Final exam PHYD38: Nonlin. Phys and Chaos, 7 Apr 2014

1 Written problems. Number pages please, put student numbers

1.1 Linear system

$$\dot{x} = x + ay$$

,

$$\dot{y} = 2 - x$$

Study it as thoroughly as you can, in the range of of parameter a from -1 to 3, finding any change in behavior in that interval.

1.2 Non-linear system

Perform the analysis of the phase plane of the autonomous system,

$$\frac{dx}{dt} = yx + ay^2$$

$$\frac{dy}{dt} = 4 - x^2$$

Find all the fixed points, their local linear type and stability. Do independently a matrix algebra method, then the method of substituting trial function of the form $x = x * +u$, $y = y * +v$ where $O(u^2) = O(v^) = O(uv)$ are to be neglected, and u, v are varying in time like $exp(i\lambda t)$. Compare results.

Assemble a global phase space patern of flow from individual surroundings of fixed points.

What kind of motion does the system perform: linear, power-law or exponential in time near the fixed point, and far from its fixed points?

Find the characteristic time scale $|u/\dot{u}|$ and $|v/\dot{v}|$ of convergence or divergence to/from sample fixed points.

1.3 2-D system

(A) For the system

$$\frac{dx_1}{dt} = x_2 - x_1 - x_1^2$$

,

$$\frac{dx_2}{dt} = x_2 - x_1 + x_1 x_2$$

find the fixed points, classify them and study their nature. Use any and all methods you have learned. Explain methods step by step, do sketches of everything (local linear behavior around fixed points, and assumed nonlocal behaviour).

(B) Do the same with

$$\dot{x} = (1 + x - 2y)x$$

,

$$\dot{y} = (x - 1)y$$

1.4 Orbits of squared iterated map and Lyapunov exponent

Estimate the Lyapunov exponent of a 1-d iterated map $x_{n+1} = F(x_n)$, where

$$F(x) = x^2 - c$$

Give the orbit analysis for $c = 0$ and any starting point x_0.
Study the cases $c = -1$, $c = 1$, for $x_0 = 0$.
If you find 1-cycle(s) and 2-cycles, try to study their stability.
If the evolution looks chaotic, store a bunch of results obtained on your calculator, and evaluate Lyapunov exponent numerically. Did you find chaos (Lyapunov exp. ¿ 0)?

2 Quiz (circle Y or N - yes or no

IF you circle N, also circle at least one word that does not fit in that sentence. You earn points only if the right reasons for sentence not baing right are given).

[...only 28 out of original 48 questions shown here]

[Y...N] You are trying to predict the future of a dynamical system to a tolerance $a = 10^{-3}$. Assume that the two trajectories of a Monte Carlo system "Road to Chaos" start at 10^{-15} apart, while the Lyapunov e-folding time is 100 orbits. How long do you need to continue the fantasy travel to achive planned tolerance?

[Y...N] Aperiodicity of behavior is not a required feature of what we call chaos

[Y...N] Exponential divergence of nearby trajectories is a required feature of what we call chaos

[Y...N] Intermittence of motion is required feature of what we call chaos.

[Y...N] In order to be called chaotic, the system must still be deterministic (no noisy, or truly randomly varying inputs)

[Y...N] Strange attractor is the one which repels instead attracting from inself, alternatively

[Y...N] Strange attractor are often fractals

[Y...N] Red noise is different from the white noise and brown noise in the sense that the power p of frequency f, describing the time-averaged power of the profile f^{-p} is neither zero nor ¿2 like in those other cases.

[Y...N] What's remarkable about chaos in meteorology is that it can be reduced to only 5-dimensional world of a Lorenz attractor. Roessler attractor has even fewr dimensions and exhibits rather similar chaos.

[Y...N] Lyapunov lived in the beginning of the 20th century in Soviet Union.

[Y...N] Discrete 1-d maps cannot show complicated behavior like multiple-periodic oscillations, because the time variable is discrete (n=1,2,3.... rather than t = real number, from a continuum of values).

[Y...N] A cobweb is a good tool to visualize the timscale (how long ago) a system was active

[Y...N] Feigenbaum has found that the ratio $(b(n) - b(n-1))/b((n-1)$ is approaching asymptotically for large n a universal value of 4.669...

[Y...N] The proof of the Feigenbaums theorem of ratio universality is simpe.

[Y...N] The linear and nonlinear stability of fixed points sometimes are two separate things. However, in most mechanical systems, for instance in R3B, linear stabillity always implies nonlinear (large amplitude) stability. An example os the motion around the triangular Lagrange points.

[Y...N] The reason to strive for optimum timestep control is not only speed of the computation but mainly its numerical accuracy.

[Y...N] Roundoff error of the processor typically shows the characteristics ofa random walk. Therefore, to destroy, say, 6 accurate digits in a resultant number, a CPU would need to do 10^{12} FLOPs (floating point operations).

[Y...N] Onset of stability of of triangular Lagrange points can be called a saddle node bifurcation: as we vary the mass parameter, stability appears symmetrically "out of nowhere".

[Y...N] Financial markets are not totally random. This is checked, e.g., by various autocorrelations

[Y...N] In financial markets, is is advantageous to analyze log(price ratio) rather than juzt day-to-day price ratio or price increment, since the particular levels of indices are not important, prices depend on the currency and so on.

[Y...N] The reason that physical systems show often close similarity to simple 1-d maps in mathematical dynamical systems (which do not even contain a continuous variable like time!) is mostly based on the fact, that the Poincare maps or the Lorentz maps of many the physical systems are looking like tent maps, ready to iterate chaotically or show period doublings before that. Roessler system is an example, however not all systems have associated tent maps.

[Y...N] Fractal means essentially that the dimension of a set is fractional according to one of the definitions of dimension.

[Y...N] Kochs curve lengthens by 1/3 in every step of its construction

[Y...N] Cantor set loses 1/3 of its points in every step of its construction. In the end it has very few points and is countable.

[Y...N] Kochs curve has finite length, unlike the idealized (neglecting atomic structure of matter) coast of Canada.

[Y...N] Similarity dimension is number n in the relation $m = r^n$, binding the number of copies of an object fitting within area of r-times larger radius or linear size (zoomed r times).

[Y...N] Reynold number is a nondimensional measure of viscosity. Changes from regular smooth flow (no vortices seen) end above several dozen ($10^{1.5}$). Re number above $1e3$ to 1e4, in a multi-dimensional flow often triggers a change from quasi-periodic shedding of vortices to the one, which is irregular in space and time.

[Y...N] A tent map is a waterproof, all-weather tourist map pioneered by Lorentz