ASTC25. PROBLEM SET #3. DUE 14 MARCH, 1PM

The first two problems use concepts introduced by French scientist Edouard Roche (1820-1883): the Roche lobe and the Roche limit of tidal disruption.

1 [25p] Can you believe it?

Having observed a transiting giant exoplanet with a short orbital period $P = 5.87$ days, circling around a star of mass $M = 0.79M_\odot$, your friend, a graduate student of astronomy, reports that in the lightcurve he sees a signature of an occasional eclipse of a small moon orbiting the planet. Before he announces the details in a press release, your friend asks you to help establish the plausibility of the discovery by showing that the orbit of the exo-moon is stable.

Compute the minimum (critical) heliocentric radius a_p and the orbital period P of a planet having a mean density $\rho_p = 0.95$ g/cm^3 (a value following from transit observations of the exoplanet, more than the density of Saturn but less than Jupiter's), which allows it to have a moon. Draw conclusions about the plausibility of the discovery of the moon.

Assume that only the moons with semi-major axes a closer to the planet than 3/11 of planet's Roche lobe radius are orbitally stable. (For instance, you can check that our Moon circles the Earth at about 1/4 of the Earth's Roche lobe radius.) In your calculation, assume that $M + m_p \simeq M$ to simplify the expression for Roche lobe radius (m_p is the mass of the planet.) Simultaneously, natural satellites held together by self-gravity must reside outside the Roche limit. Assume it is equal to 2.44 R_p, i.e. 2.44 physical radii of the planet.

2 [30p] UTSC students disprove a hoax, claim at least 11 moon months per year anywhere in the universe

ASSORTED PRESS. 1hr 25 minutes ago

TORONTO, Canada - A message from extraterrestrial civilization allegedly deciphered by TikTok user Nohan F. Knowes using a laptop connected to a parabolic satellite-tv dish, went viral world-wide. It was the second most-read news on internet on March 5, 2024. The first was titled ”On Tuesday morning, Facebook, Instagram and Threads were down. Thousands affected”.

The message from E.T.'s, in addition to elaborate greetings addressed to world governments, informed that their civilization lives on a beautiful planet with one moon. They celebrate a holiday similar to our New Year that takes place in the last moon-month, or lunar month, on their planet, which happens to be the 10th lunar month.

However, just one day after it started circulating, the message was shown to be a hoax by students from astrophysics course at the Scarborough campus of the University of Toronto. Students claim to have demonstrated that anywhere in the universe, there are at least 11 moon-months in a year (i.e., more than 11 moon orbital periods in one orbital period of the planet). This astonishing fact has so far escaped the scrutiny of astronomers and raised a few eyebrows in academia.

The proof is reportedly based on the notion of Hill stability of the moon orbit, requiring it to be within 3/11 of the so-called Hill sphere, or Roche lobe radius of the planet, as well as on the centuries-old Kepler's 3rd law. The students mentioned the fact that our Moon is nearly Hill-unstable, orbiting at the distance of 0.256 (somewhat less than 3/11) of Earth's Roche lobe radius, and completing about 13 revolutions in a year. We eagerly await the promised publication of the proof.

Spurred on by their achievement, some students reportedly also estimated the *maximum* number of moon months in one year, anywhere in the universe. Details of the second proof were not immediately available, but X platfom commentators suggested that it may be based on the concept of "Roche limit" and an assumption that the mean density of a planet cannot exceed the density of iron.

Mr. Knowes could not be reached for comment by the time of this publication.

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Can you furnish the proof(s)? For full credit, provide the complete proof of the 1st claim (minimum number of moon-months) without mistakes, or incomplete proofs of both claims (about minimum and maximum numbers of moon-months).

Take into account a subtelty of this problem. There are two different notions of a lunar month in circulation (pun intended): a sidereal and a synodic month. The number of those months in a year differ by exactly 1. More about this in `https://en.wikipedia.org/wiki/Lunar_month`. State which moon-month you are using in the derivation and express the minimum numbers using both.

3 [15p] Longer years

Years should get longer, extremely slowly, because the sun gradually loses mass. So slowly that you may have a problem trying the compute the difference of orbital period P from one year to the next on a calculator or computer. For such occasions, we have calculus.

The sun loses mass at an average rate of $2 \cdot 10^{-14} M_{\odot}$/yr by ejecting its gas as the solar wind. Additionally, every year the sun loses mass m in thermonuclear reactions: $E = mc^2$, where c is the speed of light in vacuum, and E is energy radiated away in one year, which you can compute from sun's luminosity $L = 3.846 \cdot 10^{26}$ W.

By how many microseconds will the next year be longer than the current year due to solar wind, and how many due to both effects above?

Hint: Assume that the angular momentum and the eccentricity of Earth's orbital motion are conserved in both processes.

4 [20p] Dress right for your trip to the Moon

The Moon on average is as far we are from the sun, so you'd expect a similar average temperature. But the truth is, due to the lack of atmosophere, temperature variations on the Moon are extreme. "Something I touched was hot - through the suit, so it must have been pretty hot" observed one Apollo astronaut after a Moon walk. In the shadow or during lunar night, the cold was so extreme that mechanical equipment was often malfunctioning because the grease froze solid.

Calculate $T(\theta)$, the lunar surface temperature as a function of sun angle above horizon. Assume that the sub-surface layer is insulating and conducts outward only a small thermal flux coming from the Moon's interior. That interior flux is a fraction q of the mean solar irradiation flux $F_{\odot}$, which on the Moon as on Earth is equal $F_{\odot} = 1373$ W/m^2. You can thus assume that the surface is heated only by the internal plus the solar flux (depending on sun angle θ) and cooled instantaneously by thermal re-emission that follows the Stefan-Boltzmann law. Efficiency of re-emission is 1, and of sunlight absorption is 1-A, where A=0.136 is the mean Bond albedo of the lunar surface, as measured by one satellite observatory. (The Moon has fairly dark rocks, they scatter into space only 14% of incoming light!)

Establish the unknown parameter q that gives best agreement of your $T(\theta)$ dependence with thermocouple temperature measurements of the surface by Apollo 17 astronauts, found in this picture: `https://www.workingonthemoon.com/a17psrf9-9.jpg`. Landing site on the 'shore' of the Sea of Tranquility was close (20°) to the equator and the sun was near the zenith ($\theta = 90°$) at noon. [Phase angle in the figure is a re-calibrated time, not exactly θ, but that's of no concern to you, because you are interested in temperature extrema only.]

What is the value of q? What are the minimum and maximum temperatures of the lunar surface in your model in K and °C? Plot or accurately sketch $T(\theta)$. Comment on all your results. For instance, what does q say about the amount of heat coming to the surface from the interior, relative to sun's heating?