Preparation for midterm and other interesting problems. NO SOLUTIONS.

ASTC25 (PLANETARY SYSTEMS) PREPARATION FOR MIDTERM EXAM. PART I = PROBLEMS.

There will be only one written problem in the midterm. It will not be very long, since you have to solve it in about 30 minutes. Some of the problems below are longer and more difficult, but contain diverese ideas and methods that may be helpful in exams.
Points in the square brackets give the relative weight with which the problems count toward the final score. It is way more important to provide a good path to solution, even if slightly inaccurate (due to approximations made), than to compute the final answer numerically. Please always check units before pluggin in numbers. Various physical constants are given at the end of this text. If you need additional constants or planetary data, find them online, and during exam raise your hand and the'll be provided to you.

1 Bolide, satellite and plane

You look up at zenith (straight over your head) after sunset and near each other you see: an airplane, the International Space Station satellite complex, and a bolide (big meteor burning upon the entry to the atmosphere). Calculate their angular speed in degrees per second (at zenith). Neglect the Earth's rotation speed in comparison with cosmic speeds.

For the ISS and plane, find the time of visibility above horizon in minutes, disregading any obstacles on the ground and refraction of light, but properly taking into accout the curvature of Earth. For the meteor, esimate the time to cross one radian arc in the sky.

Assume that the airplane moves at h=10.5 km, the bolide h=50 km, and the ISS at h=408 km above the ground, at constant h. Airplane's speed is v=870 km/h, and the bolide moves at 1.5 times the escape speed from Earth.

2 Orbital migration at zero eccentricity

What is the orbital migration equation in Gauss perturbation theory if acceleration f is directed backward with respect to the velocity of a satellite orbiting around central mass M? Can you obtain the same result more directly, from the conservation of angular momentum? On what timescale is the orbiting body migrating? Hint: timescale is defined as $t_a := a/\dot{a}$.

3 Solve the donkey paradox

Applying a small, **backward** directed, steady acceleration $f < 0$, we cause the orbiting body to **speed up** at a rate $d(v_K)/dt = -f > 0$, as if the force were applied forwards rather than backwards. The name of the puzzle derives from an alledged behavior of stubborn donkeys.

4 When will the Hubble fall out of the sky?

Hubble Space Telescope is a 13.2 m 4.2 m (diameter) object with 11 ton mass, currently orbiting at $h = 535$ km above Earth. In how many years will it come down, if the atmospheric density is derived from the model of atmosphere above 25 km altitude given in https://www.grc.nasa.gov/www/k-12/rocket/atmosmet.html

Solve it either analytically or numerically, integrating over time the Gauss perturbation theory prescription for the rate of change of semi-majpr axis, $\dot{a}$. Consider that at 25 km height, the satellite is already practically destroyed (broken up, burned). Neglect Earth's rotation and winds, and satellite orbit's eccentricity. Assume cross sectinal area A equal to arithmetic average of the largest and smallest such area HST presents to the flow of rarified air. Drag force can be assumed equal to

$$F = -\rho A v^2/2.$$

Compare the result with the estimate of timescale for the sinking process given by he current value of $a/\dot{a}$.

5 [2 p.] Transfer orbit of the yellow convertible to Venus

A red covertible car (sometimes called cabriolet in Europe) was sent by Elon Musk's company Tesla toward Mars and beyond several years ago. You are going to do a similar exercise of sending a yellow convertible toward Venus.

1. Design a transfer orbit to Venus-like orbit from the circular orbit of radius 1 AU, but nor from an immediate vicinity of Earth, i.e. disregarding both planet's gravitation. It will involve the use of thrusters able to change the speed by a given amount, using appropriate amount of fuel.

How much extra linear speed do you need to give to the spacecraft and in what direction so that it reaches Venus at pericenter (= perihelion) of the orbit, and by how much (and in what direction) do you change the speed at pericenter a.k.a. perihelion, to put the spacecraft on a circular orbit there?

Compare these results with the additional speed to go from near-Earth orbit to Mars (where you reach apocenter not pericenter).

2. Fuel needed (This is a more difficult calculation. Do not expect it at midterm!)

How much mass of fuel do you need in order to add speed Δv to a spacecraft of decreasing mass equal at time t to $M + m_f(t)$, where $m_f(t)$ is the decreasing fuel+oxidant mass and M the other, fixed mass of the spacecraft (structure, payload, equipment, etc.).

Assume that the exhaust gases escape from he nozzles at speed $v_f = 3.1$ km/s (which is valid for kerosine-based fuels); denote the infinitesimal decrease of spacecraft mass by dm_f, and the increase of spacecraft speed by dv. Consider the problem exactly, not as a rough estimate: formulate the equation of momentum change, where on the l.h.s. you write the infinitesimal change of the spacecraft momentum and on the r.h.s. the momentum flowing in the same time interval out of the nozzles with exhaust gases. Solve that equation to obtain Ciokowski or Tsiolkovsky (or the rocket) equation. With a correct solution in hand, you will successfully apply for jobs at NASA as a rocket scientist.

What is the initial mass fraction m_f/M needed for the transfer orbit? For the escape orbit?

6 Thermal temperature of orbiting gas falling onto accretion shock

[part A, 3p.] Derive an expression for the circular Keplerian velocity v_K of a body on a circular orbit from the centrifugal and gravitational force balance. What would be the kinetic energy E_k of a hypothetical particle (molecule) with mass μm_H, circling a star of 1 solar mass near its surface?

[part B, 2p.] Suppose that all this energy was converted into heat (i.e., was thermalized, e.g. in an accretion shock at the surface) according to the expression $E_k = (3/2)kT$, (k is Boltzmann constant). What thermal temperature T would the gas made of such particles achieve in the post-shock layer?

[part C, 1p.] Evaluate numerically that temperature for the sun.

7 Stable or unstable

A dark molecular cloud core at a distance 100 pc from Earth has the angular diameter of 33 arcminutes. Its temperature is $T = 25$ K, and the estimated mass is $M = 207 \pm 22 M_{\odot}$. Is the core gravitationally stable?

8 Simple orbital kinematics

What is the eccentricity and semi-minor axis of a heliocentric orbit, which spans the range of radii from 42 AU to 80 AU? What are the perihelion and aphelion speeds? What is the aphelion speed?

9 [4 p.] Simple Kepler 22b planet properties

What is the gravitational acceleration in Earth units, mean density, and the escape speed from the Kepler 22b planet (http://exoplanet.eu/catalog-transit.php)? What is the rough prediction for the black-body temperature on the planet, based on stellar data (luminosity $L = 0.79 L_{\odot}$)?

Calculate the expected mass if the planet has the same minerals and the same mean density as Earth, then increase the estimate of mass by 10% to approximately reflect the larger compression of rocks in the core and mantle of a more massive planet. (Only the upper limit on planet mass: $m < 31 M_{Earth}$ is known from Doppler spectroscopy.)

Is any water on the planet fluid or frozen, assuming 1 bar pressure in the atmosphere?

10 [2 p.] Simple Kepler 22b humanoid properties

[This problem is really for fun, not really for preparation to midterm.There will be no task involving bio-mechanics.]

Suppose that human-like creatures live on Kepler 22b, which are simply rescaled humans (keeping all the building materials & proportions of the body the same). They must be rescaled overall, in order for their bones to be as resilient as ours, relative to their weight. The maximum load on a bone is simply proportional to its cross-sectional area. By what factor n would those aliens be rescaled?

11 [2 p.] Do they use Skype22 on Kepler 22b?

The humanoids on Kepler 22 b want to talk using a communications program Skype22 with their friends on the other side of the planet in real time (without noticeable delay). Speculate on whether they'd enjoy doing this like we do here on Earth, assuming the same kind of technology: electromagentic waves, optical cables, electronic equipment and so on, everything limited by the speed of light and the size of the planet.

As the speed of light is the same on Kepler 22b but its size is larger, communications will have to suffer larger delays. Meanwhile, a creature n times larger/smaller than an average human, will walk and talk (we assume these processes are evolutionary linked) slower/faster if the creatures are larger/smaller than us. Since we move our extremities roughly in sync with the swing of a pendulum the length of our extremities while walking (to save energy), let's assume that the speed at which creatures on different planets think and talk is proportional to the frequency of a pendulum their size. How much faster or slower do the aliens talk on Kepler 22b, assuming those scalings?

12 [2 p.] Strange orbital mechanics

Show that the following strange result holds in the case when there is no aerodynamic drag anywhere:

Several bodies are released from a point outside the Earth with the same speed v but in different directions (such that they don't hit the planet). Gravitation of bodies other than the planet is taken to be zero. They will all come back to the launch point at exactly the same times, infinitely many times (unless they physically collide).

13 [2 p.] Mars: mountains and other features

Mars has radius = 3396 km, and mass = 0.107 Earth's, as compared to Earth's radius 6371 km.

[2p.]

Compute the mean density of Mars, both in absolute numbers (g/cm^3 would be an intuitive unit) and relative to that of the Earth.

[2p.]

Assuming that the maximum tangent of the angle at which mountain slopes are inclined to the horizon is proportional to the local gravitational acceleraton, estimate the maximum slope on Earth from your experience, and using that estimate, on Mars (radius = 3396 km, mass = 0.107 Earths).

[1p.]

Compute the length of year on Mars in days.

14 [2p.] Contact the rover

If you send a signal to a Martian rover, and it responds after 1.0 second of receiving the signal, then how long will you wait for the answer? Give precise minimum and maximum values.

15 [3p.] Orbital facts

If you brake the motion of a satellite on a Keplerian circular orbit by 1% of its speed, at the pericenter of the new orbit it will have an increased speed, instead of staying at a 1% slower speed. By how many % of the original circular speed? What if you increase the speed from circular by 1%? By how much do you change the size of the orbit in each of the above cases?

16 [4p.] Compute the tidal force

Find the tidal force pulling you apart, when you stand on the surface of a planet or another body with mass M and radius R. Compute the acceleration difference at your head and your feet, you may of course approximate the formula using the smallness of a human compared with the size of a planet. Express the answer in units of $g_E = GM_E/R_E^2$, where index E refers to the Earth, and evaluate that answer for the Earth's surface.

How small would a $M = 1M_\odot$ or sun-like mass body have to be in order to destroy yourself standing on the surface of that body? Assume the acceleration $a = 100g_E$ is enough to kill you.

17 [6p.] Chelyabinsk meteor explosion

A few days ago, a very big meteor entered the atmosphere and exploded over the Siberian city of Chelyabinsk. About 15 km above the city, it released energy equivalent to approximately 500 ktons of TNT (500e+6 kg of TNT).

A. What was that energy in Joules? (Find the equivalent energy of 1 kg TNT; find first what TNT is if you don't know!). If the Hiroshima bomb was rated at 12 ktons TNT, how much larger energy was released a few days ago? (Give a computation, don't just quote wiki!). That certainly explains the more than a thousand people injured by shards of glass.

B. If the meteor had a speed of 18 km/s, estimated from videos, before disruption, how much mass did it have and what size of a body was it, assuming stoney sphere with density 3 times as much as water density?

D. Assume the meteor's orbit had a perihelion at 0.8 AU, while its aphelion was in the astroid belt at 2.2 AU from the sun. Compute the speed of the meteor like so: From the formula for total orbital energy on the one hand and the instantaneous sum of kinetic and potential energies on the other hand, compute the the speed of the asteroid/meteoroid at $r = 1$ AU, but before it caught up with the Earth. Then take into account the depth of Earth's potential well (in terms of test particle speed, it's equal to the second cosmic speed or escape speed from Earth of 11.3 km.s) and adjust the speed up, because of Earth's gravity. What is that speed? After a vectorial subtraction of the Earth's speed, what relative speed is possible while entering the stratosphere? Draw a sketch of the orbit and the relevant speeds. Discuss your results.

[Chelyabinsk has been a subject of many jokes stressing the mythical thoughness of its residents. People of this city are all like Chuck Norris of the Siberia. It is said, for instance, that students are so though that the military recruiting committees run away from them. Cosmonauts from Chelyabinsk visiting the Space Station step outside without spacesuits to smoke cigarettes. And yes, the meteor did not explode, it was blown up by its inhabitants when they realized which city they approach.]

18 [5p.] Two planets

Two planets circling clockwise the same star of mass M in the same plane (co-planar), interact gravitationally and modify their orbits in time. If they never approach closely, then the orbits evolve very slowly, changing orbital eccentricities and the angular momenta given by the formulae

$L_1 = m_1\sqrt{GMa_1(1-e_1^2)}$

where m_1, a_1, e_i are the first planet's mass, semi-major axis and eccentricity. Planet 2 has angular momentum L_2 given by an analogous formula with subscript 2. The total angular momentum of the system is conserved.

The orbits will precess, all the time keeping their initial energies and semi-major axes.

Suppose that one planet has the mass equal to $m_1 = 3$ Earth masses and at present has $a = 2$ AU and $e = 0.1$. The second planet has $m_2 = 1.5$ Earth masses, $a = 3.5$ AU and $e = 0.25$. What is the highest eccentricity that planet 1 can ever achieve? What is the highest eccentricity that planet 2 can ever achieve? What is the minimum distance ever between the planets?

19 Some possibly useful constants

If you don't have a calculator, state it in your solution and provide your calculation rounded off to 2 significant figures (numerical error less than $\sim$10% will not lower your score.) Otherwise, at least three significant figures should be carried.

$c = 2.99792 \cdot 10^8$ m/s, $= 2.99792 \cdot 10^{10}$ cm/s (speed of light)

$G = 6.67259 \cdot 10^{-11}$ m^3 kg^{-1} s^{-2} = $6.67259 \cdot 10^{-8}$ cm^3 g^{-1} s^{-2} (gravity)

$k = 1.3807 \cdot 10^{-23}$ J/K $= 1.3807 \cdot 10^{-16}$ erg/K (Boltzmann)

$m_H = 1.66054 \cdot 10^{-27}$ kg $= 1.66054 \cdot 10^{-24}$ g (hydrogen mass)

$a = 7.5646 \cdot 10^{-16}$ J/K^4/m^3 $= 7.5646 \cdot 10^{-15}$ erg/K^4/cm^3 (radiation const.)

$\sigma = 5.67051 \cdot 10^{-8}$ J m^{-2} s^{-1} K^{-4} $= 5.67051 \cdot 10^{-5}$ erg m^{-2} s^{-1} K^{-4} (Stefan-Boltzmann)

$M_\odot = 1.9891 \cdot 10^{30}$ kg = $1.9891 \cdot 10^{33}$ g

$R_\odot = 6.9598 \cdot 10^8$ m $= 6.9598 \cdot 10^{10}$ cm

$L_\odot = 3.8515 \cdot 10^{26}$ J/s $= 3.8515 \cdot 10^{33}$ erg/s

1 AU = $1.496 \cdot 10^{11}$ m = $1.496 \cdot 10^{13}$ cm

1 yr = $3.1558 \cdot 10^7$ s; 1 pc = 206265 AU.

Earth's radius is $R_E = 6371$ km, Jupiter's mass is about 1/1000 $M_\odot$ or 316 Earth masses. Thus, Earth has $\sim 3e-6$ times the sun's mass.