Lecture L13 & 14 – ASTC25
Formation of planets

1. **Top-down hypothesis (Giant Gaseous Protoplanets) and its difficulties**

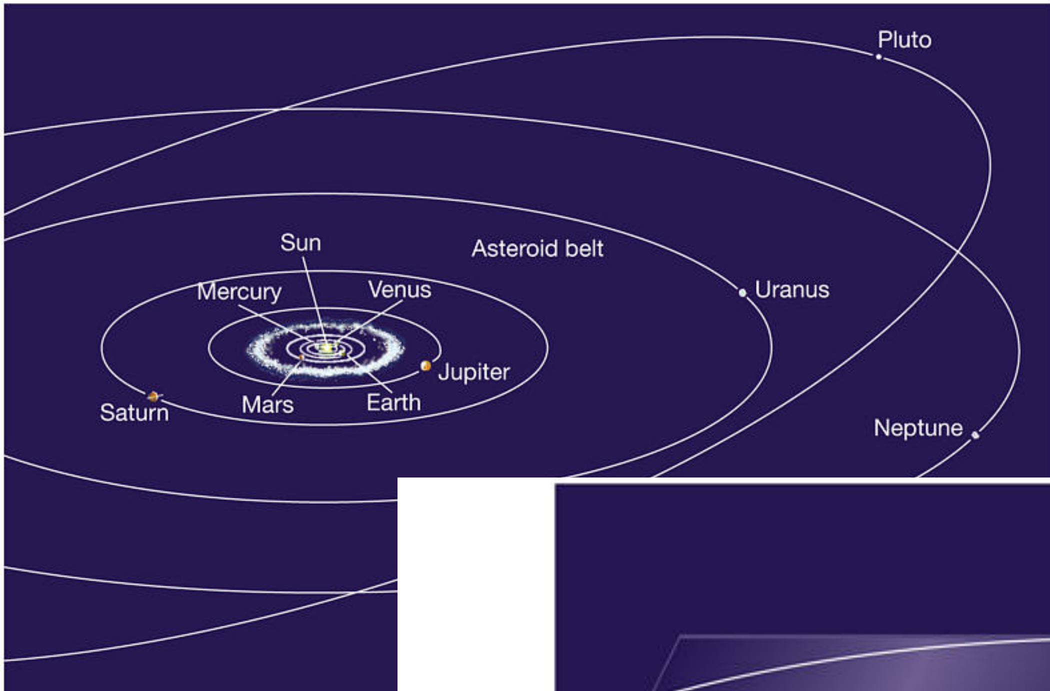
2. Standard (core-accretion) scenario

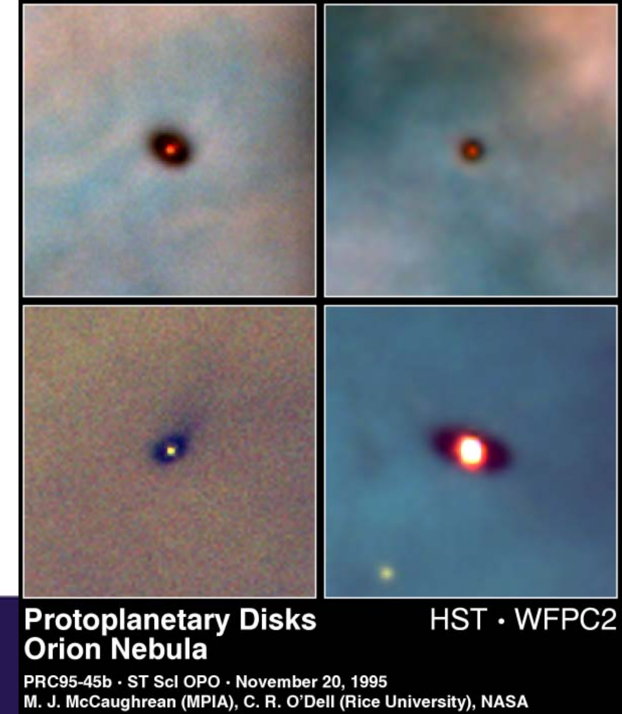
(a) Two ways from dust to planetesimals

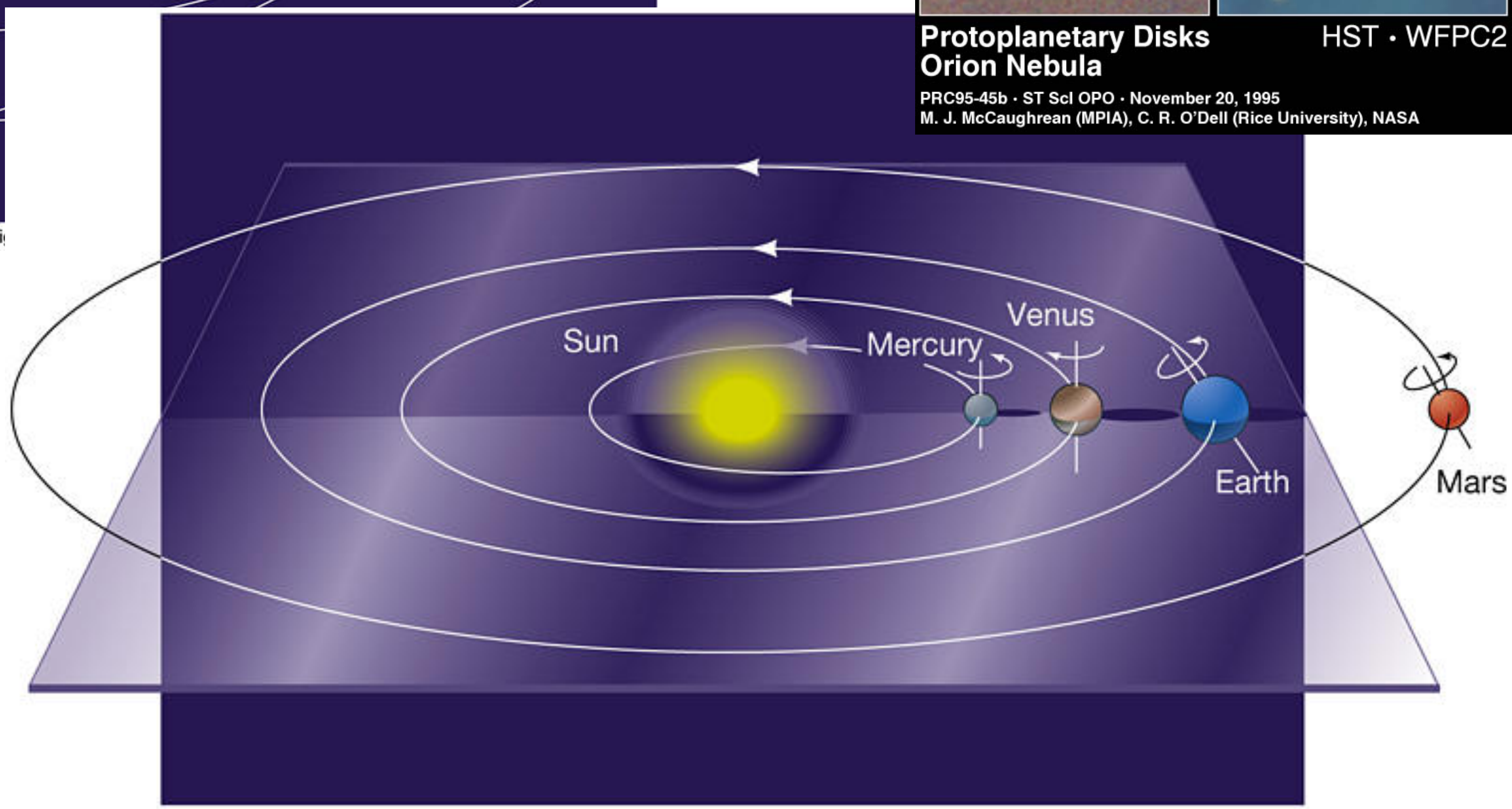
(b) From planetesimals to planetary cores. How many planetesimals?

(c) Safronov number, runaway growth, oligarchic growth of protoplanets

Only theories involving disks make sense...







How do giant gaseous planets form?

Bottom-up

core-accretion scenario
(standard scenario)

Giant planets form by gas accretion onto solid cores (envelope unstable when its mass > mass of core ~ 8-10 Earth masses)

or

Top-down

gravitational instability

Disk breaks up in a dynamic gravitational instability

One major difference:
time of formation
of giant protoplanets:
3-10 Myr (core-accretion)
0.1 Myr (disk breakup)

There are two main possible modes of formation of giant gaseous planets and exoplanets:

- bottom-up, or accumulation scenario for rocky cores (a.k.a. standard theory) predicts formation time ~(3-10) Myr (V.Safronov, G.Kuiper, A.Cameron)

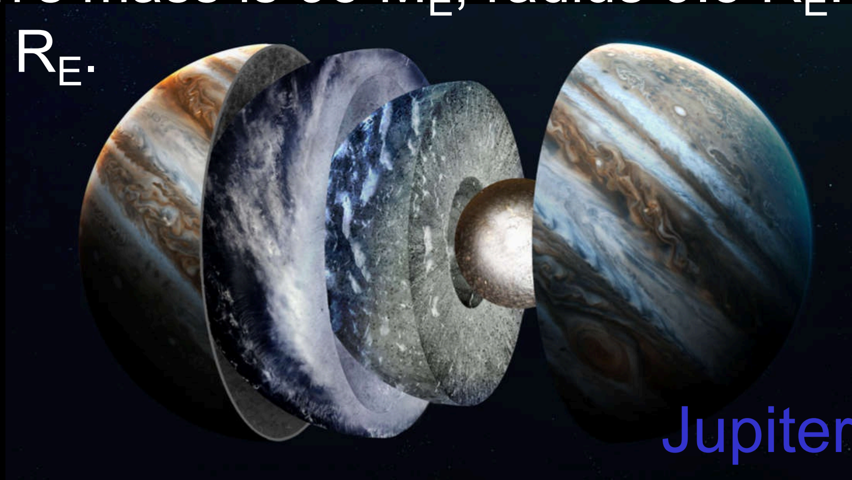
- ◆ top-down, by accretion disk breakup as a result of gravitational instability of the disk. A.k.a. GGP = Giant Gaseous Protoplanets formation time < 0.1 Myr (I.Kant, G.Kuiper, A.Cameron)

To understand the perceived need for ◆, we have to consider *disk evolution and observed time scales*.

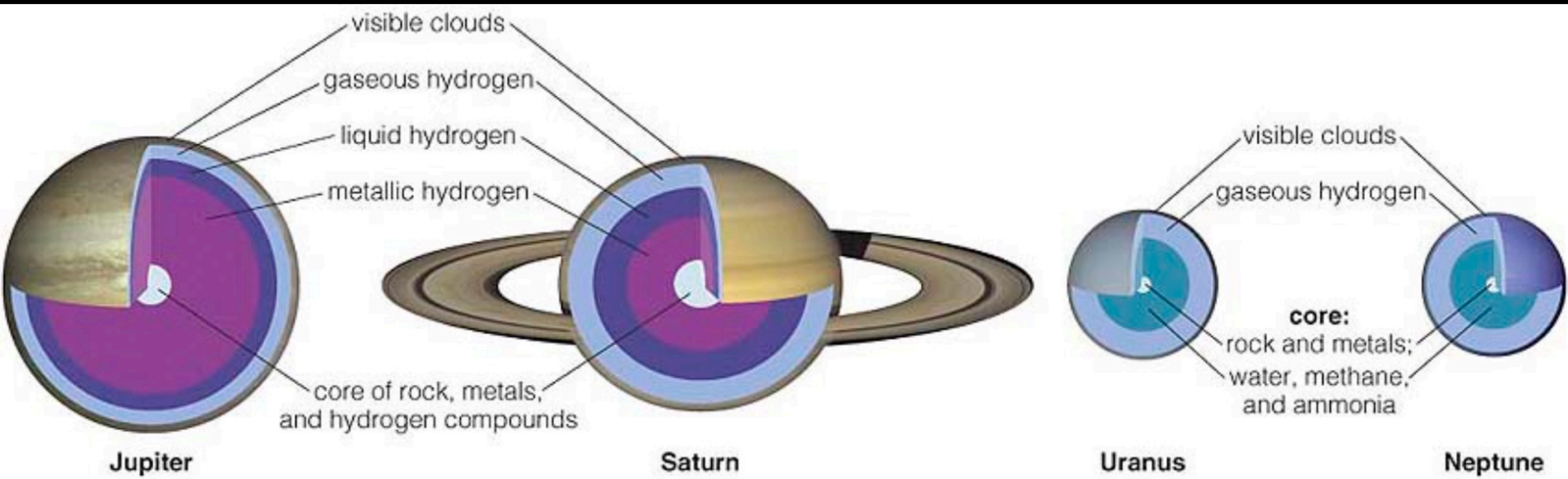
To understand the physics of ◆, we need to study the *stability of disks against self-gravity waves.*

Gravitational Instability and the Giant Gaseous Protoplanet hypothesis

In terrestrial units, mass of Jupiter is 318 M_E, radius = 11 R_E. Saturn's mass is 95 M_E, radius 9.5 R_E. Uranus/Neptune are: 14/17 M_E, and 4 R_E.



All giant planets in the Solar System have cores of compressed rock *and* ice (mostly H_2O). Core mass ~10 M_E



Fragmentation of disks

Self-gravity as a destabilizing force for the epicyclic oscillations (radial excursions) of gas parcels on slightly elliptic orbits

To study waves in disks, we substitute into the equations of hydrodynamics the wave in a WKBJ (WKB) approximation, also used in quantum mechanics:
it assumes that waves are sinusoidal, tightly wrapped, or that *kr* >>1. All quantities describing the flow of gas in a disk, such as the density and velocity components, are Fourier-analyzed as

$$X(r,\theta,t) \sim X_0 + X_1(r)\exp[i(m\theta + \int k\, dr - \omega t)]$$

$$\omega = \omega(k) = frequency\ of\ the\ wave\ inertial\ frame$$

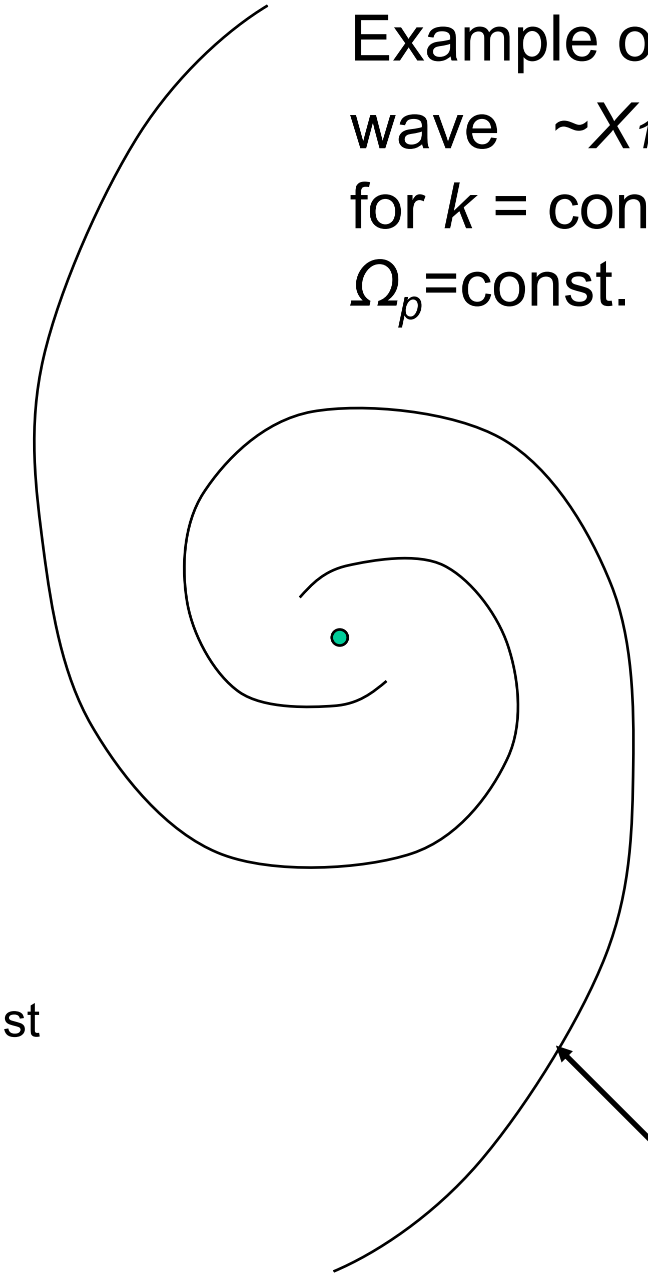
$$k = wave\ vector$$

Some history — WKB applied to Schrödinger equation (1925)

Gregor Wentzel (1898-1978) German/American physicist
Hendrick A. Kramers (1894-1952) Dutch physicist
Léon N. Brillouin (1889-1969) French physicist
} 1926

Harold Jeffreys (1891-1989) English mathematician, geophysicist, and astronomer, established a general method of approximation of ODEs in 1923

Example of a crest of the spiral wave $\sim X_1\, e^{\,ikr + im(\theta - \Omega_p t)}$
for k = const > 0 , m = 2,
Ω_p=const. = pattern speed



This spiral pattern has constant shape and rotates with an angular speed equal to $\omega = \Omega_p/m$ = const

The argument of the exponential function is constant on the spiral wavecrest

Dispersion Relation for non-axisymmetric waves in disks

tight-winding (WKB) local approximation

Doppler-shifted frequency | **epicyclic frequency** | **self-gravity** | **gas pressure**

$$(m\Omega - \omega)^2 = \kappa^2 - 2\pi G\Sigma\,|\,k\,| + c^2k^2$$

$m = number\ of\ arms\ (azimuthal\ number)$

$\Omega = angular\ orbital\ speed$

$\omega = frequency\ of\ the\ wave\ in\ the\ inertial\ frame$

$\kappa = epicyclic\ frequency\ (natural\ radial\ freq.\ in\ disk)$

$\Sigma = surface\ density\ of\ gas$

$c = soundspeed$

$k = wave\ vector\ (length)$

In Keplerian disks, i.e.disks around point-mass objects, $\kappa = \Omega = angular\ Keplerian\ speed$ a.k.a. mean motion *n*

The dispersion relation is, as in all the physics, a relation between the time and spatial frequencies, $\omega = \omega(k)$

Though it looks more frightening than the one describing the simple harmonic (sinusoidal) **sound wave in air**:

$$\omega = \omega(k) = ck = 2\pi c / \lambda$$

$$\lambda = wavelength\ of\ the\ wave$$

Don't apply THIS formula to disks! It's a simple example of sound waves in a room

you can easily convince yourself that in the limit of vanishing surface density in the disk (no self-gravity!) and vanishing epicyclic frequency (no rotation!), the full dispersion relation assumes the above form. So the waves in a non-rotating medium w/o gravity are simply pure pressure (sound) waves. The complications due to rotation lead to a spiral shape of a sound wave, or full self-gravitating pressure wave.

Dispersion Relation in disks with axisymmetric (m=0) waves

$$(m=0) \quad \boxed{\omega^2 = \kappa^2 - 2\pi G\Sigma\,|k| + c^2 k^2}$$

$$\omega^2 \mapsto \min \quad \Leftrightarrow \quad \partial_k \omega^2 = 0 \quad \Leftrightarrow \quad |k_{cr}| = \pi G\Sigma / c^2$$

and if we plug the above most unstable (or critical) k,

and take $\kappa \cong \Omega$, then the smallest ω^2 is

$$\omega^2 = \kappa^2 - (\pi G\Sigma)^2 / c^2$$

Finally, $\omega^2 \mapsto 0$ corresponds to the loss of stability

$$\boxed{Q = \frac{\kappa c}{\pi G\Sigma}} \quad \textit{Safronov} - \textit{Toomre number} \qquad \text{(1960,1964)}$$

decides about the stability : $Q < 1$ means gravit. instability

(Reminder: here *c* is the soundspeed, not the speed of !)

Gravitational *local* stability requirement

$$Q = \frac{\kappa c}{\pi G \Sigma} \quad Safronov-Toomre\ number$$

$Q > 1$ Local stability of a disk, spiral waves may grow Q =1....2

$Q < 1$ Local linear instability of waves, clumps form, but their further evolution depends on equation of state of the gas.

E.g., at a locally stable value Q ~1.5, there are however unstable global modes consisting of waves sloshing back & forth in the disk, closing an amplification cycle. Amplification of waves is happening in the Corotational Zones of the pattern of spiral waves.

Question: Do we have to worry about disk self-gravity/instability?

Using $\frac{c}{\Omega r} \simeq \frac{z}{r}$, instability criterion becomes

$$Q \simeq \left(\frac{z}{r}\right) \frac{\Omega^2 r}{\pi G \Sigma} \cdot \frac{r^2}{r^2} = \left(\frac{z}{r}\right) \frac{M}{\pi r^2 \Sigma} =: \left(\frac{z}{r}\right) \frac{1}{q_d}$$

$\Omega^2 r^3 = GM_* \equiv GM$

$$q_d = \frac{\pi r^2 \Sigma}{M} \sim \frac{M_d}{M_*} \quad \text{disk/star mass ratio}$$

1° The minimum mass solar nebula at $r = 5\,\mathrm{AU}$: $\begin{cases} q_d \sim 0.002 \\ z/r \sim 0.05 \end{cases}$

$\Rightarrow Q \simeq 25$, very stable gravitationally **Ans: No**

2° Massive primordial nebula following protostellar collapse of molecular cloud

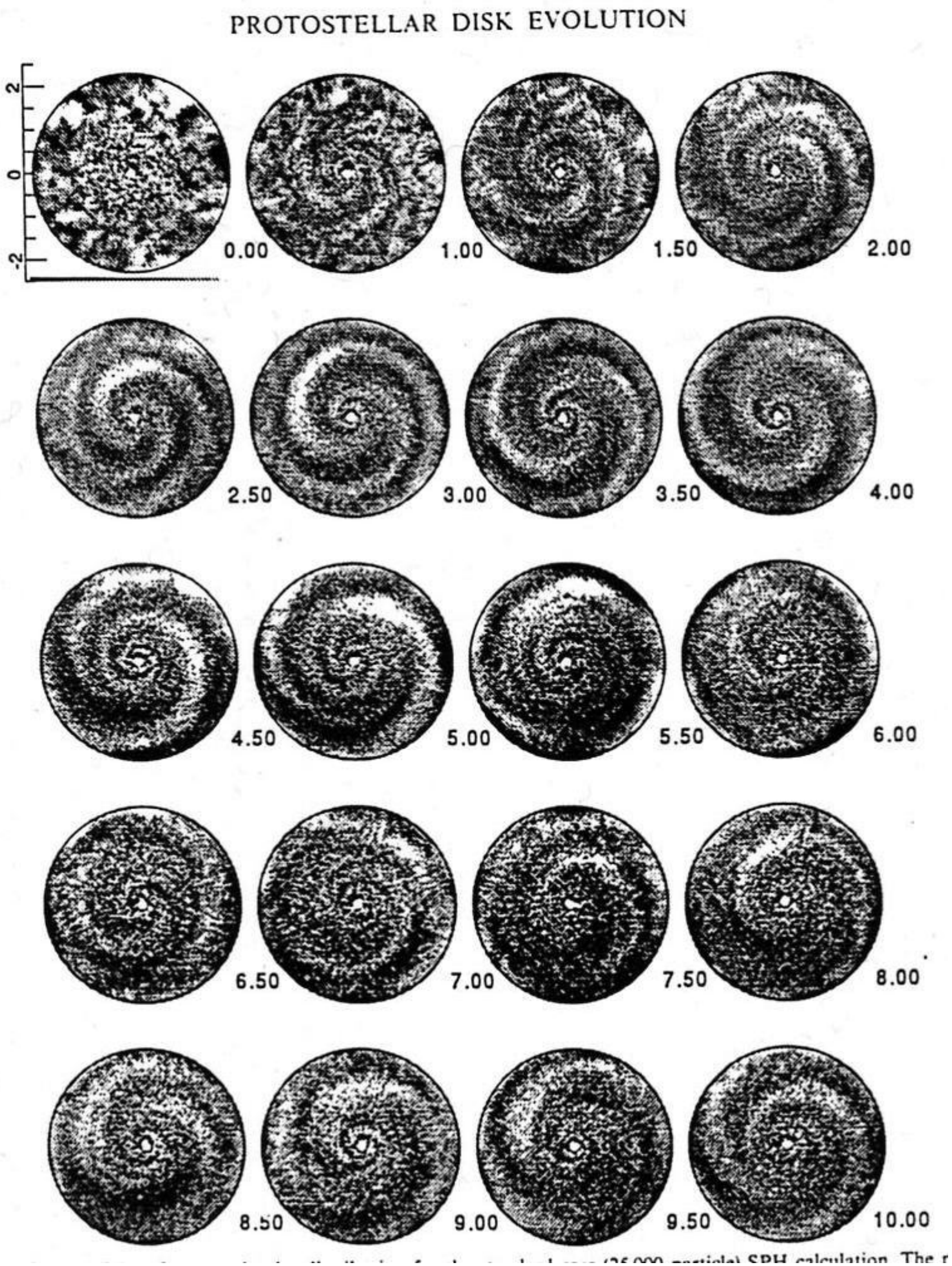
$z/r \sim 0.1$

$q_d > 0.1$

$$Q \simeq \frac{(\sim 0.1)}{(\gtrsim 0.1)} \lesssim 1 \Rightarrow \text{instability}$$

Ans: Yes

From: Laughlin & Bodenheimer (2001)



Disk in this SPH simulation initially had Q~1.8.

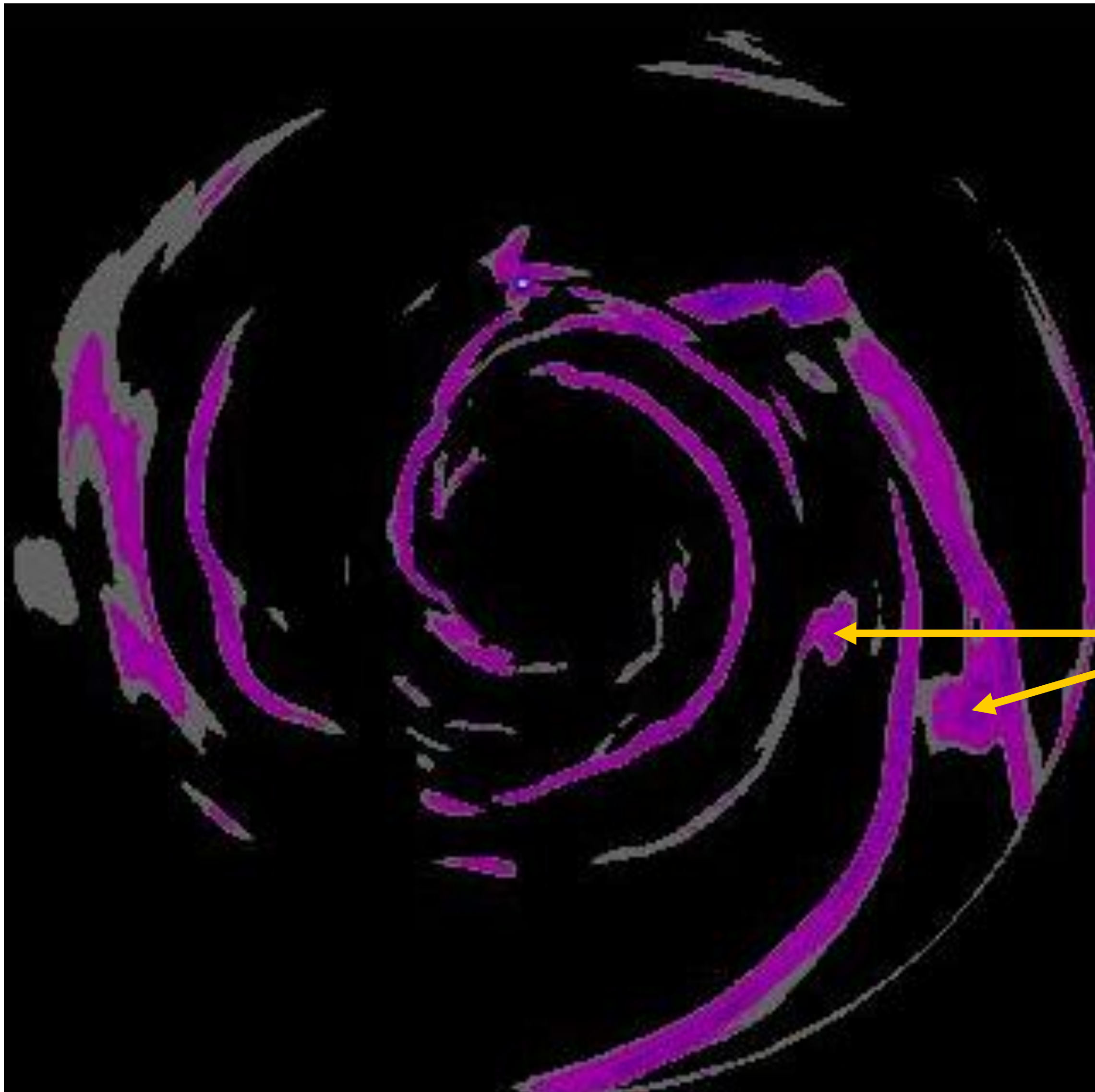
The m-armed global spiral modes of the form

$$\exp[i(m\theta + \int k\,dr - \omega t)]$$

grow and compete with each other.

But the waves in a stable Q~2 disk stop growing and **do not** form small objects (GGPs).

At the end of 20th century, A. Boss revived the half-abandoned idea of disk fragmentation


Alan Boss, Carnegie Institution

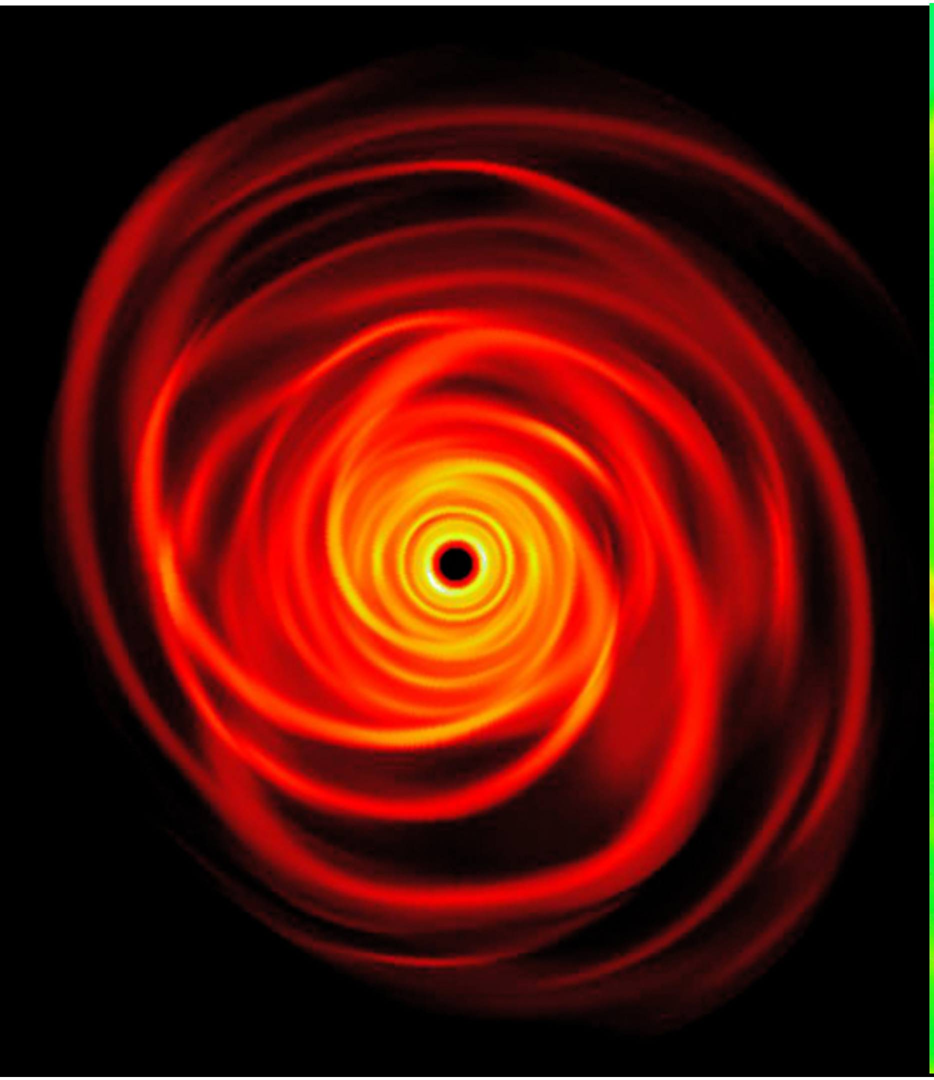
Clumps forming in a gravitationally unstable disk ($Q < 1$)

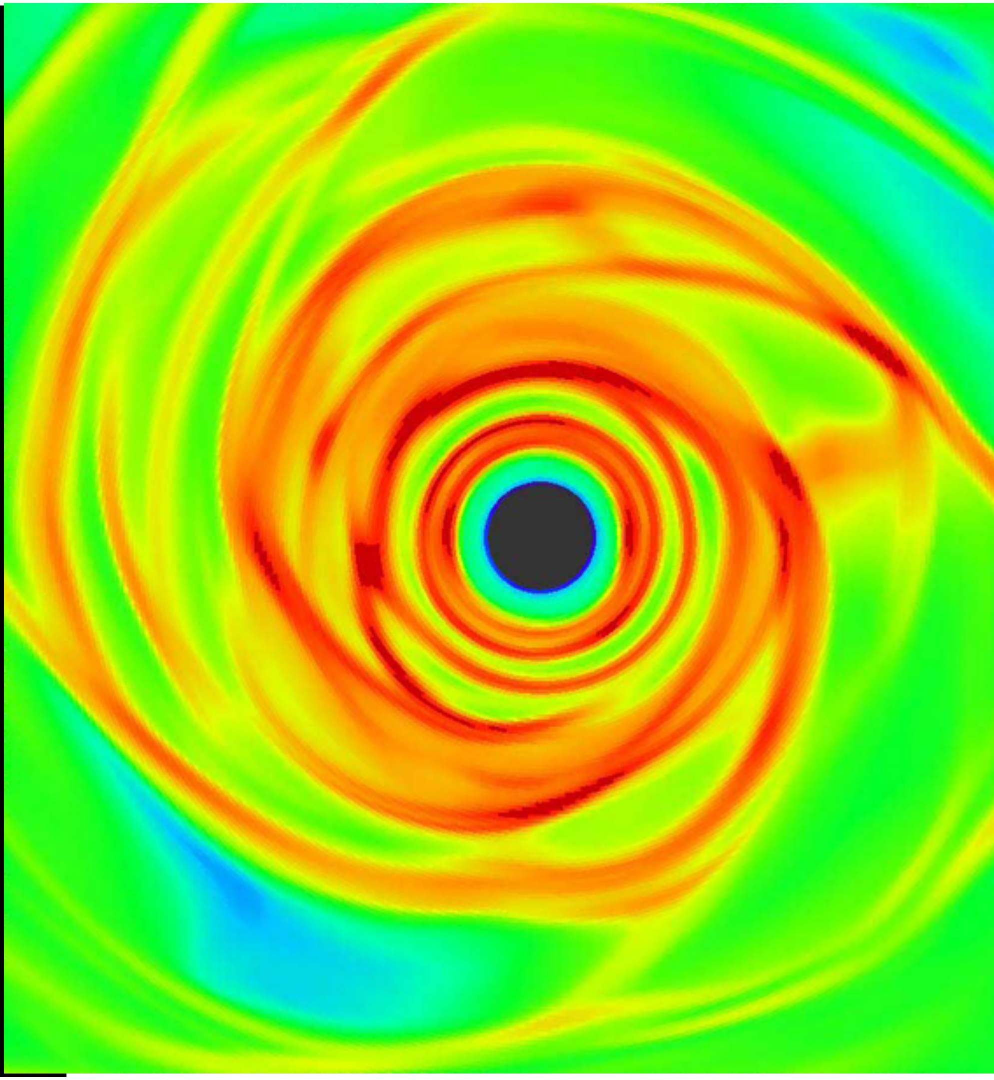
giant gaseous protoplanets?

not quite...

Two examples of formally unstable disks **not** willing to form objects immediately

Durisen et al. (2003)

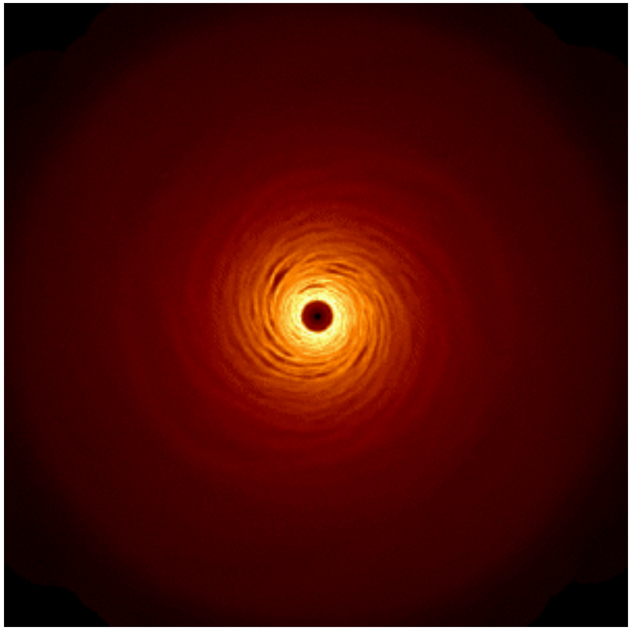




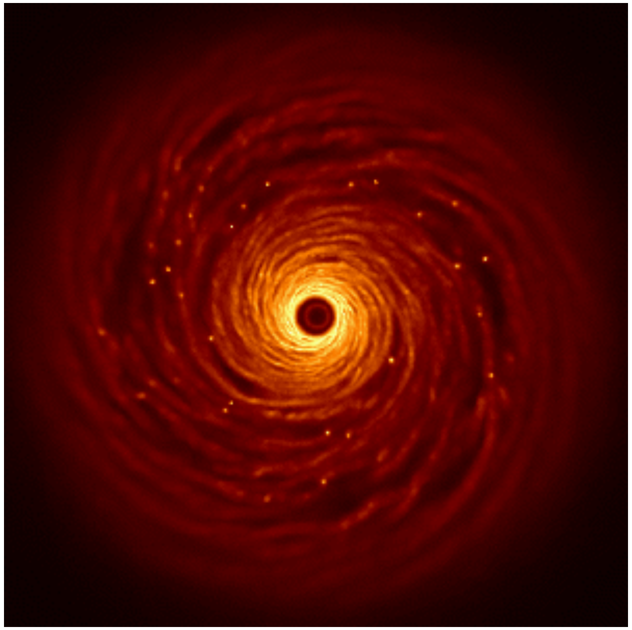
Break-up of the disk depends on the equation of state of the gas, and the treatment of boundary conditions.

Armitage and Rice (2003)

Simulations of self-gravitating objects forming in the disk (with grid-based hydrodynamic codes) shows that rapid thermal cooling is crucial



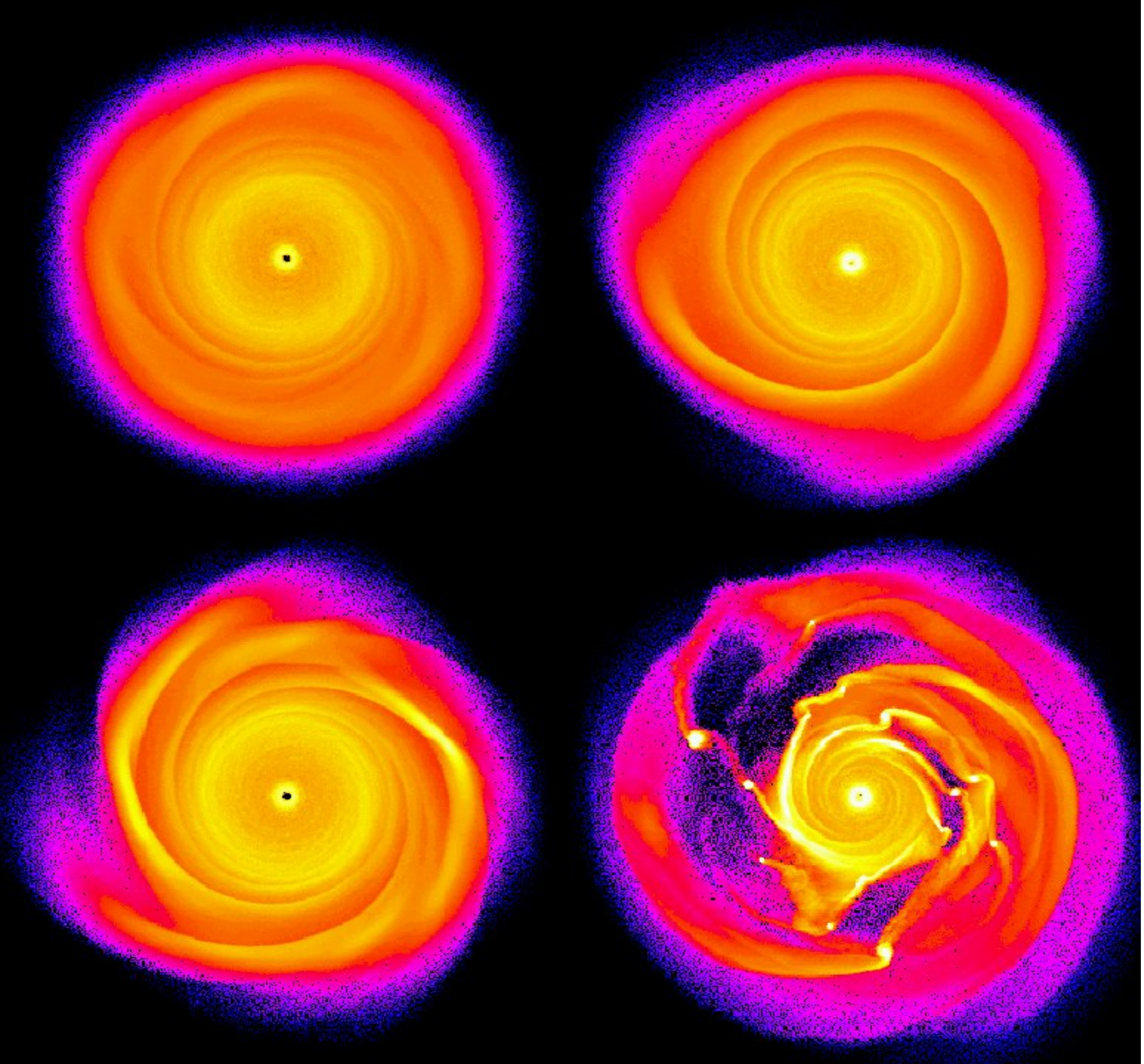
Disk not allowed to cool rapidly (cooling timescale > 1 P)



Disk allowed to cool rapidly (on dynamical timescale, < 0.5 P)

$$Q = \kappa c_s / \pi \Omega \Sigma$$

Mayer, Quinn, Wadsley, Stadel (2003)



SPH =
Smoothed
Particle
Hydrodynamics

with 1 million
particles

← Isothermal
(infinitely
rapid cooling)

Giant Gaseous Protoplanet hypothesis = disk fragmentation scenario (A. Cameron in the 1970s)

Main Advantages: forms giant planets quickly, avoids possible timescale paradox; planets tend to form at large distances amenable to imaging.

MAIN DIFFICULTIES:

1. Non-axisymmetric and/or non-local spiral modes start developing not only at $Q < 1$ but already when Q decreases to Q ~1.5…2
   They redistribute mass and heat the disk => increase Q (stabilize disk).
   Empirically, this self-regulation of the effects of gravity on disk is seen in disk galaxies, all of which have Q ~2 and yet don't split into many baby galaxies.

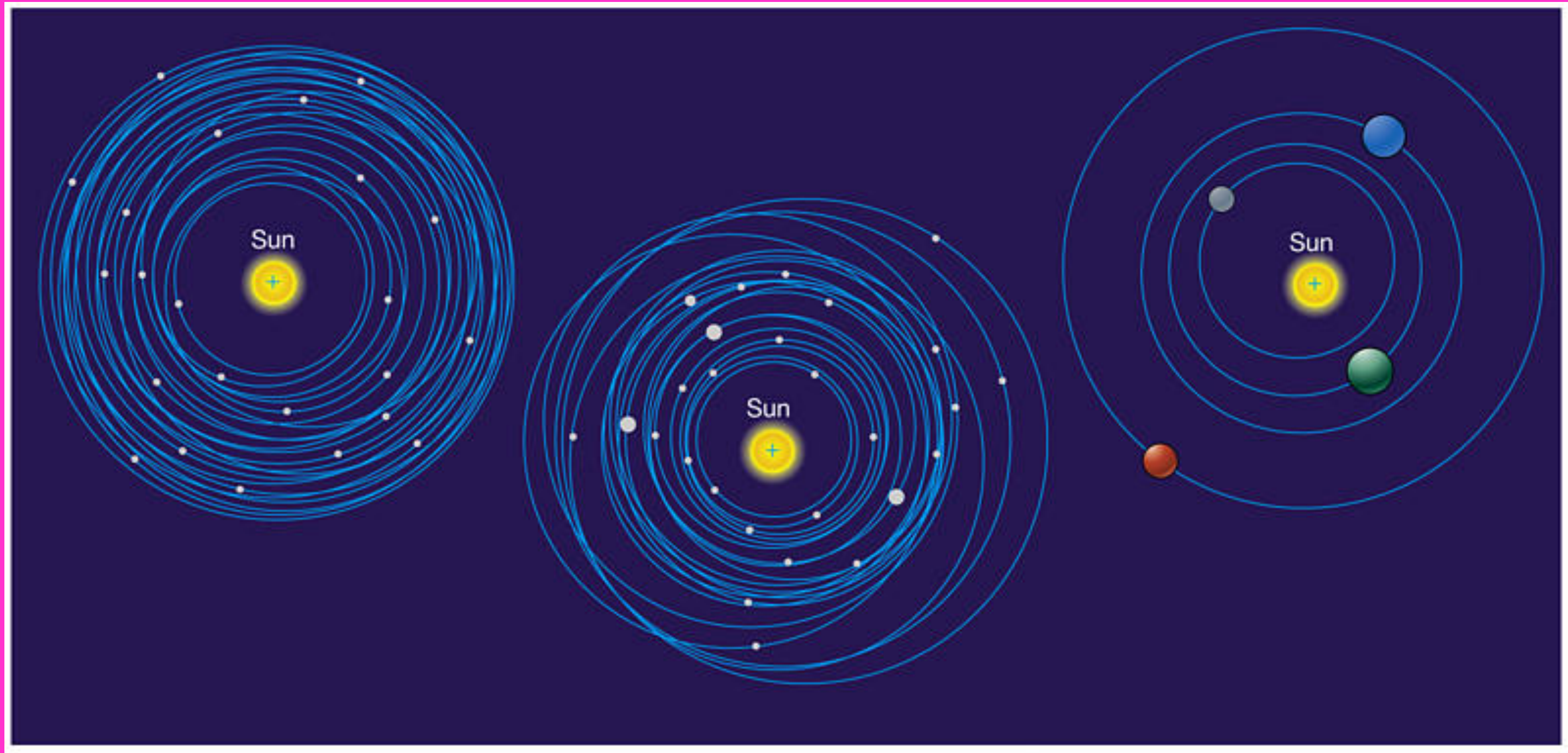
2. The only way to force the disk fragmentation is to lower $Q \sim c_s / \Sigma$ by a factor of 2 in just one orbital period. This is impossible, except very far from the star.

3. Any clumps in disk may in fact shear and disappear rather than form bound objects. Durisen et al. have found that the equation of state and the correct treatment of boundary conditions are crucial. They could not confirm the fragmentation except in the isothermal gas.

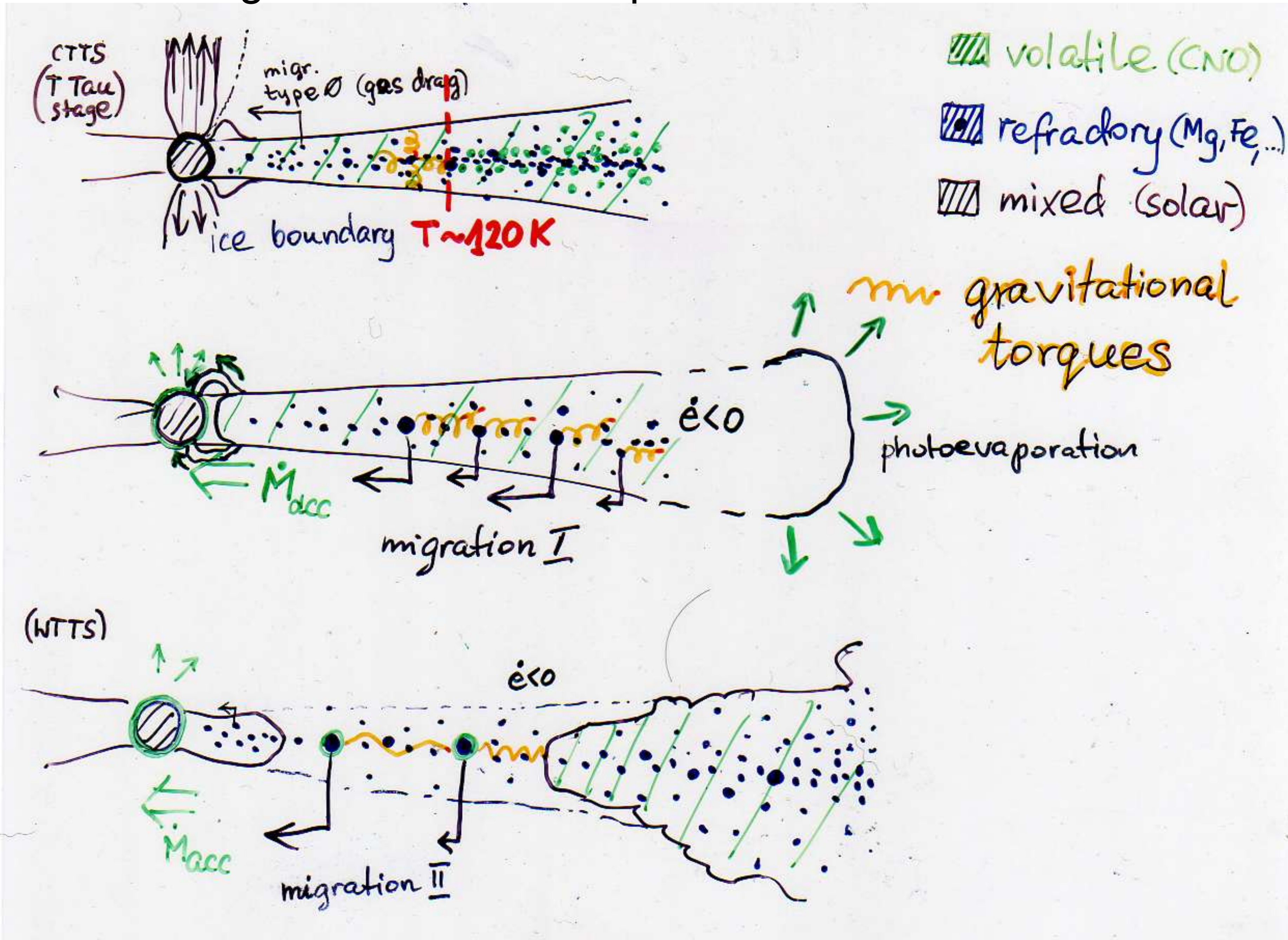
4. GGP hypothesis is difficult to apply to Uranus & Neptune. Final masses after accretion would be in the brown dwarf not the planetary range

7. GGPs cannot easily explain the similarity of core masses of planets and exoplanets, nor the chemical correlations (stars with more metallicity form planets much more frequently!)

Standard Accumulation Scenario
a.k.a.
core-accretion scenario



Two-stage accumulation of planets in disks



STANDARD ACCUMULATION SCENARIO

PLANETESIMAL FORMATION

↓

ORDERLY VS. RUNAWAY GROWTH OF PLANETARY CORES

↓

ISOLATION AND GIANT IMPACTS

↓

INSTABILITY OF PRIM. ATMOSPH. (GIANT PLANET FORMATION)

↓

GAP OPENING & migration type II

MIGRATION type I, ECCENTRICITY EVOL., FINAL MASS

Planetesimal = solid body >1 km

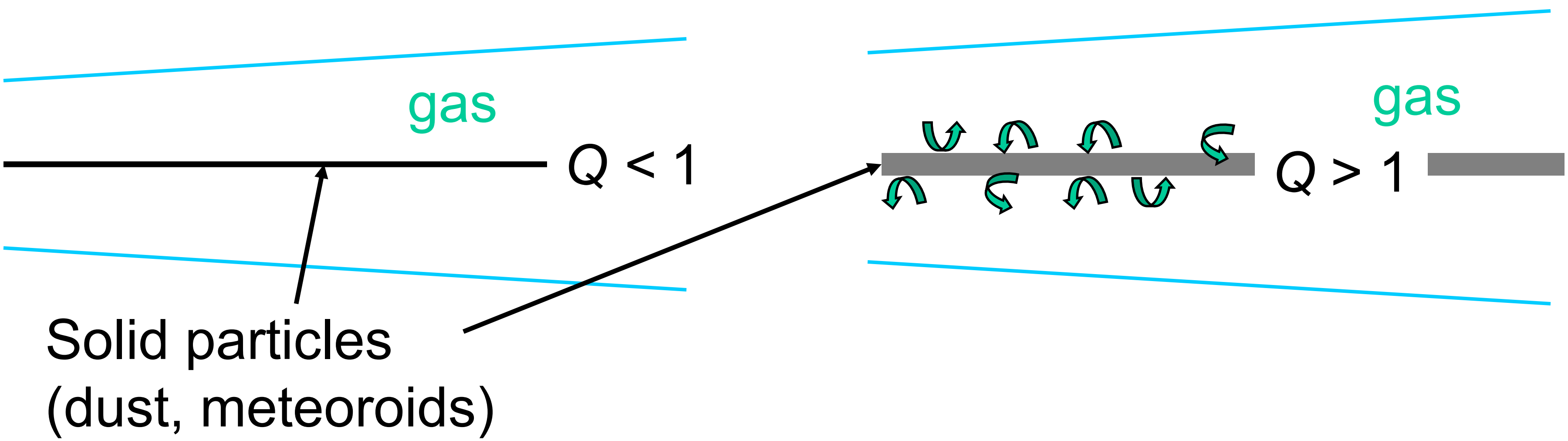
M_{core}=10 M_E(?) => contraction of the atmosphere and inflow of gas from the disk

(issues not addressed in the standard theory so far)

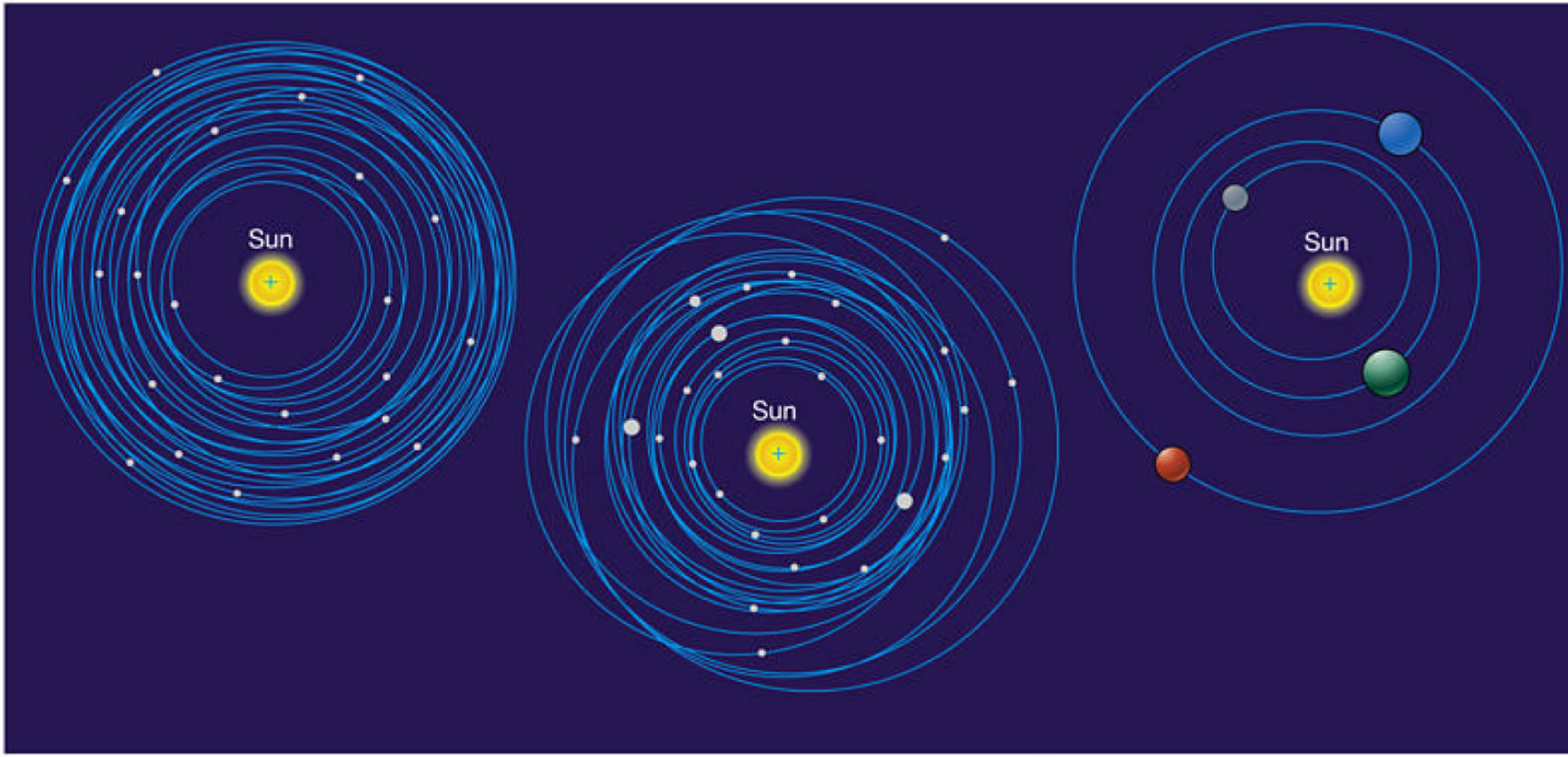
Two scenarios proposed for planetesimal formation

Particles settle in a very thin sub-disk, in which $Q < 1$, then gravitational instability in dust layer forms planetesimals

Particles in a turbulent gas not able to achieve $Q < 1$, stick together via non-gravity forces.



How many planetesimals formed in the solar nebula?



How many planetesimals formed in the solar nebula?

Gas mass ~ 0.02 - 0.1 $M_{\odot}$ > $2{\cdot}10^{30}$kg Z = $4{\cdot}10^{28}$ kg ~ 10^{29} kg

Dust mass ~ 0.5% of that ~ 10^{27} kg ~ 100 Earth masses

(Sun's metallicity Z=0.02 but volatile elements do not condense easily)

Planetesimal mass assuming s =1 km = 10^3m,

$m \sim (4/3)\pi s^3\ (10^3\ \mathrm{kg/m^3}) \quad \Rightarrow \quad m \sim 10^{13}$ kg

Assuming 100% efficiency of planet formation

N = 10^{27} kg / 10^{13} kg = $\underline{10^{14}}$ =100000000000000, s = 1 km

↓

N = 10^{27} kg / 10^{16} kg = 10^{11} = 100000000000, s = 10 km

↓

N = 10^{27} kg / 10^{19} kg = 10^{8} = 100000000, s = 100 km

↓

N = 10^{27} kg / 10^{22} kg = 10^{5} = 100000, s = 1000 km

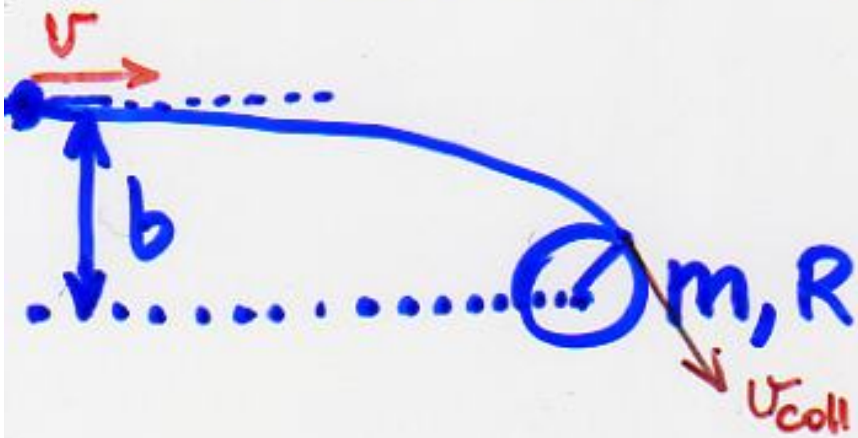
↓

N = 10^{27} kg / 10^{25} kg = 10^{2} = 100, s = 10000 km (rock/ice cores)

↓

N = 10^{27} kg / 10^{26} kg = $\underline{10}$, s~20000km (~10 M_E rock/ice cores of giant planets)

ORDINARY, GRAVITATIONALLY ENHANCED GROWTH OF BODIES



$$\begin{cases} l = v \cdot b = v_{coll} \cdot R \\ E = \frac{v^2}{2} - \frac{Gm}{\infty} = \frac{v_{coll}^2}{2} - \frac{Gm}{R} \end{cases}$$

$$\Rightarrow \frac{v^2}{2} = \frac{v^2}{2} \cdot \frac{b^2}{R^2} - \frac{Gm}{R}$$

$$\frac{b}{R} = \sqrt{1 + \frac{2Gm}{R} \cdot \frac{1}{v^2}}$$

Gravitational focusing factor

Safronov parameter

$$\Theta := \frac{v_{esc}^2}{2 v^2}$$

$$b = R\sqrt{1 + 2\Theta}$$

Safronov parameter

$$\Theta := \frac{v_{esc}^2}{2v^2}$$

$$b = R\sqrt{1+2\Theta}$$

$$\frac{b}{R} = \sqrt{1 + \frac{2Gm}{R} \cdot \frac{1}{v^2}}$$

$\Theta < 1 \rightarrow$ ~ no grav. focusing

$\Theta \sim 1$: if gravitational stirring balances collisional dissipation of energy

$\Theta \gg 1 \rightarrow$ large focusing, slow collisions ($v \ll v_{ess}$)

if grav. stirring is balanced by gas drag or other disk drag. $\rightarrow \Theta \lesssim 10^3$

Growth of planetary cores: $\theta = GM/(Rv^2)$ is Safronov number

$dM/dt = \pi R^2 \rho v\,(1+2\theta)$ or, using $M = \rho_{pl}(4\pi/3)\,R^3$, we can write

$dR/dt = (1+2\theta)\,v\,\rho\,/(4\rho_{pl})$

Case 1: Orderly growth with roughly constant $\theta \sim 1$. This could be described as *democratic growth* of all bodies, where velocity v (dispersion in a cloud of planetesimals) grows with their growing escape speed.

Objections :
For the proto-Earth, $dR/dt \sim 10$ cm/yr ➔ 10^8 years formation time. But that's too long! Radioisotopes & geochemistry show a timescale < 30 Myr for Earth formation. The cores of giant planets must have formed in less than 3 Myr, or else they would not have accreted gas from the solar nebula. And the Uranus core would take > age of Universe to form.

Case 2: *Runaway growth* *boosted by large gravitational focusing:*

In this view, $\theta >> 1$ and is not constant, velocity dispersion of small bodies is almost constant, and they grow democratically and slowly, while M, R, and the escape speed of the *locally largest body* grow much faster.

Neglecting 1 in $1+2\theta$,

$$dM/dt = \pi R^2 \rho v\ (1+2\theta) \approx 2\pi R\ \rho\ GM/v$$

and using $M = \rho_{pl}\,(4\pi/3)\,R^3$,

$$R^{-2}\ dR/dt = (2\pi/3)\ G\rho/v \approx const.$$

This equation can be integrated after multiplication by *dt* (separ. of variables)

$$1/R_0 - 1/R = t\,/(TR_0), \quad \text{or equivalently}$$

$$R = R_0/(1 - t\,/\,T),$$

where $T = R_0^{-1}\,[(2\pi/3)\ G\rho/v]^{-1}$ and R_0 is the initial radius R of the protoplanet.

This solution blows up at $t=T$, so clearly something's not right with our assumptions (constant v, and constant spatial density of solids in disk ρ) but at least the formation time is of order T and is much more plausible, e.g. for the Earth $T \sim 10$ Myr if: the disk of planetesimals has small aspect ratio z/r=1/100, ~3 Earth masses of solids are within 1 AU (which allows calculation of ρ), and small bodies have v~300 m/s consistent with disk thickness, and R_0~100 km.

Runaway accretion of the locally largest body (called an oligarch) solves the problem of timescales, even though in reality the velocity dispersion of small bodies will be stirred up by the large bodies, which moderates the growth of Safronov number.

This is the currently favored scenario of *oligarchic growth* of planetary embryos.

It allows many such bodies to grow, independently of each other, in the disk of small planetesimals (solid planet-forming bodies similar to comets and asteroids)

Stopping the runaway growth of planetary cores

Roche lobe radius $r_L = (\frac{\mu}{3})^{1/3} a$ grows non-linearly

with the mass of the planet, slowing down the growth of a planet as its mass (ratio) increases.

The Roche lobe radius r_L is the size of the Hill-stable disk region, divided by $2\sqrt{3}$. We derived this together with r_L in Lecture 8.

This will allow us to perform a thought experiment and compute the maximum mass to which a planet grows spontaneously by destabilizing further and further regions.

The concept of isolation mass

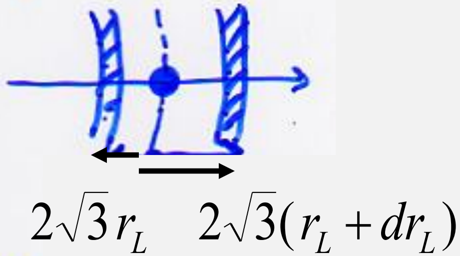
Thought experiment: let r_L grow by dr_L, and m_{pl} by dm_{pl} (or $\mu \to \mu + d\mu$).

Since $a\left(\frac{\mu}{3}\right)^{\frac{1}{3}} = r_L$, we have

$$d\mu = d\left[(3r_L)^3\right] \cdot a^{-3} = 9a^{-3} r_L^2 dr_L .$$

On the other hand, the subsequently destabilized and eventually accreted disk regions have mass ratio w.r.t. star

$$d\mu' = \frac{(2\,dr_L)\Sigma \cdot 2\pi a}{M_*} \cdot 2\sqrt{3}$$



Accretion will spontaneously continue (in a runaway manner) if $d\mu' > d\mu$, or

$$\mu \leq \mu_{max} = 3^{-\frac{5}{4}} 2^{\frac{9}{2}} \Big(\underbrace{\frac{\pi a^2 \Sigma}{M_*}}_{q_d}\Big)^{3/2}$$

This mass is called isolation mass (of embrios).

Isolation mass in different parts of the Minimum Solar Nebula

zone*	Σ (g/cm^2)	$M_{max}/M_{\oplus}$	N
Venus	12	0.0062	130
Earth	7	0.0074	135
Jupiter	15	3.3	3
Saturn	4	2.8	4
Neptune	0.5	4.4	2

* - Based on **Minimum Solar Nebula** (Hayashi nebula) =: a disk of just enough gas to contain the amount of condensable dust equal to that in rocks and ice inside planets; total mass ~ 0.02 $M_{\odot}$ & mass within 5 AU ~ 0.002 $M_{\odot}$

Conclusions:

(1) The inner & outer Solar System are **different**:
critical core=10M_E could only be achieved in the outer solar sys.

(2) There was an **epoch of giant impacts** onto protoplanets
when all those semi-isolated '**oligarchs**' where colliding.

REQUIRED READING:

Lissauer and de Pater “Fundamental Planetary Science”
the whole chapter 15 - Planet Formation

On isotopic similarities of Earth and Moon (not required):
https://astronomy.com/news/2019/05/giant-impact-hypothesis-an-evolving-legacy-of-apollo