ASTC25 (PLANETARY SYSTEMS) FINAL EXAM. PART I. PROBLEMS.

Points in the square brackets give the relative weight with which a problem counts toward the final result. This part of the exam is worth **22% of the total course score**. *Physical constants are given at the end of this section, not all necessarily useful.*

1 [15 p.] JuMBOs

J. Webb Space Telescope in October 2023 discovered dozens of strange objects in Orion Trapezium cluster, called Jupiter Mass Binary Objects (JuMBOs). These giant planets move in pairs through space, orbiting not stars but each other. Their masses have been estimated from spectra, but they are located too far to measure their sizes in images.

Assume for simplicity that a typical JuMBO has two planets exactly like Jupiter (same mass and radius), their orbit has zero eccentricity, and the semi-major axis is larger than 2 AU but smaller then 200 AU (we only know their separations projected onto the sky, so a large range is possible.) Compute the (range of) periods and the length of transits for a suitably placed observer that sees total eclipse. Should astronomers attempt to detect transits (occultations) from such a pair? Why/why not?

2 [20 p.] Donkey Paradox

Supposedly, donkeys will go backward if you push them forward, and vice versa. Hence the name of the similar orbital paradox.

A small satellite is orbiting a point mass M on a circular orbit of radius a. Acceleration f (much smaller than gravitational acceleration of M) is applied to the satellite along its orbit (acting in the direction of motion).

Eccentricity remains negligibly small, but specific energy (energy per unit mass of satellite) E, specific angular momentum L, and semi-major axis of the orbit steadily change.

A. Using orbital energy considerations, show that the orbital speed v *decreases* at a rate given by $dv/dt = -f$. I.e., the linear speed decreases at the same rate as it would increase in case of zero mass M (more precisely, in case $GM/a^2 \ll f$). Hint: Think about how dE/dt depends on f, using basic principles of mechanics.

B. Derive the same result using angular momentum instead of energy. Hint: Think about how dL/dt depends on f using the principles of mechanics.

3 [25 p.] Orbital eccentricity and heating of planets

When a planet circles a star given on a circular orbit, we can easily write a formula for the rate of its heating by energy absorption (taking into accout star's luminosity, star-planet distance $r = a$, and planet's albedo). The heating rate is proportional to inverse-square of star-planet distance, a^{-2}.

Consider another orbit for a copy of the same planet, with the same semi-major axis a, but different angular momentum and eccentricity $0 < e < 1$. Instantaneous rate of its heating is proportional to $r^{-2}(t)$ (r now changes in time). Is that planet, on average, heated more or less than the $e = 0$ planet, and therefore is it hotter, colder or the same mean temperature? What is the ratio of temperatures, assuming that a planet has very large heat capacity & conductivity, such that it keeps constant $T = const.$ throughout its orbit?

Hint: When you perform time-averaging of heating over 1 period of motion, change integration variable from t to θ.

4 [35 p.] Orbital adventures of a particle

A. A solid particle orbits around a star in a transparent gas disk, at a distance of a few AU from the star. Argue qualitatively (no need to derive exact formulae, but detailed explanation is required) that the gas disk rotates slightly slower than at Keplerian circular speed v_K, because of gas pressure gradient force. This effect is weak in thin protoplanetary disks; assume that it results in gas velocity being a factor $\eta = 0.0045$ slower than v_K (that is, $v_g = 0.9955 v_K$).

Neglecting the presence of gas, what would be the circular speed of a particle with $\beta > 0$, in units of v_K? (Parameter β is radiation pressure to gravity ratio.) Give a general formula.

Show that the particle with $\beta > 0.01$ gradually moves outward on a tight spiral, by describing the consequences of aerodynamic drag force acting on the particle in a gas disk?

B. The disk is truncated (density of gas goes to zero) at its outer radius, equal $r_d = 100$ AU. The particle, upon arriving there, changes its orbit to one that is elliptic, with periastron practically equal $r_p = r_d$. Calculate the equilibrium eccentricity $e(\beta)$ by requiring that at periastron the solid particle gas around the star exactly as fast as the gas (no push or pull from the gas). Also write the expression for $a(\beta)$.

Apply your result to silicate grain with $\beta = 1/3$ and state its e, a and apoastron (give all distances in AU).

5 Constants

$c = 2.99792 \cdot 10^8$ m/s $= 2.99792 \cdot 10^{10}$ cm/s (speed of light)
$G = 6.67259 \cdot 10^{-11}$ m^3 kg^{-1} s^{-2} = $6.67259 \cdot 10^{-8}$ cm^3 g^{-1} s^{-2} (gravity)
$k = 1.3807 \cdot 10^{-23}$ J/K $= 1.3807 \cdot 10^{-16}$ erg/K (Boltzmann)
$m_H = 1.66054 \cdot 10^{-27}$ kg $= 1.66054 \cdot 10^{-24}$ g (hydrogen mass)
$a = 7.5646 \cdot 10^{-16}$ J/K^4/m^3 $= 7.5646 \cdot 10^{-15}$ erg/K^4/cm^3 (radiation const.)
$\sigma = 5.67051 \cdot 10^{-8}$ J m^{-2} s^{-1} K^{-4} $= 5.67051 \cdot 10^{-5}$ erg m^{-2} s^{-1} K^{-4} (Stefan-Boltzmann)
$M_\odot = 1.9891 \cdot 10^{30}$ kg = $1.9891 \cdot 10^{33}$ g; $M_J \simeq 10^{-3} M_\odot$ (Jupiter's mass)
$R_\odot = 6.9598 \cdot 10^8$ m $= 6.9598 \cdot 10^{10}$ cm; $R_J = 69500$ km (Jupter's mean radius)
$L_\odot = 3.8515 \cdot 10^{26}$ J/s $= 3.8515 \cdot 10^{33}$ erg/s
$M_E \simeq 3 \cdot 10^{-6} M_\odot$ (mass of the Earth); $R_E = 6371$ km
1 AU = $1.496 \cdot 10^{11}$ m = $1.496 \cdot 10^{13}$ cm; 1 pc = 206265 AU
1 yr = $3.1558 \cdot 10^7$ s

Part II – TRUE-OR-FALSE quiz (please mark with Y or N for Yes or No in front of the statement. **Circle phrase(s) or word(s), or numbers you think are incorrect**. Disregard language errors/typos. This part is worth **22%** of course mark. Please sign your student number on top of the first page, and every loose quiz sheet, if not stapled.

[Y N] Planetary rings around giant planets in the solar system are stabilized by the precession of particles orbits caused by oblateness of the planets. Terrestrial planets are too spherical to cause similar precession.

[Y N] Runge-Kutta method allows establishing the age of a rock based on radioisotope ratios.

[Y N] Computer simulations of trajectories are used when perturbations are large, which invalidates the assumptions of analytical perturbation theory

[Y N] Jacobi energy method can guarantee that a particle will never cross zero velocity curve (or surface), and therefore be stable against an escape through that surface.

[Y N] Restricted elliptic 3-body problem does not have Jacobi energy constant.

[Y N] Triangular Lagrange points can be dynamically stable equilibria, because they are minima of effective potential of forces.

[Y N] Roche lobe is virtually the same concept as Hill sphere or sphere of gravitational influence. Hill's equations are making approximations which become exact in the limit of vanishingly small planetary mass, whereas Roche lobes also exist for large mass ratios.

[Y N] For a molecular cloud core of a given temperature and radius, Jeans mass is a critical mass exceeding which will lead to instability, collapse, and fragmentation of the core. Dynamical contraction of this kind is called Hayashi phase.

[Y N] Opacity-limited fragmentation of stars (when heat produced by nuclear reactions gets trapped in the gas because of gas opacity) leads to formation of white dwarfs.

[Y N] The smallest fragments in F. Hoyle's scenario of opacity-limited fragmentation have masses of order 80 Jupiter masses

[Y N] Brown dwarfs (13-80 Jupiter mass objects) support hydrogen fusion reactions. Their observability is due to a slow cooling from the relatively hot formation period.

[Y N] Accretion disks owe their shape to energy conservation. Energy of radial motions is not dissipated, unlike in energy of vertical motions.

[Y N] T Tauri systems are millisecond pulsars, neutron stars that rotate very rapidly. In 1992 Earth-sized planets were discovered around one of them.

[Y N] Core-accretion scenario proposes a top-down theory of giant planet formation, in which accretion disks fragment to quickly form planets (in less than ten thousand yr).

[Y N] Gravitational stability requires that Safronov-Toomre number Q = (epicyclic frequency)*(soundspeed)/[const* surface density)] is less than 1.

[Y N] Amplification of global modes does not end up in disk fragmentation, because nonlinear waves present in the disk heat the disk, stabilizing it. Only the models with inifitely fast cooling (isothermal gas models) or similar want to fragment in computer simulations.

[Y N] Standard accumulation scenario is of bottom-up type, and leads to very short timescales of formation

[Y N] If typical encounter velocities of solid bodies inside the protoplanetary disk were suddenly increases, then the gravitational focusing factor would increase as well.

[Y N] Accretion of gas through a disk forms a common thread in the history of formation of all stars. Total amount accreted during formation phase (rate * typical duration) is of order 0.05 solar mass per million years.

[Y N] We call the early (<1 Myr) massive hydrogen-helum disks primordial solar nebulae

[Y N] Transitional disks like RY Lupi have ages of 20 to 200 Myr and are devoid of gas

[Y N] Disk that is largely transparent and heats up by stellar radiation absorption by embedded blackbody dust grains will have a flaring geometry resembling a model where $z/r \sim r^{1/4}$

[Y N] T Tau disks are optically very thick (optical thickness 1e5...1e6 meters)

[Y N] MRI is a convective instability of disks. It produces $\alpha \sim 0.01$

[Y N] $z/r = c/v_K$ (z = vertical scale height od disk; r = radius, c = non-dimensional kinematic viscosity coefficient; v_K = Keplerian orbital velocity

[Y N] Runaway growth of planetesimals into planets occurs when the largest body in its zone has a much larger gravitational focusing factor than the rest.

[Y N] Ices in the comets are rich in oxygen, and some have a chemical formula $MgSiO_4$.

[Y N] Runaway and oligarchic growth require high random velocities of planetesimals.

[Y N] Angular momentum distribution in the solar system is such that just 0.002 solar masses of material in it contains 98 percent of all angular momentum. Most of angular momentum is in the form of spin of giant planets, which leads to a visible rotational flattening of their shapes.

[Y N] The interiors of all of the major planets, many asteroids and most if not all large moons are differentiated, with most of the heavy material confined to their cores, distinct from the rest of their material. E.g., Earth has a liquid iron core.

[Y N] Water condenses as pure water-ice at temperatures below 200 K. Aluminum oxides condense into ice (mixture with water ice) at somewhat lower temperatures.

[Y N] As a disk of solar-composition gas cools, various compounds condense into microscopic grains. The first substantial condensates are silicates and iron compounds (at T = 300-1500 K).

[Y N] Final epoch of rocky planet growth is characterized by large orbital eccentricities, significant radial mixing and giant impacts, such as those that punctured thin crust of the Moon and created lunar maria regions (once, lunar "oceans" of lava).

[Y N] Final epoch of giant planet growth involved their cooling, sufficient to allow very rapid (runaway) gas accretion onto a rocky core of at least 10 Earth masses.

[Y N] Thousands of minor planets of radii $>$ 10 km orbit between Mars and Jupiter, making the total mass of the asteroid belt larger than 1 Earth mass.

[Y N] Chemical differentiation of minor and regular planets requires heat source. Heat was produced by (i) accretion of solids, (ii) decay of short-lived radioactive isotopes.

[Y N] When the body reaches distances of greater than 50 AU, perturbations from the tidal pull of the galaxy can lift its perihelion out of the planetary region, and the body can thus be stored in the Oort cloud.

[Y N] Protoplanetary disks are very rare around stars older than 10 Myr.

[Y N] In Oort cloud all comets pursue prograde orbits (circling the sun in the same direction as planets).

[Y N] Moon's bulk composition resembles Earths mantle (the ligher fractions). Collision between the proto-Earth and at least a Mars-sized protoplanet ejected enough crushed and molten mantle rocks into Earths orbit, for the Moon to assemble quickly.

[Y N] During the collision that formed the Moon, volatiles and water were dispersed in space and did not re-assemble.

[Y N] Radiation pressure on dust in the absence of gas in disk causes solid particles to be pushed away from the star on spiral trajectories.

[Y N] Dust avalanches in young planetary systems have a spiral form

[Y N] Crystalline grains with regular crystal structure are derived from amorphous grains with disordered structure by annealing effect, requiring temperature up to 1000 K.

[Y N] By number, most exoplanets have been discovered by the Hubble Space Telescope in the early 2000s.

[Y N] Finding density of an exoplanet requires the use of Doppler technique in combination with direct imaging.

[Y N] We currently find that there are about 10 planets per one star in our Galaxy. A typical example is provided by the solar system.

[Y N] Having high metallicity and having planets are highly inversely correlated characterists of a star. This is explained by a limited total budget of rocks and ices.

[Y N] Protoplanets migrate because they interact gravitationally with and distort the protoplanetary disk. They launch waves and tides, and open gaps.

[Y N] We once thought that Jupiter could grow in a protoplanetary nebula to a much larger mass than 1 Jupiter mass, but now we think that gap opening cuts off the flow of gas onto a planet.

[Y N] In 2017-2020 astronomers discovered two interstellar objects, traveling on hyperbolic orbits in and out of the Solar System: I/1 Oumuamua and I/2 Comet Borisov.

[Y N] Burgess shale in Canadian Rockies is a rich source of information on the earliest life forms on Earth from 500-570 Myr ago.

[Y N] Type III migration happens in a partially open gap, acts very rapidly (in 100 orbital periods), and starts when mass deficit in the gap is larger than the mass of the planet.

[Y N] Life may have started neither from a protein not from a DNA molecule, but with an RNA molecule. RNA molecules are small bacterial cells (like SARS-Cov2 virus, which is RNA-based).

[Y N] About one new planetary system per day forms somewhere in the Universe, on average.

[Y N] The most common mineral types in the universe are silicates olivine (Mg,Fe)SiO4 and pyroxene (Mg,Fe)SiO3. They are also common on Earth.

[Y N] In 2020 NASA scientists Urey and Miller announced the experiment where they have created artificial life from the amino acids found in Martian meteorite ALH 84001, which landed in Antarctica some 13000 years ago.