of photons.

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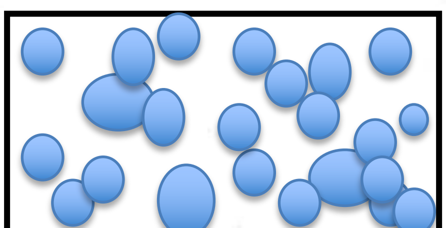
change of P_{rad} in a slab of optical thickness $d\tilde{\tau}$ equals

$$dP_{rad} = \frac{dF}{c}$$

where F = flux of energy of photons

$$dF = -F\,d\tilde{\tau}$$

$$\frac{dP_{rad}}{ds} = -\frac{F}{c}\frac{d\tilde{\tau}}{ds}$$



The derivation is not a required material, only the final result

$$\frac{dP_{rad}}{ds} = -\frac{F}{c}\frac{d\tilde{\tau}}{ds}$$

From $P_{rad} = aT^4/3$, we obtain

$$\frac{d(\sigma T^4)}{d\tilde{\tau}} = -\frac{3F}{4}$$

where $\sigma = \frac{ac}{4}$ = Stefan-Boltzmann constant

Integrating from the midplane of a disk ($\tilde{\tau}=0$, $z=0$) to its surface ($z = z_0$ and $\tilde{\tau} = \tilde{\tau}_0$)

$$\sigma(T_0^4 - T_{eff}^4) = \frac{3}{4}F\tilde{\tau}_0$$

Optical half-thickness of the disk τ_0

On the surface $T = T_{eff}$ and $F = \sigma T_{eff}^4$, so approximately, $T_0^4 - T_{eff}^4 = \frac{3}{4}\tau_0 T_{eff}^4$,

$$\frac{T_0^4}{T_{eff}^4} = \frac{3}{4}\tau_0 + 1$$

We now show that $\tau_0 \gg 1$ in early T Tau disks, and hence $T_0 \gg T_{eff}$

$$\tau_0 = \int_0^\infty \kappa \rho \, dz \approx \kappa \int_0^\infty \rho \, dz = \frac{1}{2} \kappa \Sigma$$

$\kappa = ?$ $[\kappa] = \frac{cm^2}{g}$ opacity of dust + gas

Let's estimate the opacity coeff. of dust first.

$$\kappa_{dust} = \frac{\int \pi s^2 \, n(s) ds}{\int \rho_{dust} \frac{4\pi}{3} s^3 \, n(s) ds} = \frac{\langle A_1 \rangle}{\langle M_1 \rangle},$$

$n(s) = s^{-3.5} ds$

standard size distrib. like in ISM.

$\langle A_1 \rangle$ – dominated by $s \sim s_{min}$

$\langle M_1 \rangle$ — " — $s \sim s_{max}$

Take $\left\{ \begin{array}{l} s_{min} \sim 0.1\ \mu m \\ s_{max} \sim 1\ cm \end{array} \right\}$ in early T Tau disk

$\kappa_{dust} \sim \dfrac{3 \cdot 10^{-10}\ cm^2}{4 \cdot 10^{-15}\ g} \cong 10^5\ \dfrac{cm^2}{g}$. Since dust : gas $= 1:100$,

we have $\kappa \sim (1:100) \cdot \kappa_{dust} \sim 10^3\ cm^2/g$

$\Sigma = ?$ Minimum mass solar nebula ;

$\Sigma \sim \left\{ \begin{array}{l} 10^3 \\ 10^2 \end{array} \right. g/cm^2$ @ $r = 1$ AU, 10 AU $\Rightarrow \tau_0 \sim \left\{ \begin{array}{l} 10^6 \text{ @ 1AU} \\ 10^5 \text{ @ 10 AU} \end{array} \right.$

High!

$\dfrac{T_0}{T_{eff}} \sim \left(\dfrac{3}{4}\tau_0\right)^{1/4} \sim \left\{ \begin{array}{l} 10^{1.5} \\ 10^{1.25} \end{array} \right\} \sim \left\{ \begin{array}{l} 30 \\ 20 \end{array} \right\}$ at $\left\{ \begin{array}{l} 1 \text{ AU} \\ 10 \text{ AU} \end{array} \right\}$,

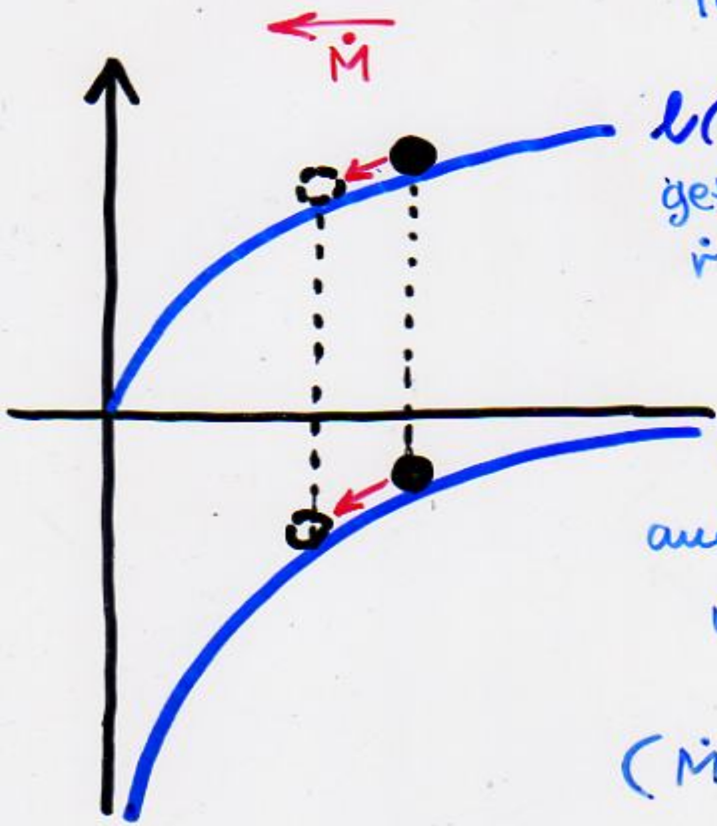
in early T Tau disk (MMSN).

(on the other hand, in debris disks which don't have a lot of gas and much less dust as well, both the opacity of dust and the surface density of matter are much lower, so that the optical depth is tau_0 << 1 in every direction.)

The source of **energy** is the effective **viscosity** described by coefficient ν [length × velocity]

At the same time viscosity has the important task of **angular momentum** transport,

(Lynden-Bell & Pringle 1974, Pringle 1981 in ARA&A) which causes **mass transfer** toward the star.



Test mass moves at speed $v_r = \dot{r}$

$\ell(r) = \sqrt{GM_* r}$

getting rid of ℓ at the rate

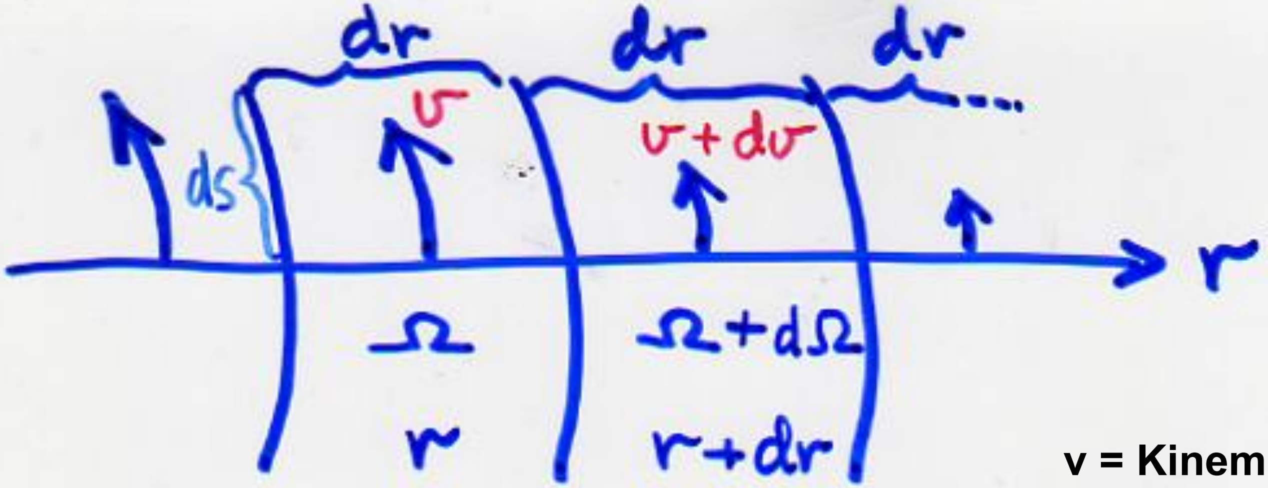
$\dot{r}\frac{d\ell}{dr}$ = torque

$e(r) = -\frac{1}{2r} \cdot GM_*$

and losing energy at rate

$\dot{r}\frac{de}{dr}$ = radiation flux $2F$

($\dot{M} = 2\pi r \Sigma v_r$)



v = Kinematic viscosity coeff.

How differential rotation causes both energy release ($\mathcal{E}$) and torque ($T \cdot r$) in the presence of viscosity (ν)

viscous force per (ds) $\quad T = \nu \Sigma \underbrace{r \frac{d\Omega}{dr}}_{\frac{dv}{dr}}$,

(that's definition of ν!)

where dv = velocity difference between cylinders

Don't memorize this derivation! It's optional...

$$\varepsilon = \frac{\text{Frictional power}}{\text{mass}} = \frac{\text{force} \times \text{velocity diff.}}{\Sigma \cdot dr \cdot ds} =$$

$$\varepsilon = \nu \left(r \frac{d\Omega}{dr} \right)^2 \underset{\uparrow}{=} \frac{9}{4} \nu \Omega^2$$

↑ Keplerian rotation

← allows vertical structure calculation if ν known.

$$2F = \int_{-\infty}^{\infty} \varepsilon \rho \, dz \approx \Sigma \nu \cdot \frac{9 \Omega^2}{4}$$

Comparing the positive transfer of angular momentum from radius $r - dr$ with negative gain due to radius $r + dr$, we obtain that the whole ring of width dr gains L at rate

$$d\dot{L} = dr \frac{\partial}{\partial r} \left(2\pi r^3 \frac{d\Omega}{dr} \Sigma \nu \right)$$

Angular momentum of a ring, $L = 2\pi r^3 \Sigma dr \Omega$, can also change in time because of radial variations in the mass flow rate $\dot{M} = 2\pi r v_r \Sigma$, i.e., due to local density increase or decrease.

The balance of gains and losses reads

~net gain of ang. mom. ~ang. mom. advected ~torque

$$\frac{\partial}{\partial t}(r^3 \Omega \Sigma) = -\frac{\partial}{\partial r}(r^3 \Omega v_r \Sigma) + \frac{\partial}{\partial r}\left(r^3 \frac{d\Omega}{dr} \nu \Sigma\right)$$

or, after simplification brought by continuity eq.,

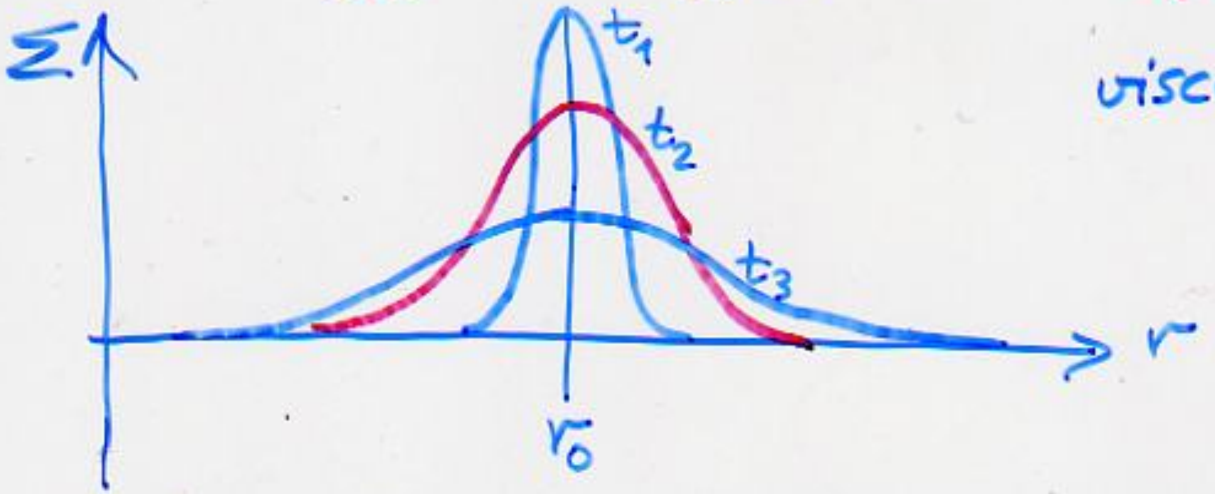
$$\boxed{\frac{\partial \Sigma}{\partial t} = 3 \frac{1}{r} \frac{\partial}{\partial r}\left\{ r^{1/2} \frac{\partial}{\partial r}\left(r^{1/2} \nu \Sigma\right)\right\}}$$

$\nu(\Sigma, r, t) \longrightarrow \Sigma(r, t)$ (disk evolution)

Notice that it is a diffusion equation.
Locally, whenever $\nu\Sigma$ changes rapidly with r,

$$\frac{\partial \Sigma}{\partial t} \cong 3\,\frac{\partial^2(\nu\Sigma)}{\partial r^2}$$

like heat diffusion equation, etc.



viscous ring spreading

Characteristic spreading time = viscous time =

$$= t_\nu = \frac{r_0^2}{\nu}$$

(a time scale for most of the disk to spiral in toward the star)

The ratio of viscous to dynamical time is called Reynolds number (***Re***).
It is a very large number in astrophysics, here on the order ~10^5, which means a very slow spiraling of gas toward the star (along a tight spiral).

The analytical solutions (Pringle 1981)

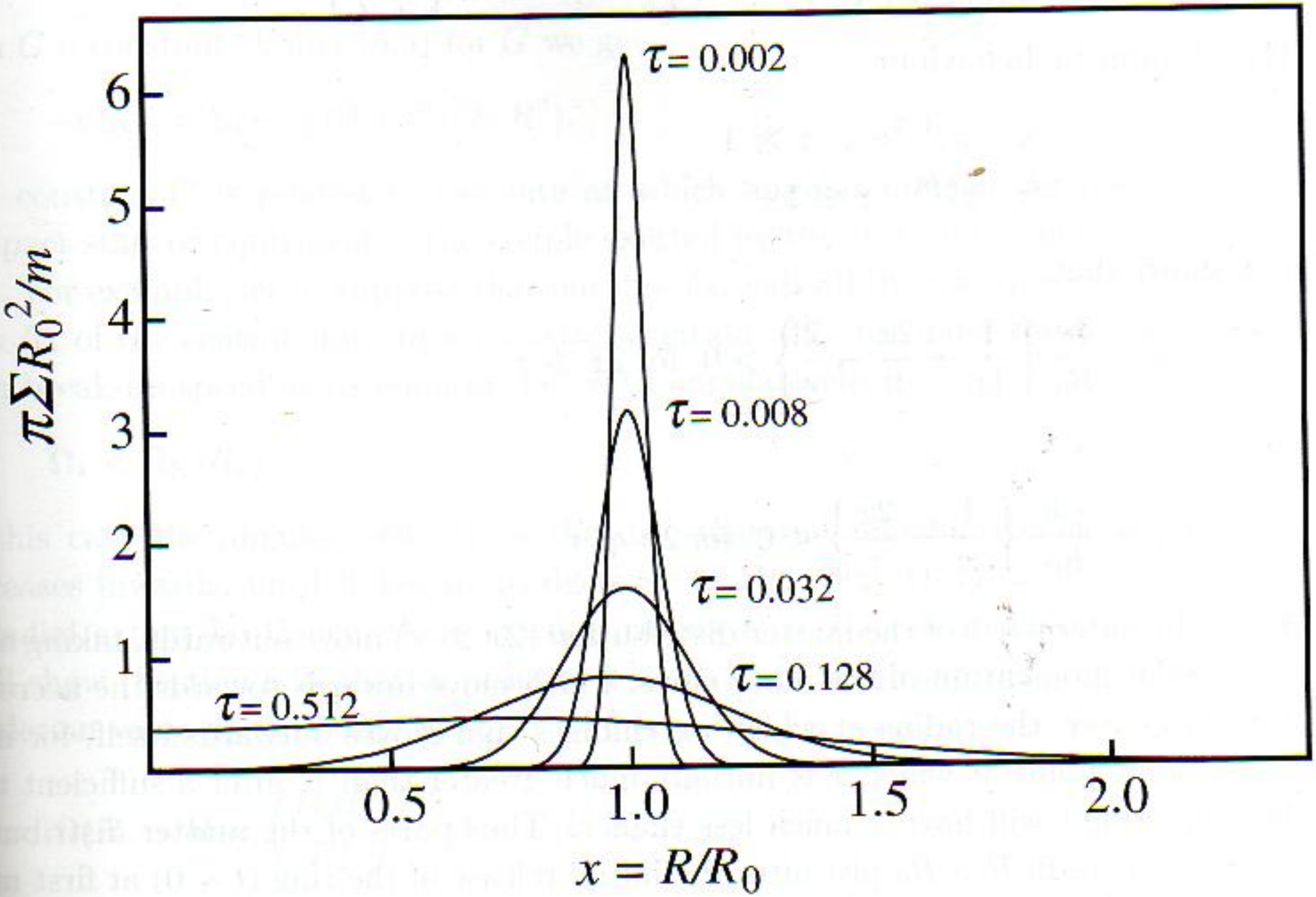
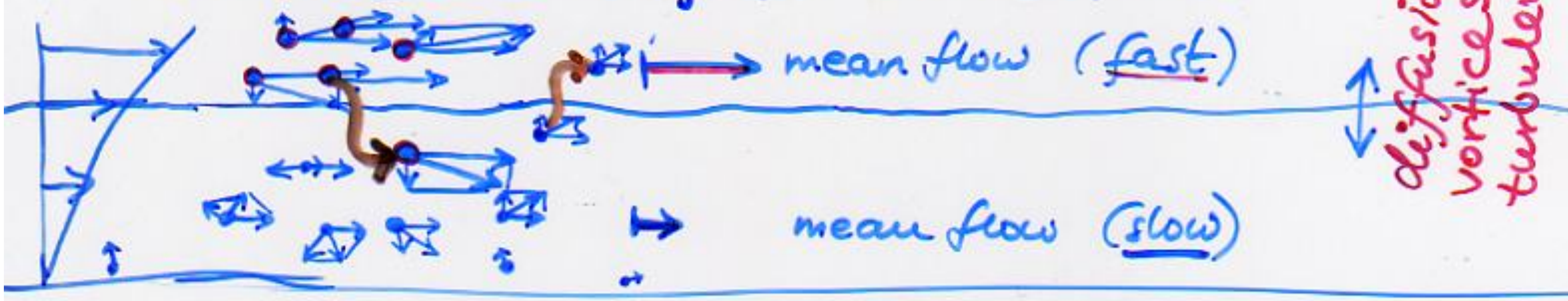


Fig. 5.1. A ring of matter of mass m placed in a Kepler orbit at $R = R_0$ spreads out under the action of viscous torques. The surface density Σ, given by equation (5.10), is shown as a function of $x = R/R_0$ and the dimensionless time variable $\tau = 12\nu t R_0^{-2}$, with ν the constant kinematic viscosity.

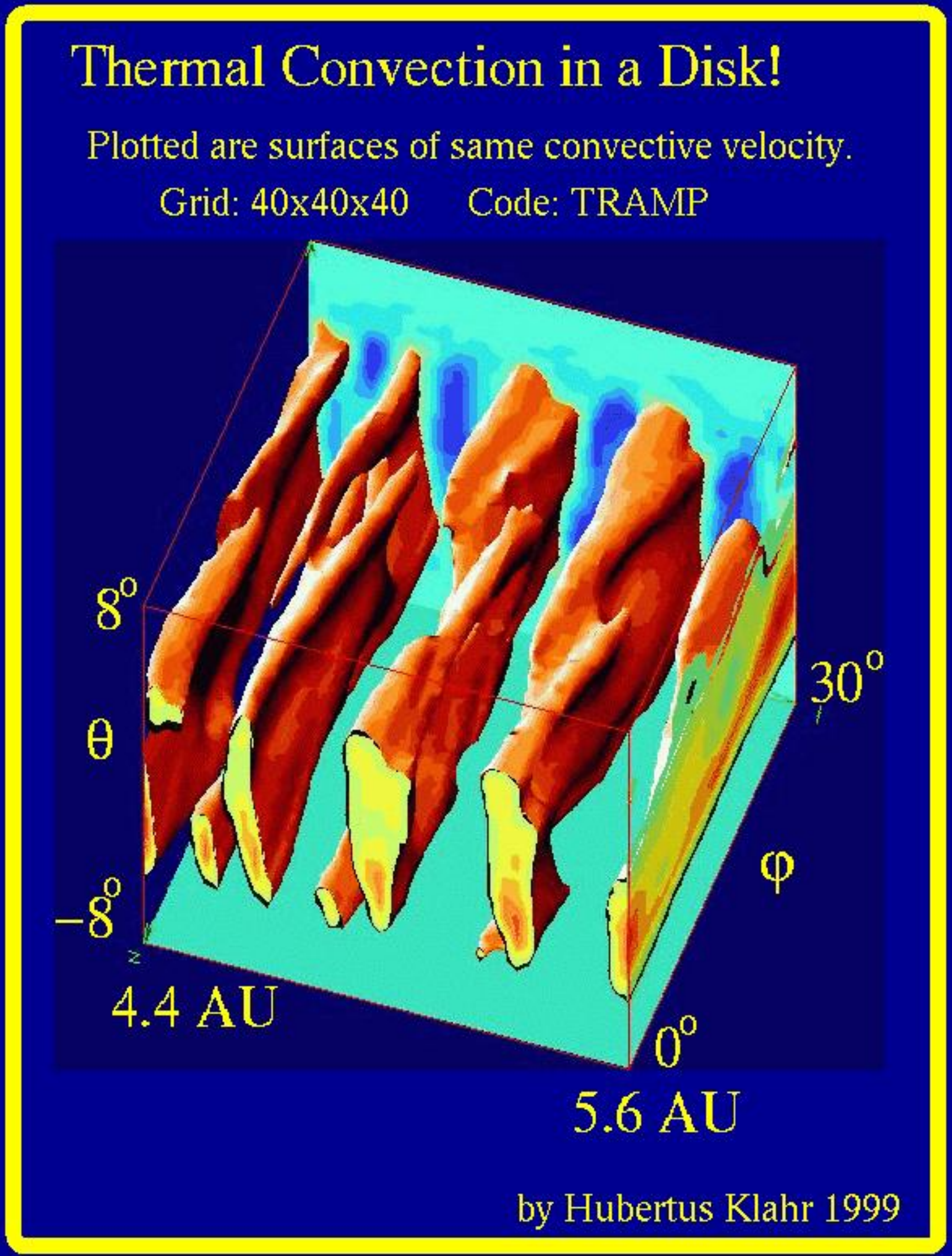
THE VISCOSITY

All kinds of viscosity are based on (microscopic) exchange of mass elements with differing momentum or angular momentum



It is believed that turbulence is responsible for anomalous viscosity in astrophysical disks

(anomalous because molecular viscosity of gas is extremely small-scale and thus weak, as quantified by $Re \sim 10^{12} \ggg 1$, i.e. causes negligible spreading or accretion of solar nebula in Hubble time).



It was initially thought that convective rolls provide the turbulent mixing that creates the anomalous viscosity.

Problem: convection transports angular momentum *inwards*

It is believed that turbulence is responsible for anomalous viscosity in astrophysical disks

(anomalous because molecular viscosity of gas is extremely small-scale and thus weak, as quantified by $Re \sim 10^{12} \ggg 1$, i.e. causes negligible spreading or accretion of solar nebula in Hubble time).

The cause of turbulence is not known, but there are several known possibilities:

- ✗ convection (≡ convective instability)
- → magnetic instabilities **MRI**
- fluid-dynamical instabilities (?)
- local selfgravitational instabilities

From dimensional arguments or by analogy with molecular viscosity

$$\nu \cong v_t \cdot s_t$$

v_t: turnover velocity

s_t: characteristic scale of turbulence; largest eddies

α - disks

Shakhura-Sunyayev (1973)

Non-dimensional parameter

c = soundspeed

z = disk scale height

$$\nu = \alpha c z \quad , \quad \alpha < 1$$

Idea: gather all uncertainties in one parameter α:

$$\nu \cong \alpha \left(\frac{z}{r}\right)^2 l$$

because

$$\frac{c}{\Omega r} \simeq \frac{z}{r}$$

$$l = \Omega r^2$$

l **= Specific angular momentum**

Reynolds number:

$$Re = \frac{l}{\nu} = \frac{1}{\alpha}\left(\frac{r}{z}\right)^2 \sim \frac{10^3}{\alpha} \sim 10^5$$

$$t_\nu = \frac{r^2}{\nu} = Re\, \Omega^{-1} = Re\, t_{dyn} \sim 10^5\, t_{dyn}$$

$$|v_r| = \frac{3}{2\,Re} v_K \sim 10^{-5} v_K$$

(spiralling of gas very much slower than v_K, Keplerian vel.)

- Mysterious viscosity in disks:

  Disks need to have Shakura – Sunyayev alpha

  α ~ from 0.001 to 0.1, in order to be consistent with observations such as UV veiling, Hα emission line widths etc., which demonstrate sometimes quite vigorous accretion onto central objects.

  [**$\nu = \alpha c z$** can be computed from the theoretical prediction of stationary disk theory that $dM/dt = 3\pi\nu\Sigma$]

- What is the ***a priori*** prediction for the Shakura-Sunyaev **α** parameter, which so cleverly combines all our ignorance into a single dimensionless number?

That depends on the mechanism of instability!

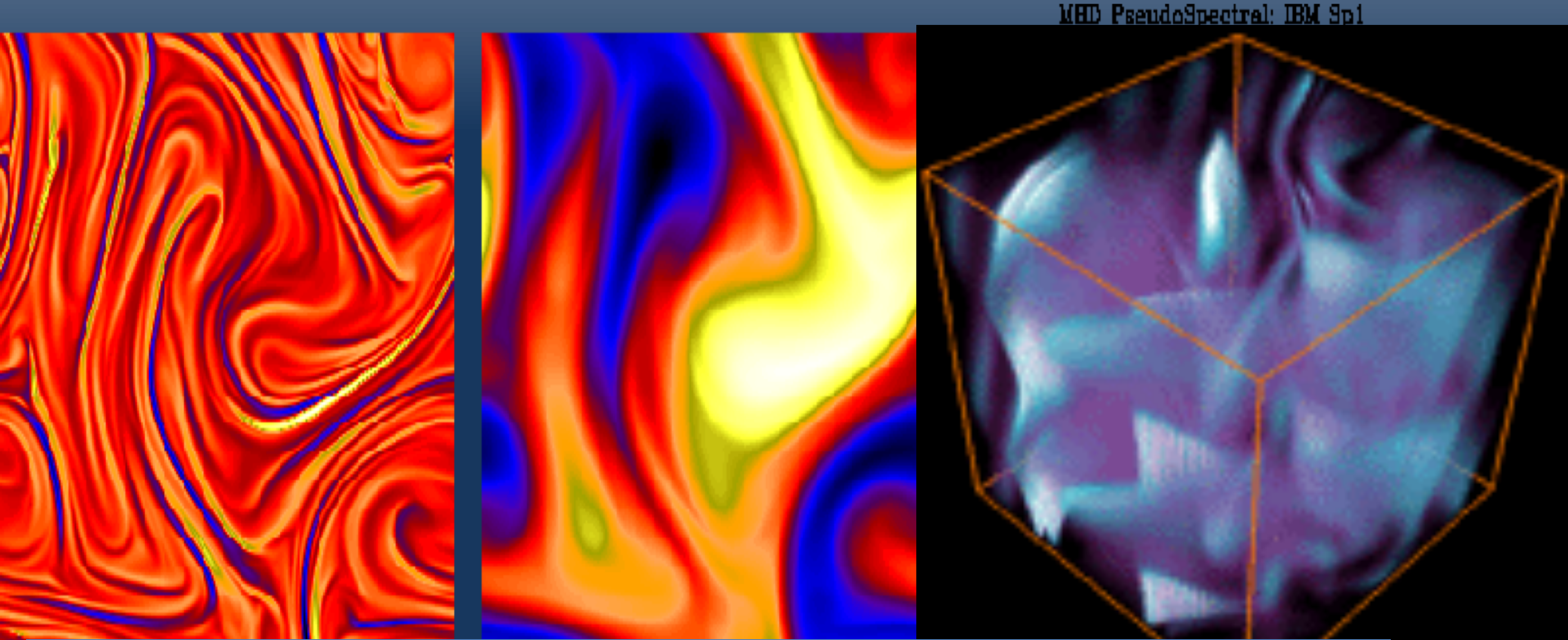
Magneto-rotational instability (MRI) as a source of viscosity in astrophysical disks.

Velikhov (1959), Chandrasekhar (1960), later re-discovered by Balbus and Hawley (1991).

Disk conditions: *gas ionized; magnetic field dragged with gas*

magnetic field energy and pressure << gas energy, pressure

differential rotation (angular speed drops with distance)



2-D and 3-D simulations of Magnetic turbulence inside the disk

Basic equations are complicated...

$$\frac{d\ln\rho}{dt}+\nabla\bullet v=0,$$

Magnetic pressure

Magnetic line tension

$$\frac{dv}{dt}+\frac{1}{\rho}\nabla\left(P+\frac{B^2}{8\pi}\right)-\frac{1}{4\pi\rho}(B\bullet\nabla)B+\nabla\phi=0,$$

$$\frac{\partial B}{\partial t}-\nabla\times(v\times B)=0.$$

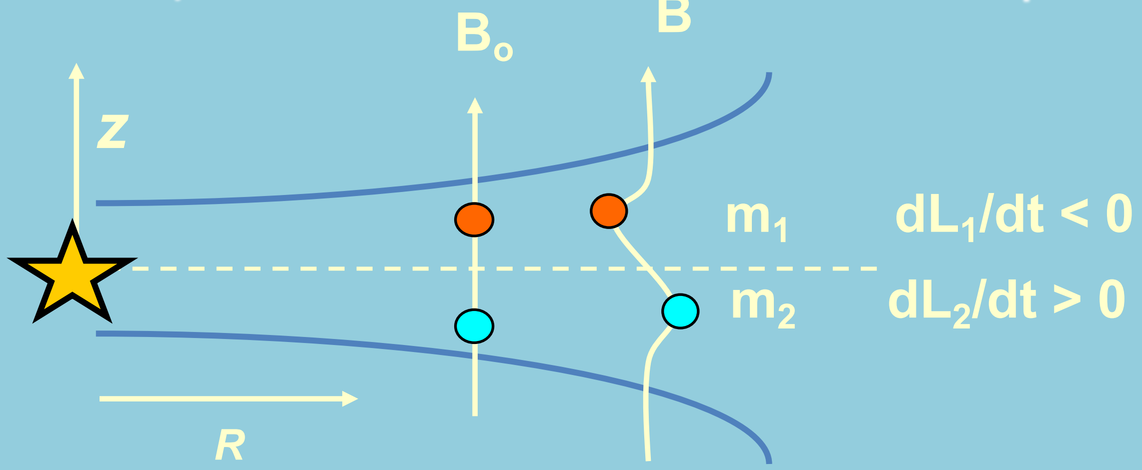
Consider perturbations $e^{i(k_R R+k_z z-\omega t)}$

Using approximations:

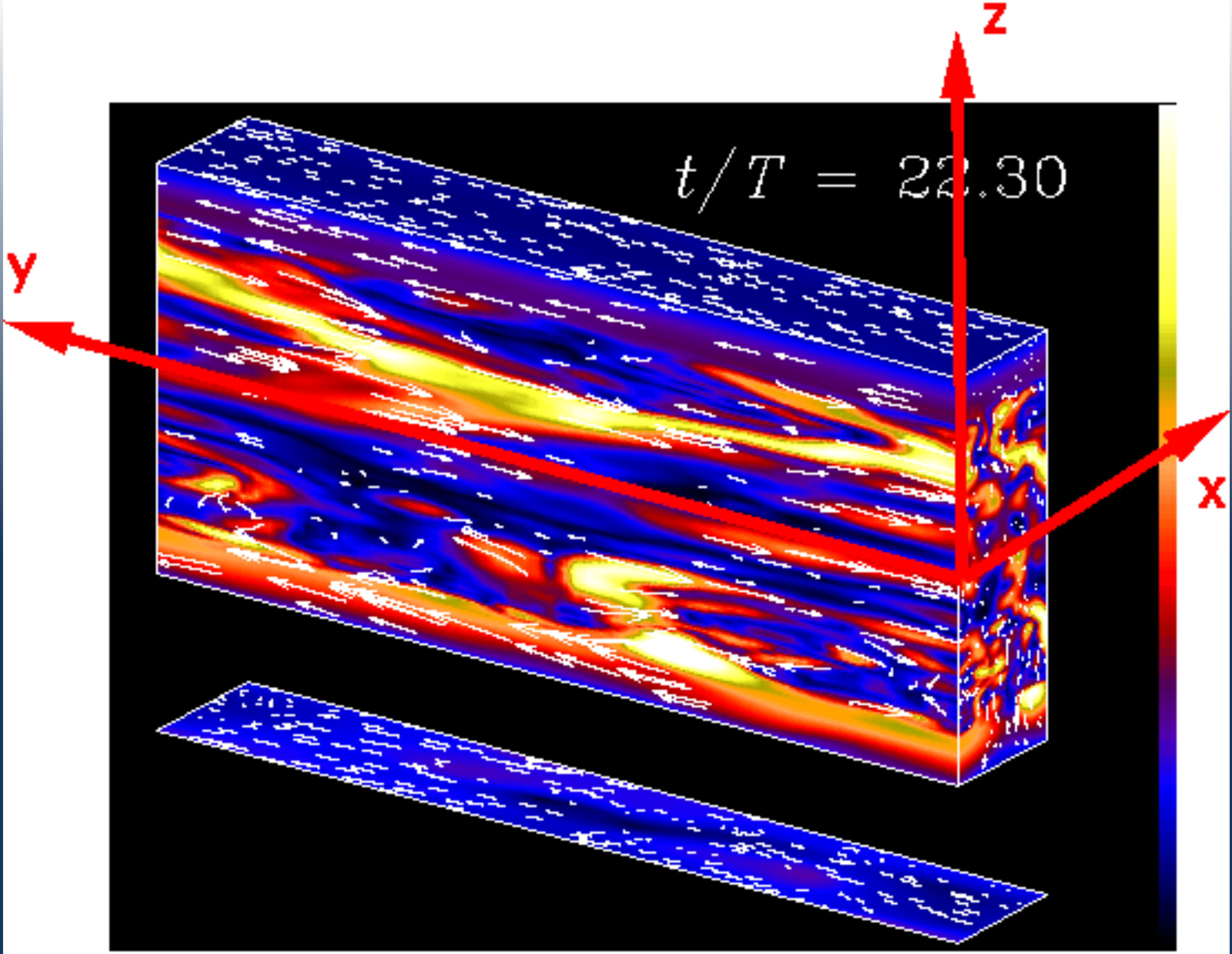
1. Boussinesq Apprximation: ignore δP/P. 2. Adiabatic 3. B is Poloidal

..but MRI instability can be explained by magnetic field tension (you must imagine the orbital motion going into the plane of the picture)

- Two fluid elements, in the same orbit, are joined by a field line (B_o). The tension in the line is negligible.
- If they are perturbed, the line is stretched and develops tension.



- The tension acts to reduce the angular momentum of m_1 and increase that of m_2 (since $L \sim r^{1/2}$). This further increases the tension and the process "runs away".



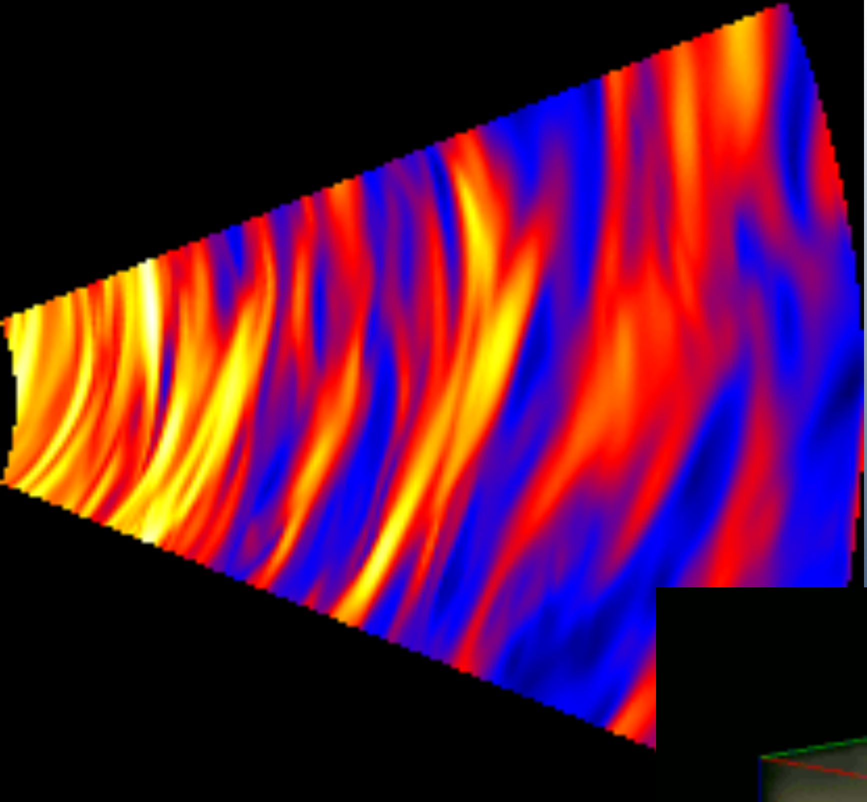
The magnetic flux density at the periphery of the computational box.
Yellow indicates strong magnetic fields.

Chris Reynolds et al.

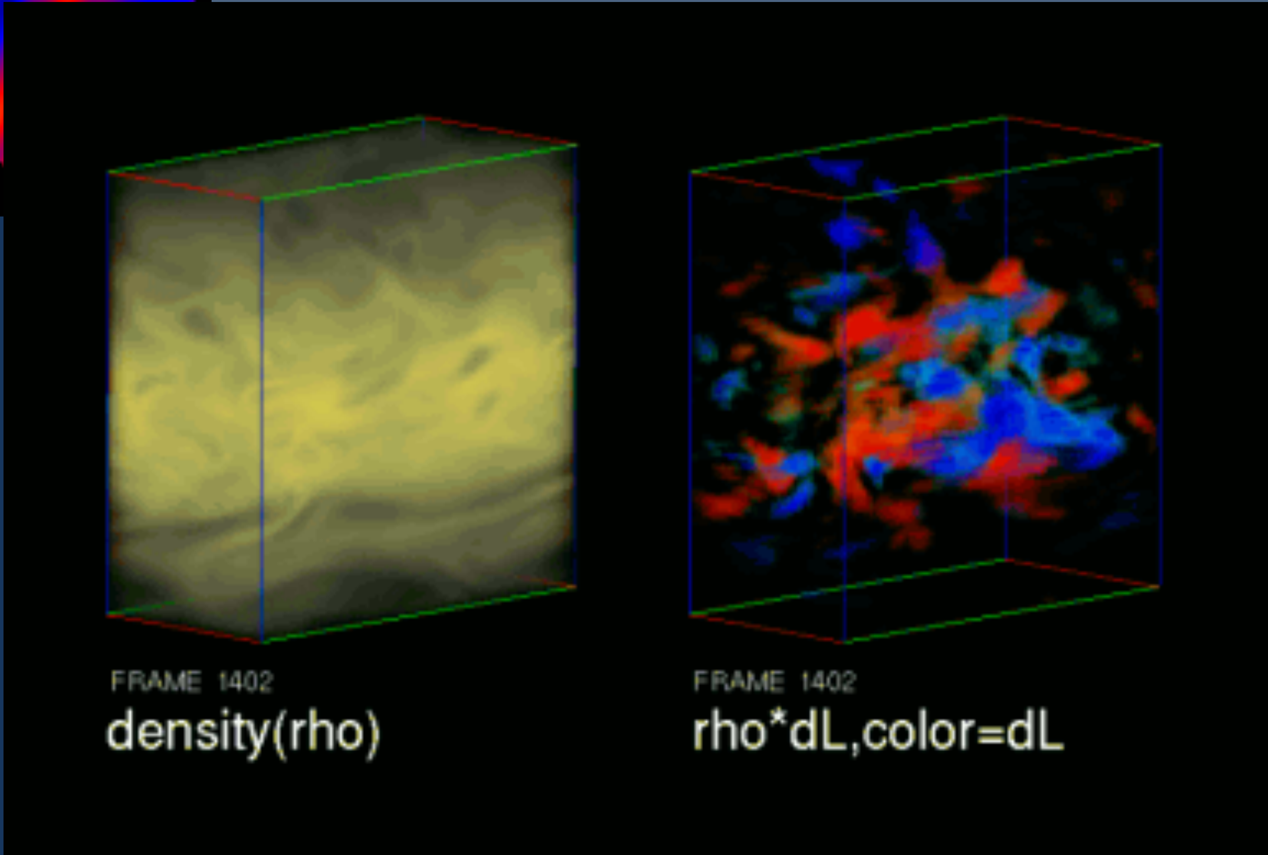
Results: alpha computed ab initio, sometimes not fully self-consistently, often not in a full 3-D disk:

$\alpha \sim 10^{-3}$

(the work on MRI is ongoing… also on whether the disks have sufficient ionization for MRI).



Charles Gammie et al.



Alpha estimate from observations...

TTau: $\dot{M} \sim 10^{-8}\, \frac{M_\odot}{yr} \left(\frac{t}{Myr}\right)^{-(1.5\div 2.7)}$

total $dM = \int dt\, \dot{M} = \mathcal{O}(1) \cdot 10^{-2} M_\odot \sim$

~ observed total mass of the disk out to $R \sim 10^2$ AU

(phenomenological) theory: $\dot{M} \sim \frac{M_d}{t_\nu}$

$$t_\nu = \frac{R^2}{\nu} = \frac{\Omega^{-1}}{\alpha}\left(\frac{R}{z}\right)^2$$

$t_\nu \sim 1$ Myr @ $R \sim 10^2$ AU

$\alpha \sim 10^{-2}$

..Agrees with pure theory: $\alpha = (\text{a few}) \times 10^{-3}$ from multi-D MHD calculations of MRI

Observations

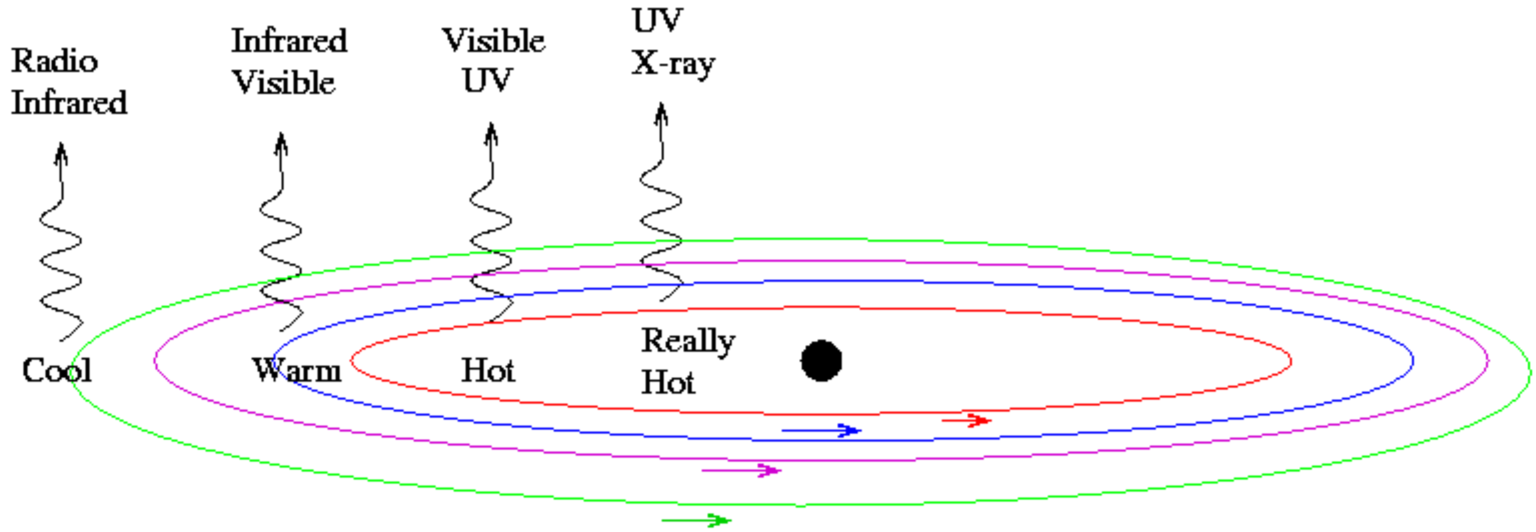
Modeling of observations

↑

Compares OK

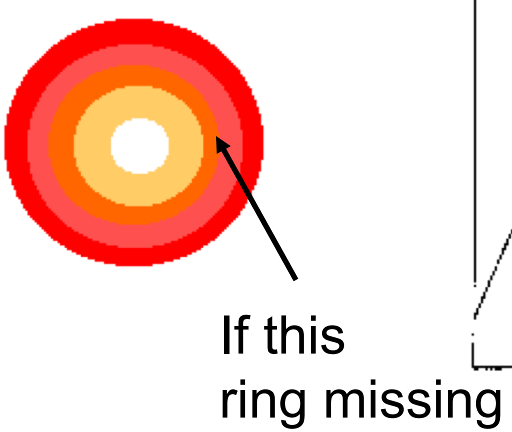
↓

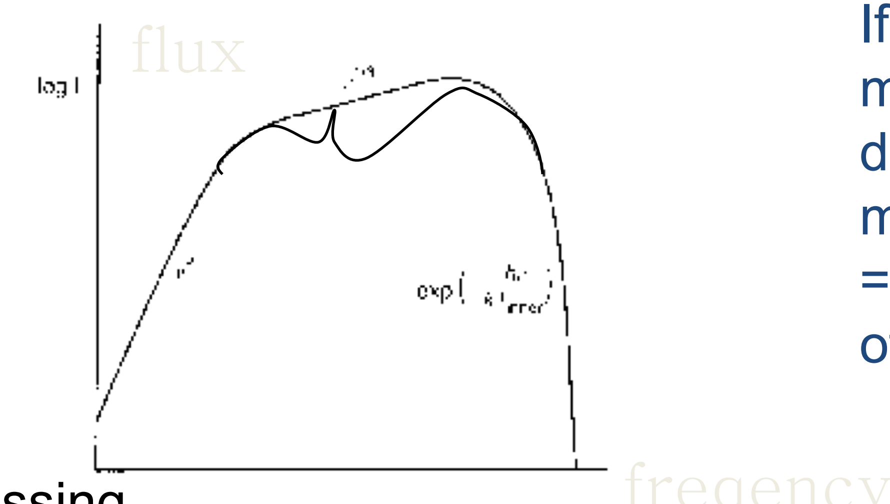
Ab-initio calculations (numerical)



Thin disk approximation

If each disk annulus radiates as a blackbody,
overall spectrum has a slope of 1/3 in frequency





If part of the disk is
missing => spectr. en.
distrib. (SED)
may show a dip
=> possible diagnostic
of planet(s).

Summary of the most important facts about accretion disks:

These disks are found in:

- quasars – in their central engines
- active galactic nuclei (AGNs), galaxies
- around stars (cataclismic variables, dwarf novae, young stars)
- around planets.

Disks drain matter inward, angular momentum outside.
Release gravitational energy as radiation, or reprocess radiation.

Easy-to-understand vertical structure: $z/r \approx c/v_K$

Radial evolution due to some poorly known viscosity,
parametrised by $\alpha << 1$.

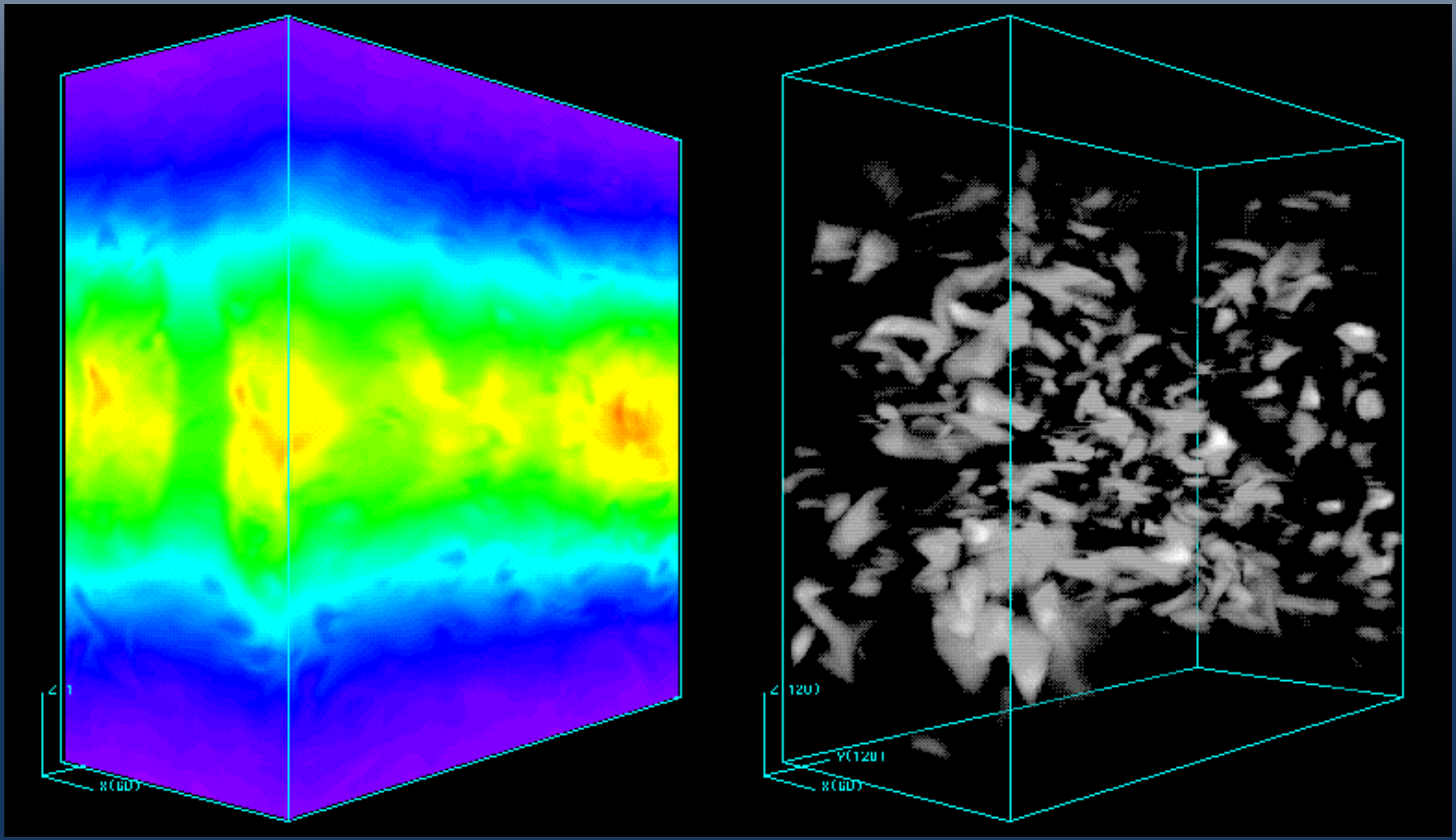
The best mechanism for viscosity is MRI (magneto-rotational instability), an MHD process of growth of tangled magnetic fields at the cost of mechanical energy of the disk.

Simulations give α = a few * 10^{-3}

The following few pages are optional, the information is not required for ASTC25 exam, but you may find it interesting.

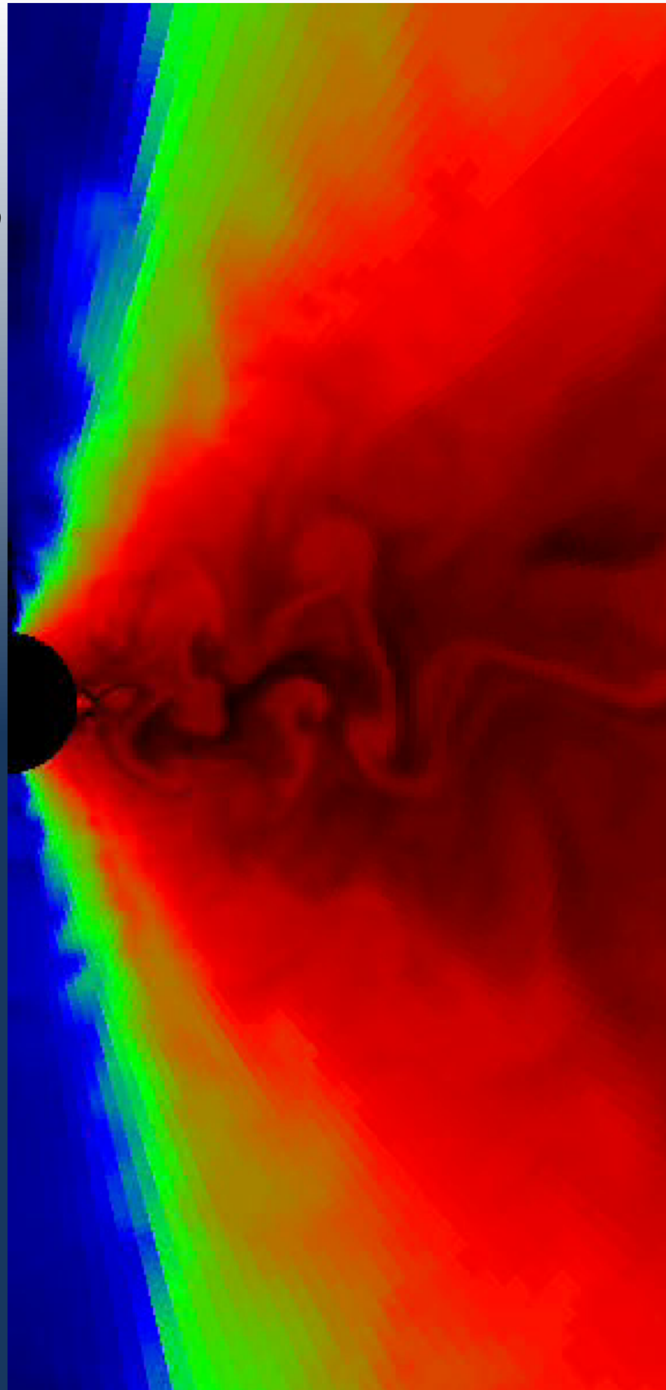
Recent simulations and their problems

Shearing box: useful but distort results

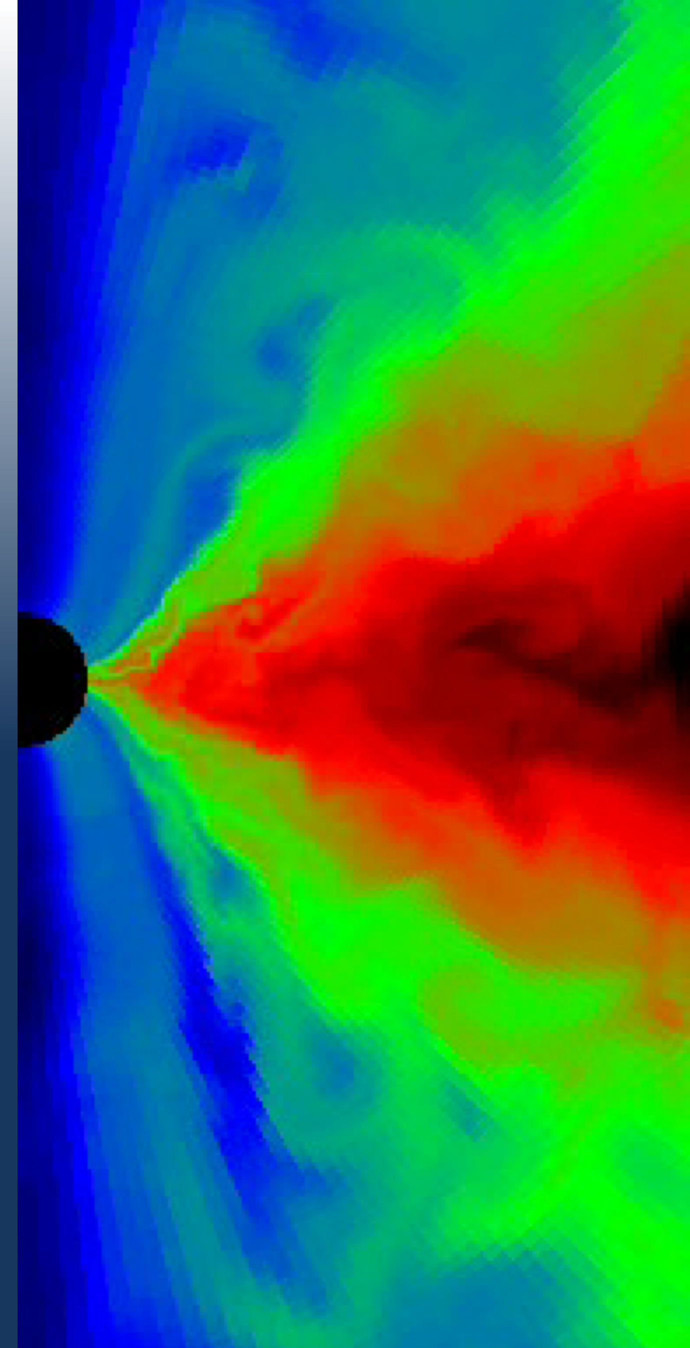


Stone, Hawley, Balbus & Gammie, 1996, ApJ 463, 656

Non-magnetic convection



MRI MHD



Original estimates of strength (α) of angular momentum and mass transport - very optimistic

- Balbus and Hawley (1990s) :
  depending on the geometry of the external field,
  could reach $\alpha = 0.2\text{-}0.7$ if the field is vertical, or 10x less if toroidal.

- Taut and Pringle (1992) : $\alpha \sim 0.4$

- Usually, non-stratified cylindrical disks are assumed

More recently…

- much reduced estimates of maximum alpha: $\alpha \sim 10^{-3}$
- In the past, special non-zero total fluxes and configurations of B field were assumed; local - periodic boundaries, no vertical stratification
- (e.g. Fromang and Papaloizou 2007; Pessah 2007)
- This caused a dependence of α on these rather arbitrary assumptions
- They can be relaxed, i.e. something like a disk dynamo can occur in a total zero flux situation (cf. Rincon et al 2007)

Possible non-MRI Sources of Turbulence (α)

– **Molecular viscosity** (far too weak, orders of magnitude)
– **Convective turbulence** (Lin & Papaloizou 1980, Ryu & Goodman 1992, Stone & Balbus 1996)
– **Electron viscosity** (Paczynski & Jaroszynski 1978)
– **Tidal effects** (Vishniac & Diamond 1989)
– **Purely hydrodynamical** instabilities: Dubrulle (1980s) and Lesur & Longaretti (2005) – anticyclonic flows do not produce efficient subcritical turbulence
– **Gravito-turbulence** (Rafikov 2009)
– **Baroclinic instabilities** (Klahr et al. 2003)
– **Modes** in strongly magnetized disks (Blockland 2007)