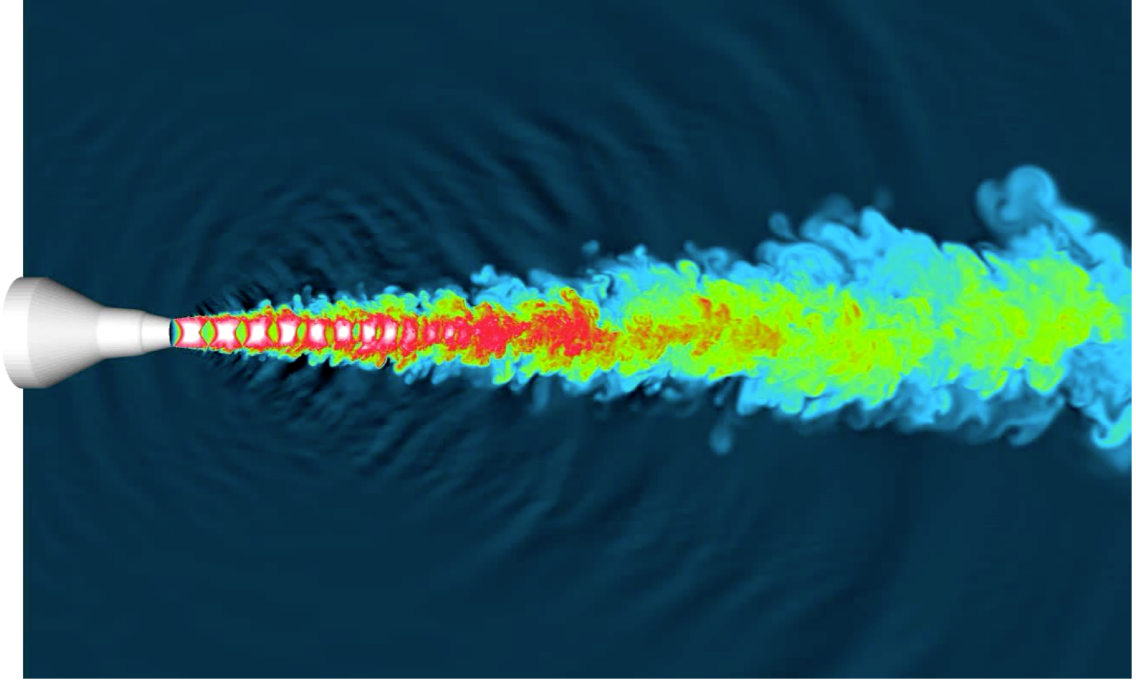
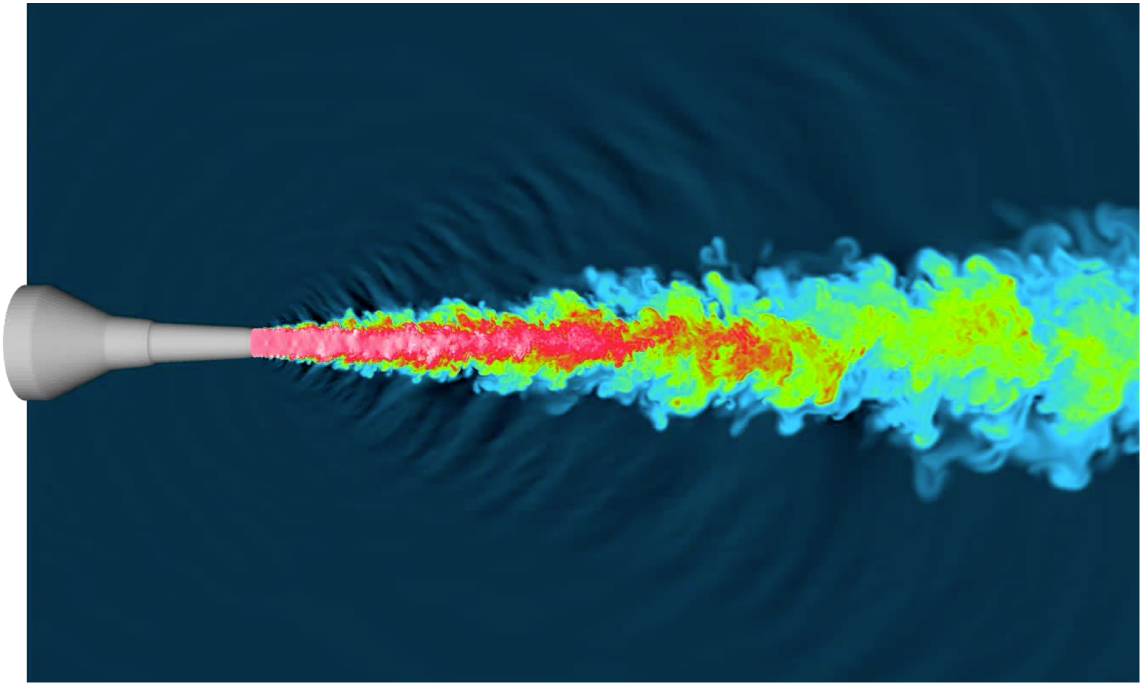
PHYD38 – Lecture 23

Turbulent jets

1. Examples and universal facts about jets
2. Similarities with instabilities in simplified dynamical systems
3. Physics of jets: entrainment of ambient fluid
4. Proof of the universal opening angle



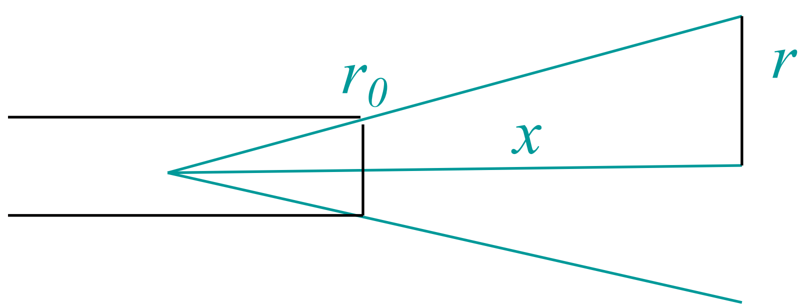


Cf. also part 4/5 of the presentations on turbulent jets

https://www.youtube.com/watch?v=1syjH7p2jyw

Basic facts about a turbulent jet:

- The u(r) cross-jet average profile of x-velocity component becomes self-similar a few nozzle diameters past the nozzle. Opening angle of the jet is constant.
- Momentum flux through consecutive cross-sections, that is through different x=const. sections, is conserved, because the ambient fluid that is entrained initially lacks momentum, and momentum doesn't pile up anywhere (steady-state jet)
- The shape is cylindrical cone with a tip being a virtual point inside the pipe/nozzle (the flow is still parallel there, but behaves *outside nozzle* as if it was coming from the tip of a cone.



$$r = x/B$$

$$B \sim 6 = const.$$

=> the full opening angle of the cone is always close to $\sim 20^o$

- Distance x is counted from that virtual source point.

Proof of B = const:

- $A(x)$ = cross sectional area of the jet = $\pi\, r^2$
- ρ = const. density (assumed incompressibility of the fluid)
- $u(x)$ = top speed at distance x, on the axis ($r = 0$)
- Flux of momentum through section x is constant:

$$\rho\, A(x)\, u(x)^2 = \text{const., so} \qquad r(x)\, u(x) = \text{const.} = r_0 u_0$$

- Turbulent diffusion is widening the jet. It is a random walk process, and results in the average radius r growing in time according to diffusion relationship: $r^2 = \nu$
- ν is the kinematic coefficient of diffusion, its units are m^2/s.

We will now assume that diffusion coefficient is constant, and show that, as a consequence, the jet has the empirically observed scaling $B := x/r = \text{const.}$

Proof:

For a constant diffusion coefficient ν, on average, the jet's half-width follows $r^2 = (r_0 u_0 / u)^2 = \nu\, t$.

Proof (cont'd): $r^2 = (r_0 u_0 / u)^2 = \nu t$

$$r = r_0 u_0 / u = (\nu t)^{1/2}$$

$$u = dx/dt = r_0 u_0 / (\nu t)^{1/2}$$

which integrates by separation of variables to

$$x(t) = (2 r_0 u_0 / \nu^{1/2}) \; t^{1/2}$$

Since both $r(t)$ and $x(t)$ grow as ~ $t^{1/2}$, their ratio is constant:

$$B = x/r = 2 r_0 u_0 / \nu \, . \qquad (1)$$

But in the theory of diffusion, ν = V L /3, where L is the mixing length, a concept introduced by Ludwig Prandtl, denoting the average distance of turbulent transport. L is simply the size of a typical eddy, which Prandtl proposed to take equal to L=r. V is the aver. speed of eddy transport., equal to V = L ($\delta u / \delta r$), or using the estimate $\delta u / \delta r = u/r$, so V = u. Therefore, the viscous diffusion coefficient equals

$$\nu = u \, r /3 = u_0 r_0 /3, \qquad \text{and} \qquad u_0 r_0 = 3 \nu = const.$$

Substituting ν into (1), we obtain the promised result: B = 6

=> 20 degrees full cone angle.

The full opening angle ϕ of the cone is constant:
B = 6. So: $\phi = 2\tan^{-1}(1/B) = 2 * 9.46^o \sim 19^o$

Empirically it is closer to ~22°, but that's a minor difference.
Constant diffusion coefficient hypothesis works well !
It explains the universal conical shape of the jet, with full opening angle of a bit more than 19 degrees.

- Mass flux (dm/dt) is NOT constant, since the ambient fluid is gradually mixing in, but at what rate?

$$u = u_0\, r_0/r,$$

$$dm/dt\,(x) = \rho\, A(x)\, u(x) = \pi\, \rho\, u_0\, r_0\, r(x) \sim x.$$

Mass transported in a jet grows linearly with distance from the virtual jet origin. At the distance from orifice = Br_0 = 3 diameters of the nozzle, it is already 2 x mass outflow rate from the nozzle: 50% of jet fluid and 50% of ambient, entrained (mixed-in) fluid. At 2x larger distance from orifice, mass flowing in the jet is 3x mass outflow, in the ratio 2:1 ambient : injected fluid. And so on. As a consequence, dilution & cooling of a warm jet to an ambient temperature is quick.

PHYD38 – Lecture 24

Nonlinear astrophysical gas & particle dynamics. Supercomputing

Pawel Artymowicz

+ former UTSC undergrad & UofT graduate students:

- prof. Jeffrey Fung (Clemson U., in 2020)
- Fergus Horrobin (Tesla, car dynamics simulation div. leader, in 2022)

1. Exoplanets. Origin, migration.
2. Dust disk instabilities
3. Supercomputing
4. Other topics

The worlds come into being as follows: many bodies of all sorts and shapes move from the infinite into a great void

they come together there and produce a single whirl, in which, colliding with one another and revolving in all manner of ways

they begin to separate like to like.

Leucippos (480 - 420 BC),

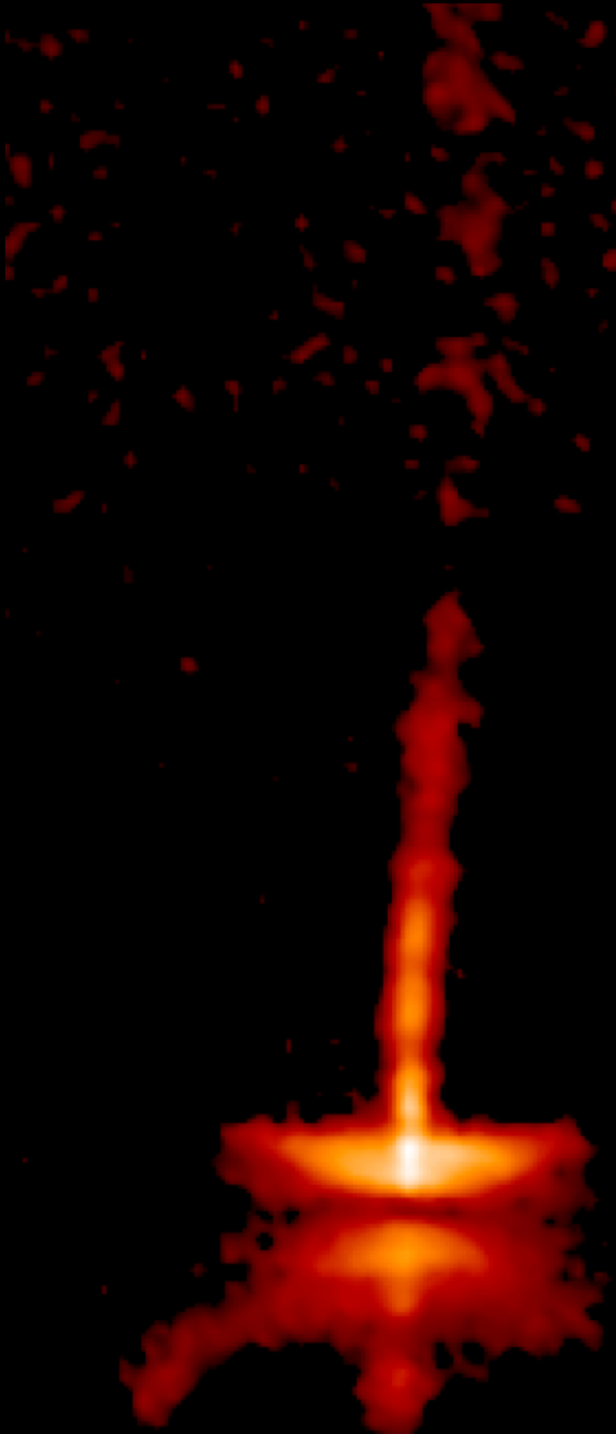
cited by Diogenes Laertios (180 - 240 AD)

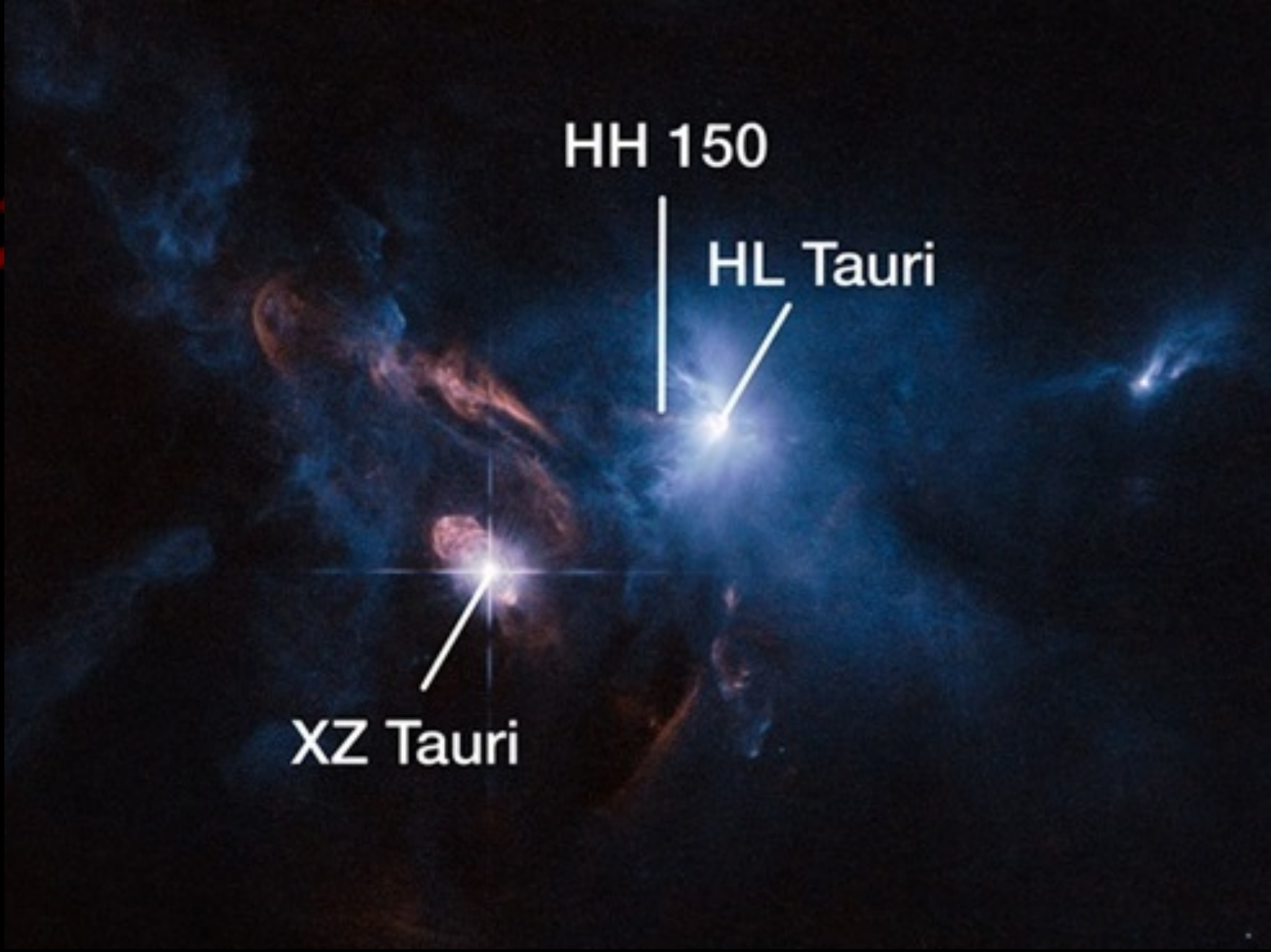
In some worlds there is no Sun and Moon, in others they are larger than in our world, and in others more numerous.
In some parts there are more worlds, in others fewer (...); in some parts they are arising, in others failing.

There are some worlds devoid of living creatures or plants or any moisture.

Democritus (ca. 460-370 B.C.)

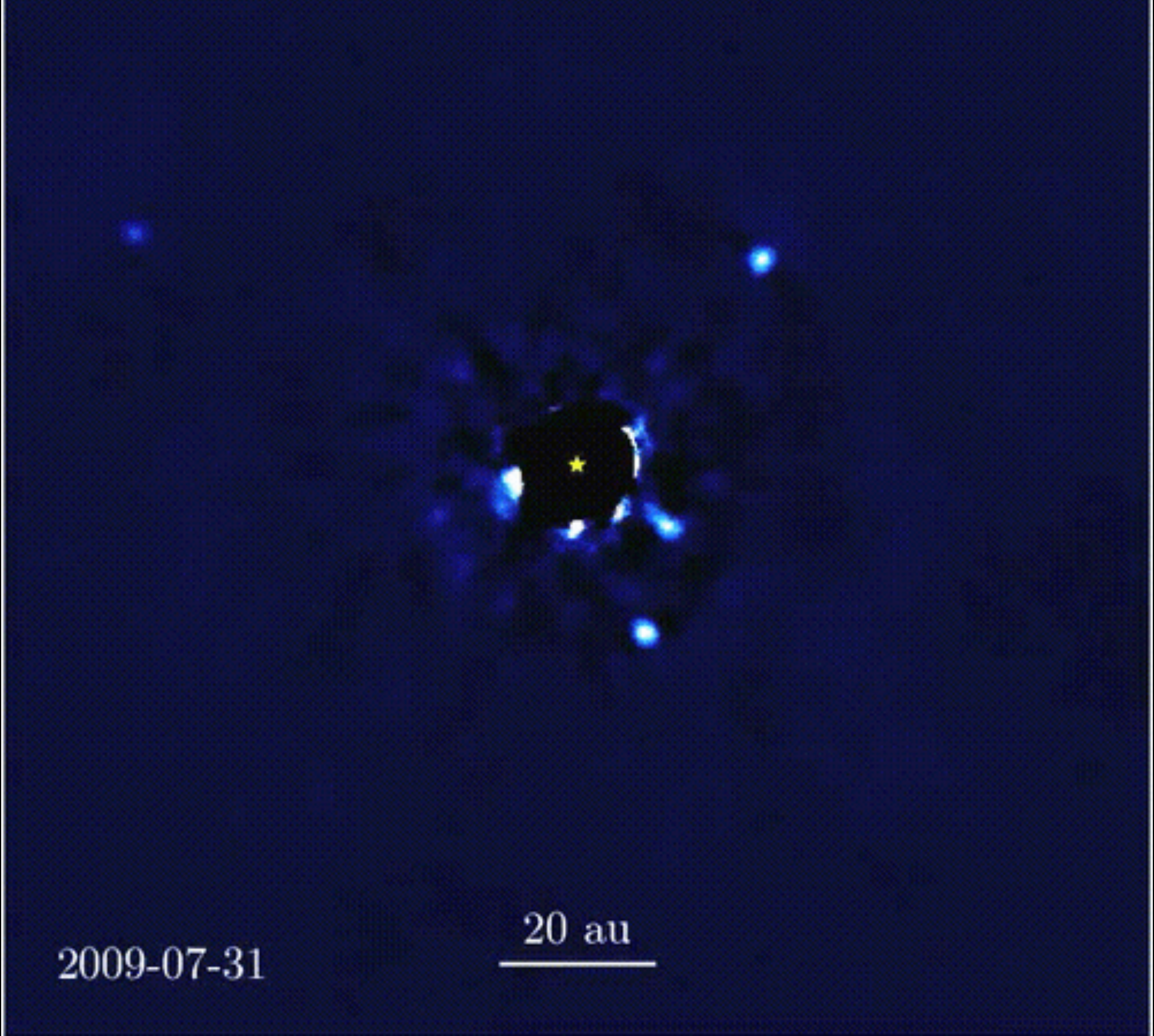
In the last 30 years we've found thousands of proofs of this prescient thinking

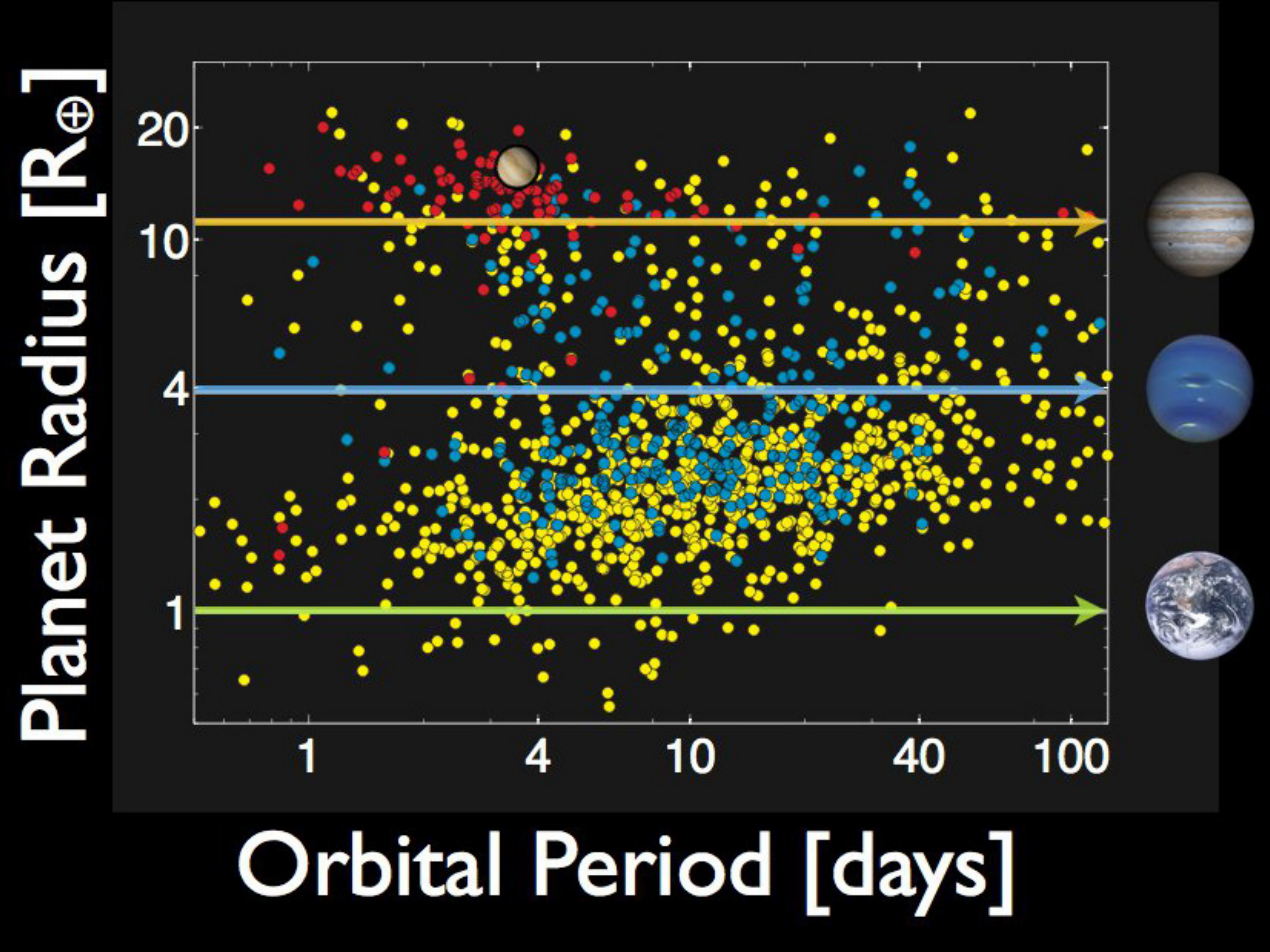




Disks in star-forming regions
produce stars & planets
(as by-product)

Imaging of exoplanets of HR 8799




Planet Radius [R⊕]
20
10
4
1
1
4
10
40
100
Orbital Period [days]

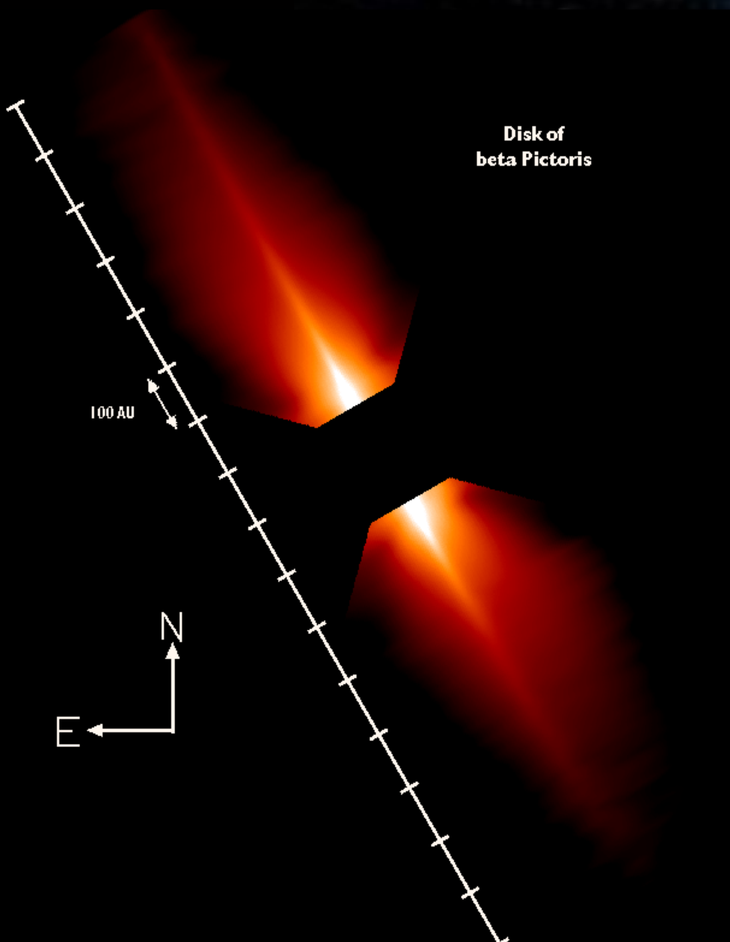
ALMA = Atacama
Large Millimeter Array

HL Tauri disk
0.5 Myr age

T Tau disks are primordial
– they have lots of H and He, which formed the star

We also need to understand the dusty disks around 1/3 of normal stars

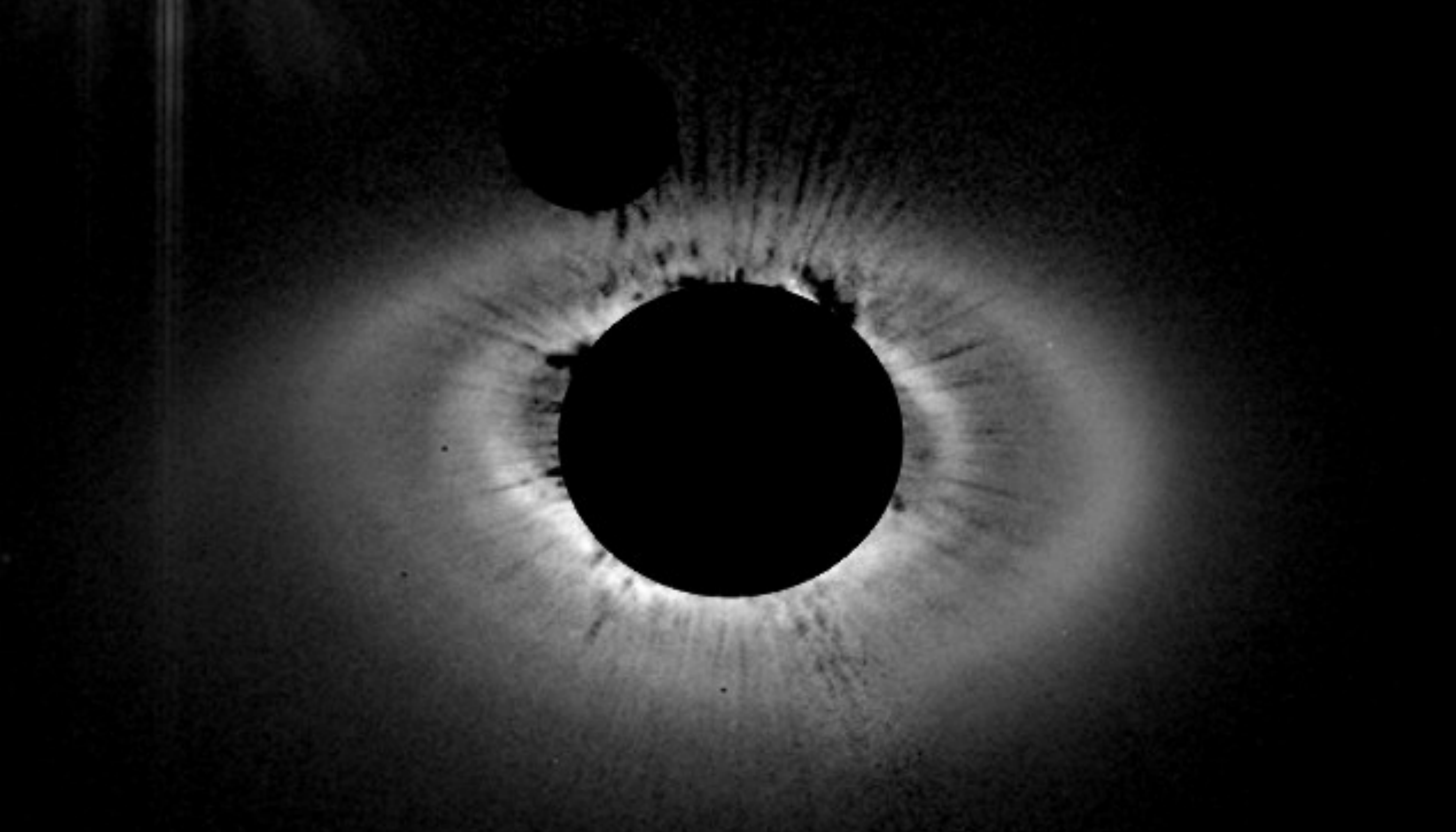
AU Microscopii –
a dusty disk in a
planetary system



Beta Pictoris - a prototype of such debris disks

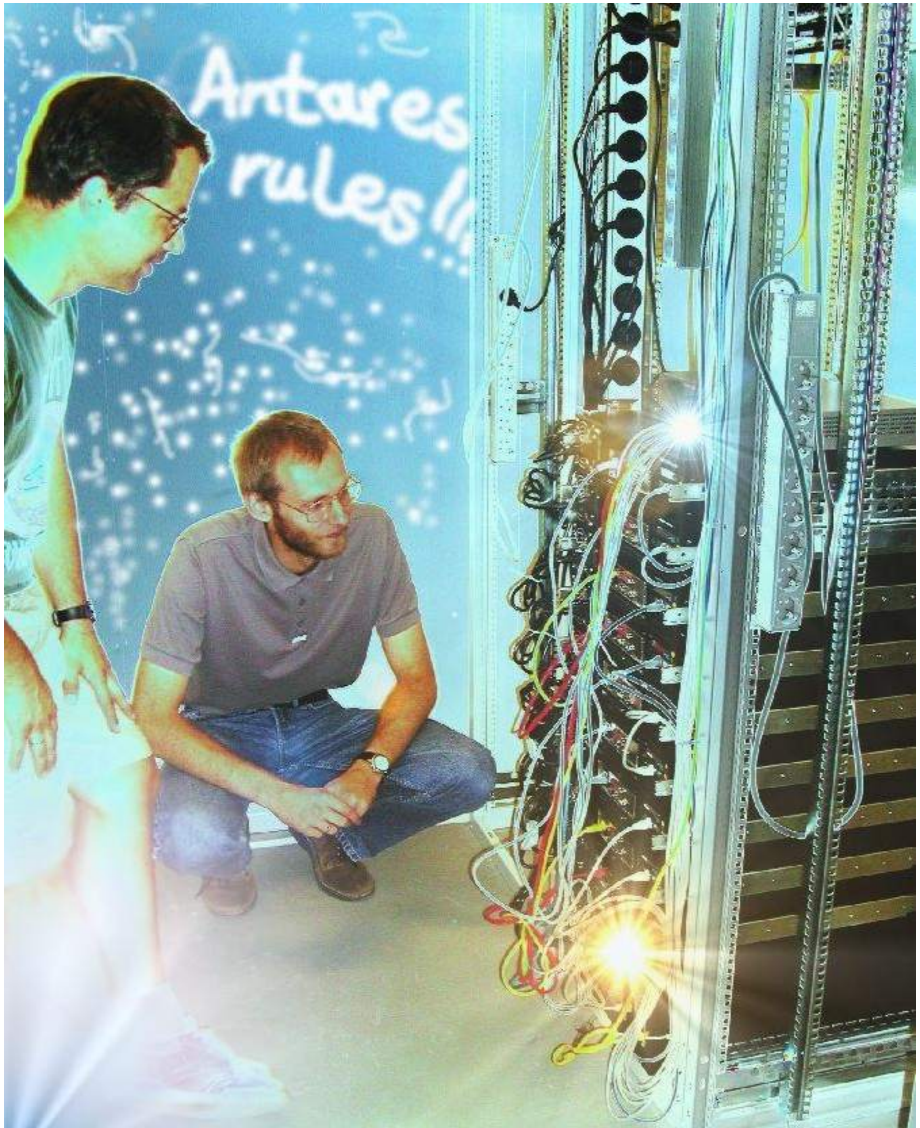
(the two disks are seen edge-on)

HD 14169A disk with a gap and a set of different spiral features



What produces the intricate morphology: planets or dust+gas+radiation ?

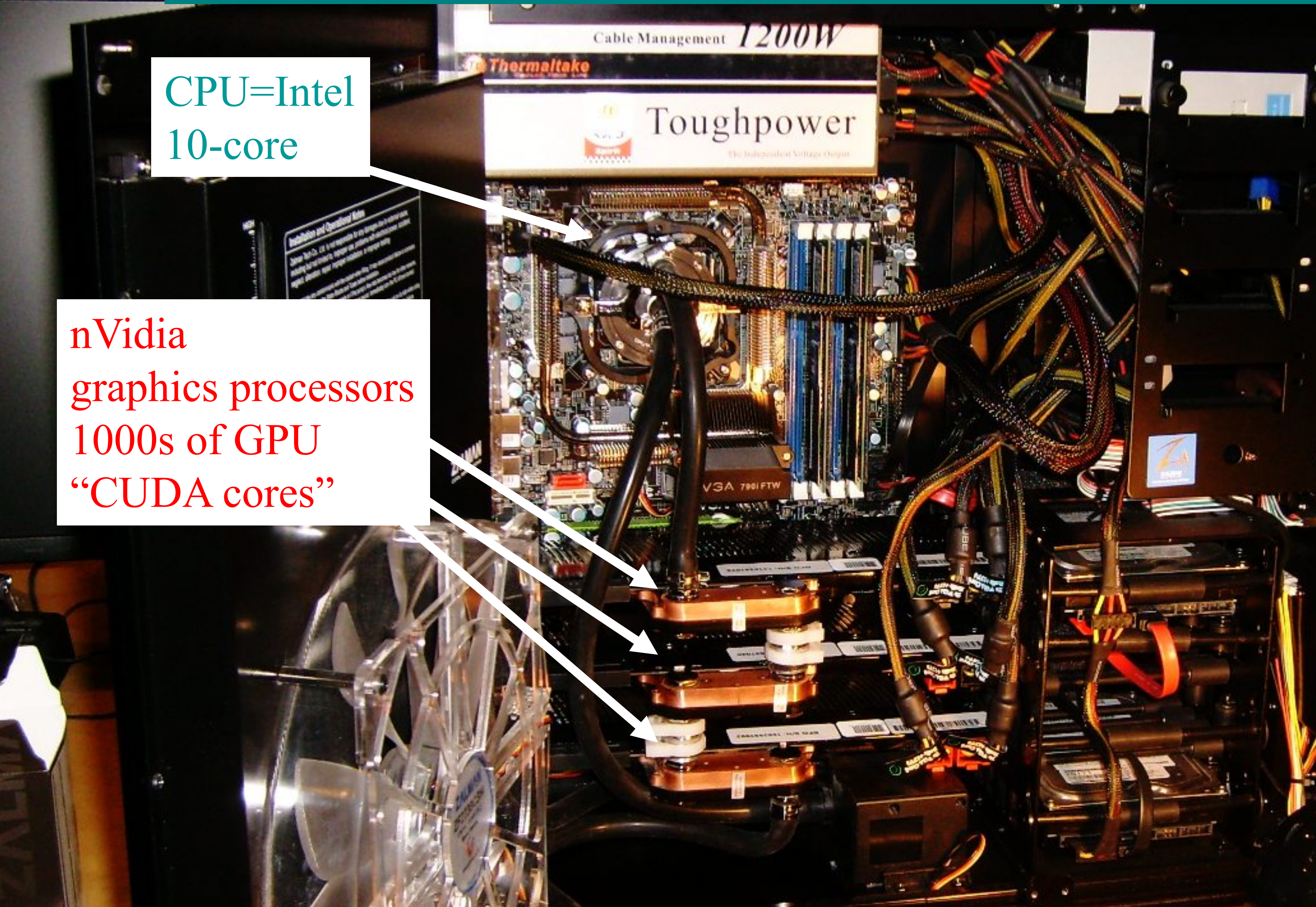
1990s and 2000s was the era of clusters



MPI (message passing interface) for parallelization

2007-2010 = beginning of an era of GPU or “graphics computing”

GPU-based PERSONAL SUPERCOMPUTE



CPU=Intel
10-core

nVidia
graphics processors
1000s of GPU
“CUDA cores”

Calculations on

(Nvidia) GPUs

…

GTX 970
GTX 1080ti

…

RTX 3090

…

RTX 5090

Supercomputing ~2016

CPU **GPU** **MIC**

(Intel Xeon Phi)

3 different types of compute units:

CPU = Central Processor Unit

GPU = Graphics Processor Unit

MIC = Many Integrated Cores = like CPU but many more, simple cores

Supercomputing in 2025

CPU **GPU** **ASIC**

2 different types of compute units, since Xeon Phi line merged with Intel CPUs.

ASIC = application-specific integrated circuit (custom-designed)

Yet another platform: Xeon Phi coprocessor, Many-Integrated-Core (MIC)

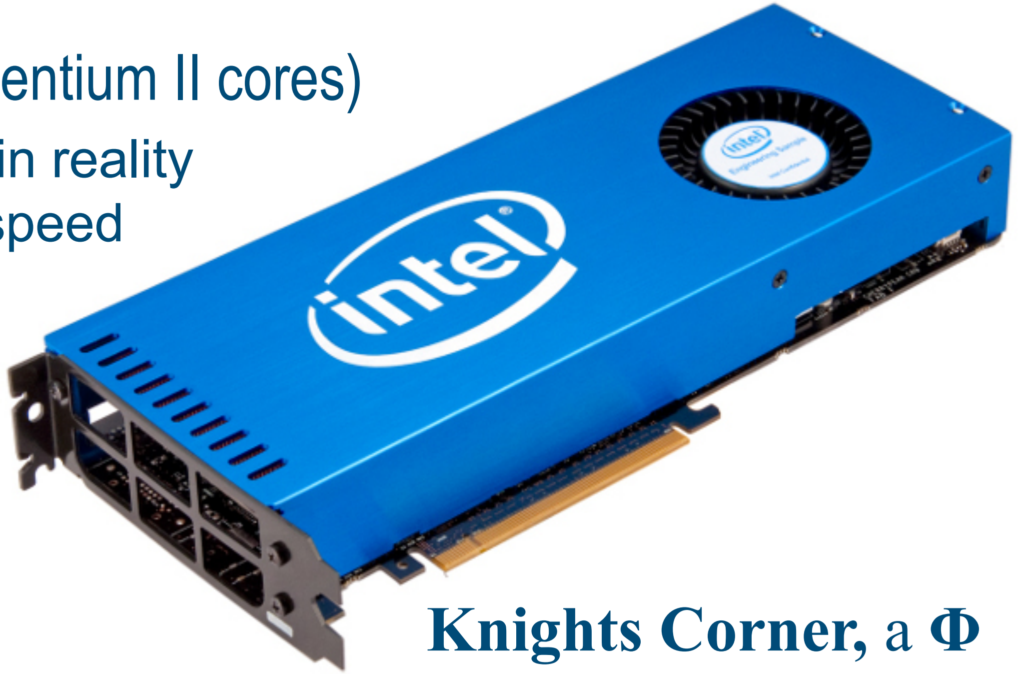
IXPs architectures: Knights Corner or KNC (*soon Knights Landing, 3x faster*)

- Not unlike GPU, ~1 TFLOP theor. max throughput in double prec., ~2 TF sp.
- Power consumption similar to GPU: 200-300W

  (200W for 400^3 grid CFD on Φ; while ~250W the same code on a GPU)
- Similar physical format, cooling methods
- Similar amount of DDR5 memory, 6GB on GPU vs. 8GB on Φ; similar bandwidth

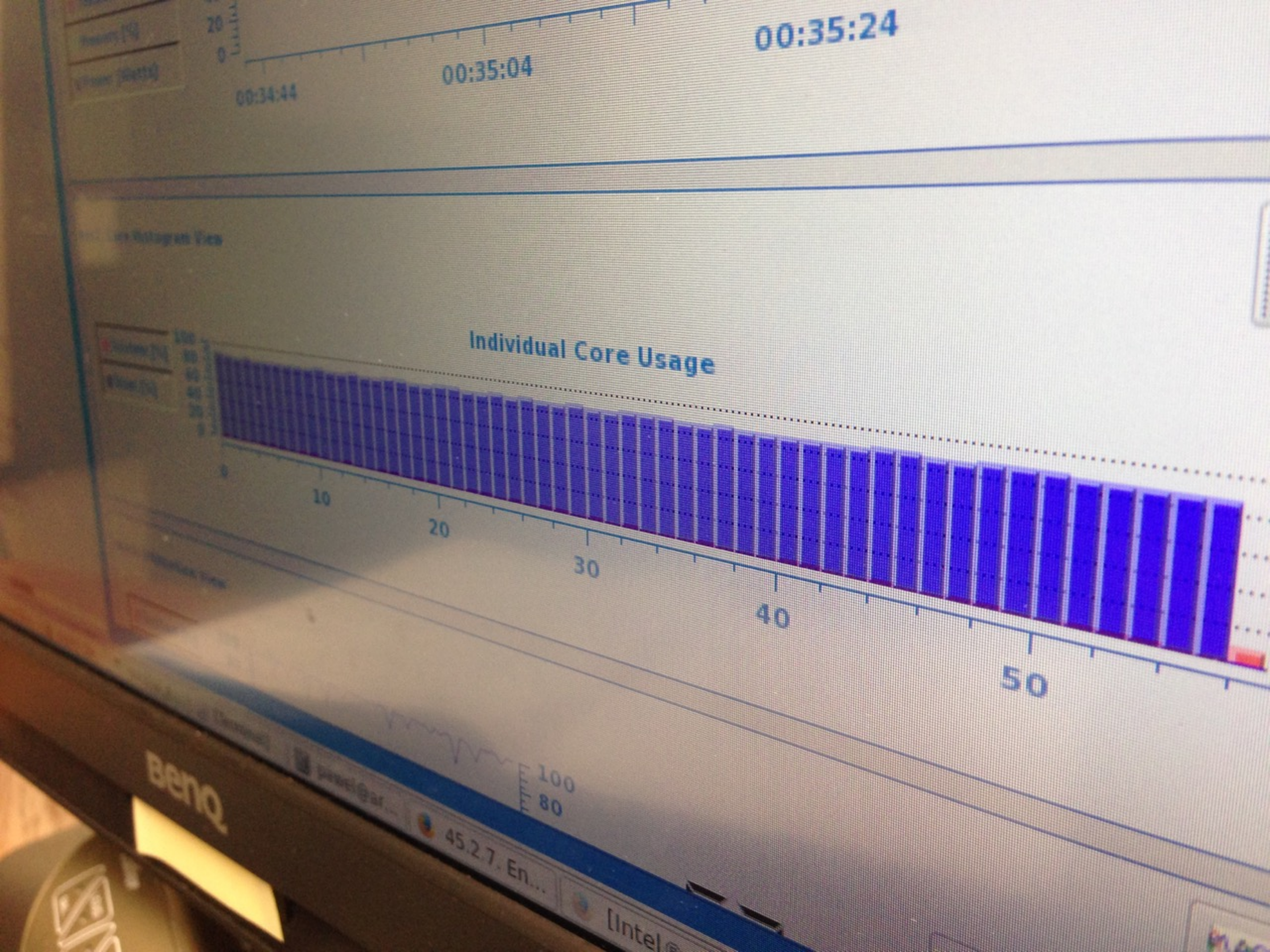
☐ 57-60 Intel cores (more modern Pentium II cores)

☐ 1500 to 2600 cores" on GPU but in reality only 8-16 multiprocessors of clock speed ~1 GHz (= Phi) (CUDA cores really do not exist)



☐ programing is very different:
CUDA on GPU is more complex;
there is no free CUDA Fortran, only CUDA-C/C++

Knights Corner, a **Φ**
a.k.a. IXP or MIC


00:34:44
00:35:04
00:35:24
Individual Core Usage
0
10
20
30
40
50
100
80
45.2.7. En...
[Intel]
BenQ

The list of fastest supercomputers in the world, **2015** edition.
The top platforms were: **Φ**, GPU & **CPU**, in that order.
Intel later merged CPU and **Φ** platforms.

SITE	platform:	SYSTEM	CORES	RMAX (TFLOP/S)	RPEAK (TFLOP/S)	POWER (KW)
National Super Computer Center in Guangzhou China	48k **Φ**'s	**Tianhe-2 (MilkyWay-2)** - TH-IVB-FEP Cluster, Intel Xeon E5-2692 12C 2.200GHz, TH Express-2, Intel Xeon Phi 31S1P NUDT National Univ. of Defense Tech., Peoples Liberation Army, Peoples Rep China	3,120,000	33,862.7	54,902.4	17,808
DOE/SC/Oak Ridge National Laboratory United States	7k **Titan GPU**	**Titan** - Cray XK7 , Opteron 6274 16C 2.200GHz, Cray Gemini interconnect, NVIDIA K20x Cray Inc.	560,640	17,590.0	27,112.5	8,209
DOE/NNSA/LLNL United States	**CPU**	**Sequoia** - BlueGene/Q, Power BQC 16C 1.60 GHz, Custom IBM	1,572,864	17,173.2	20,132.7	7,890
RIKEN Advanced Institute for Computational Science (AICS) Japan	**CPU**	K computer, SPARC64 VIIIfx 2.0GHz, Tofu interconnect Fujitsu	705,024	10,510.0	11,280.4	12,660
DOE/SC/Argonne National Laboratory United States		**Mira** - BlueGene/Q, Power BQC 16C 1.60GHz, Custom IBM	786,432	8,586.6	10,066.3	3,945
Swiss National Supercomputing Centre (CSCS) Switzerland	**Titan GPUs**	**Piz Daint** - Cray XC30, Xeon E5-2670 8C 2.600GHz, Aries interconnect , NVIDIA K20x Cray Inc.	115,984	6,271.0	7,788.9	2,325

CPU?

On the other hand, scientists are usually guilty of not squeezing the full power from their CPUs.

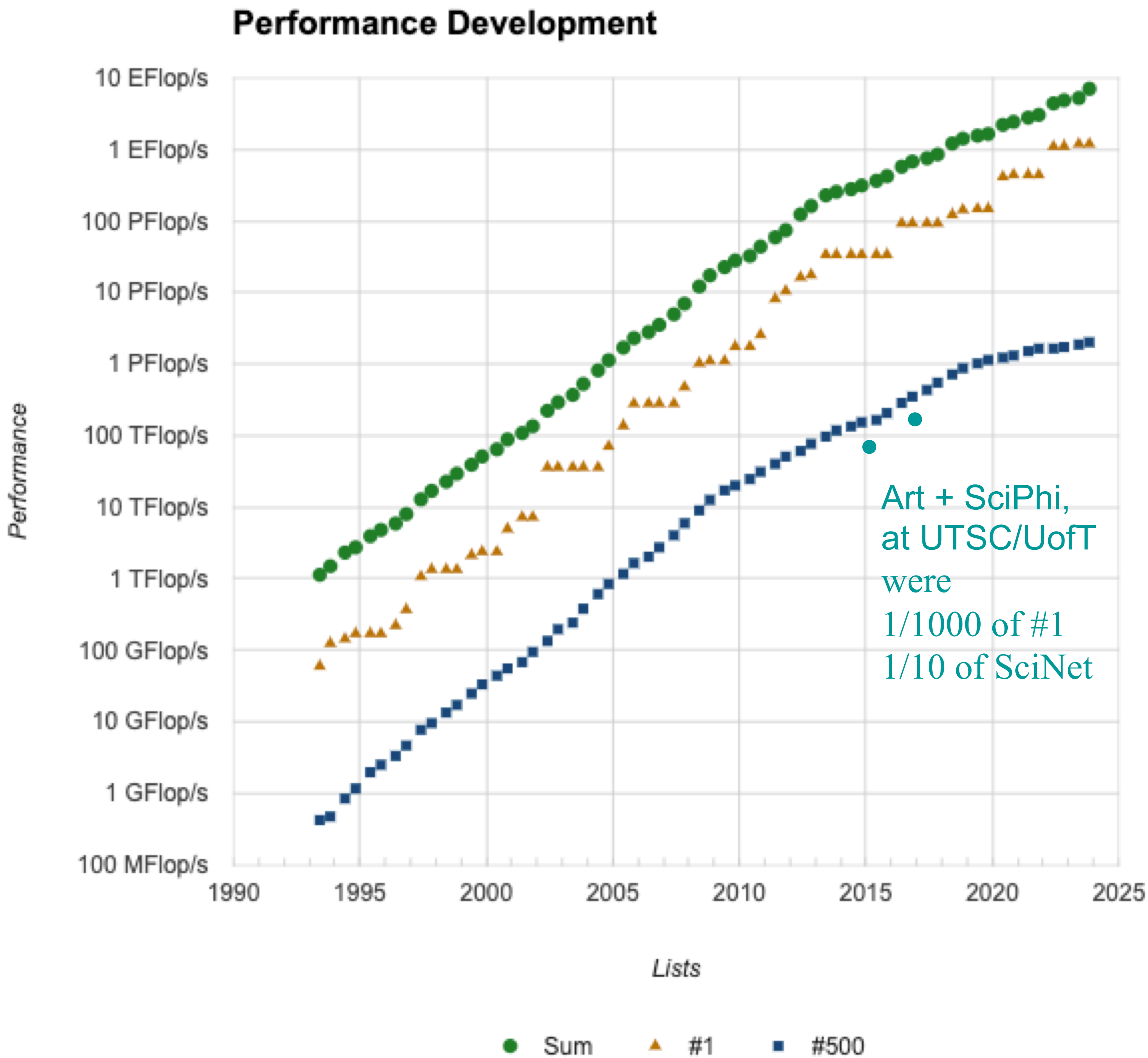
Many of us rely on compiler optimization switches & use MPI to connect the nodes of a cluster (to compute in parallel).
But, as a rule, we:
- don’t achieve a linear speedup on multi-core CPUs because we
- don’t do fully efficient openMP (multithreading) ←
- don’t spend time to optimize the code on the level of one thread
- don’t vectorize. We never bother to learn where and how to use freely and straightforwardly available AVX (advanced vector extensions on Intel proc’s).

AVX descends from similar tools called MMX and SSE, SSE-2. Vectorization is sometimes called SIMD (single instruct., multiple data) processing.

The list of fastest supercomputers in the world, **Nov. 2023** edition.
The top platforms are: **CPU (=MIC) & GPU**, in that order.

	Rank	System	Cores	Rmax (PFlop/s)	Rpeak (PFlop/s)	Power (kW)
64-core CPUs	1	**Frontier** - HPE Cray EX235a, AMD Optimized 3rd Generation EPYC 64C 2GHz, AMD Instinct MI250X, Slingshot-11, HPE DOE/SC/Oak Ridge National Laboratory United States	8,699,904	1,194.00 ➢ 1 EFLOP ➢ (exa-scale HPC)	1,679.82	22,703
52-core CPUs	2	**Aurora** - HPE Cray EX - Intel Exascale Compute Blade, Xeon CPU Max 9470 52C 2.4GHz, Intel Data Center GPU Max, Slingshot-11, Intel DOE/SC/Argonne National Laboratory United States	4,742,808	585.34	1,059.33	24,687
GPUs	3	**Eagle** - Microsoft NDv5, Xeon Platinum 8480C 48C 2GHz, NVIDIA H100, NVIDIA Infiniband NDR, Microsoft Microsoft Azure United States	1,123,200	561.20	846.84	
48-core CPUs	4	**Supercomputer Fugaku** - Supercomputer Fugaku, A64FX 48C 2.2GHz, Tofu interconnect D, Fujitsu RIKEN Center for Computational Science Japan	7,630,848	442.01	537.21	29,899


UTSC
EVGA GTX 480


Performance Development
10 EFlop/s
1 EFlop/s
100 PFlop/s
10 PFlop/s
1 PFlop/s
100 TFlop/s
10 TFlop/s
1 TFlop/s
100 GFlop/s
10 GFlop/s
1 GFlop/s
100 MFlop/s
Performance
1990
1995
2000
2005
2010
2015
2020
2025
Lists
Sum
#1
#500
Art + SciPhi,
at UTSC/UofT
were
1/1000 of #1
1/10 of SciNet

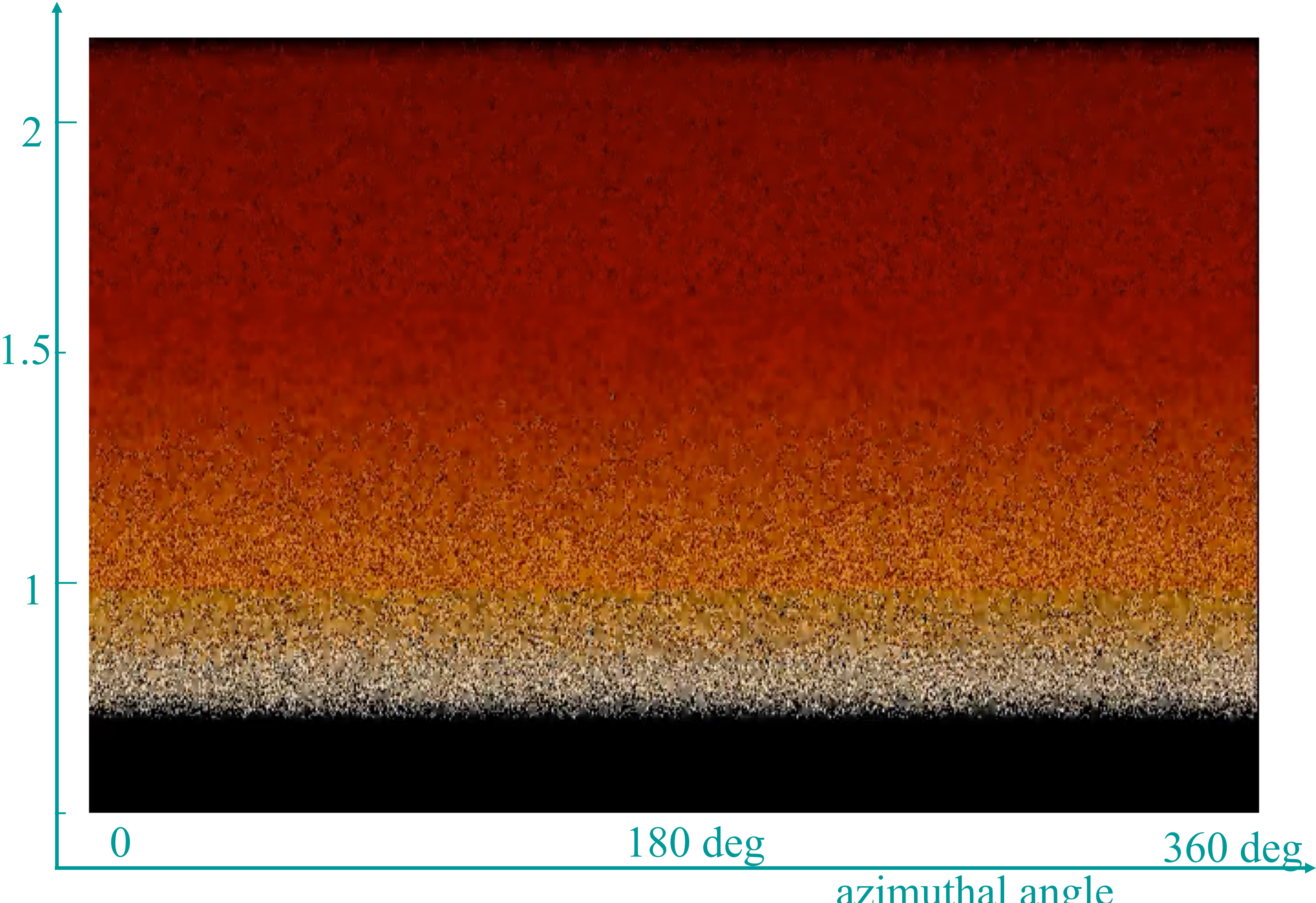
Some results of UTSC supercomputing of **nonlinear astrophysical processes:**

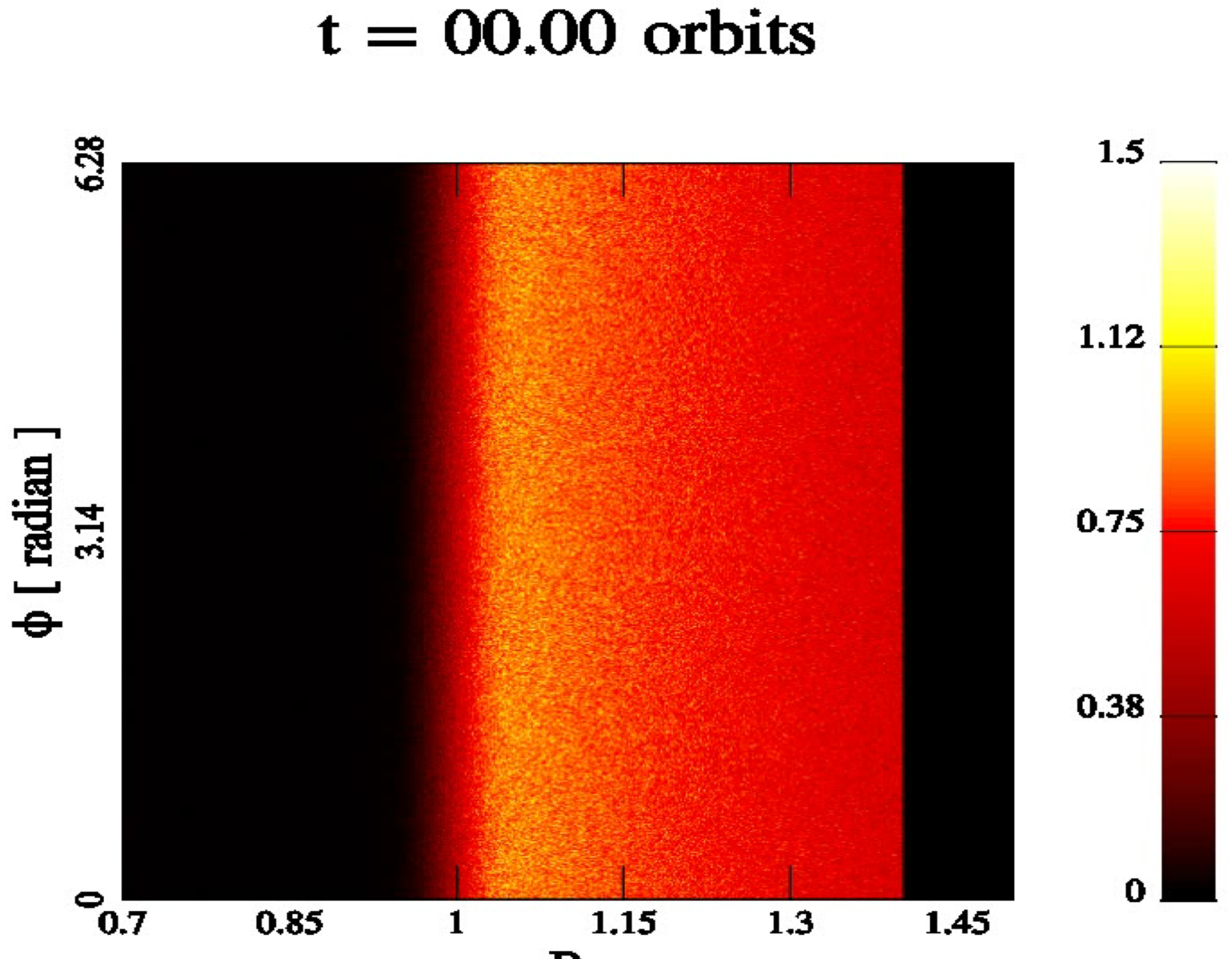
Irradiation instability of opaque dust disks(IRI)

Migration of giant protoplanet in disks

3-D gas flow around an Earth-like planet

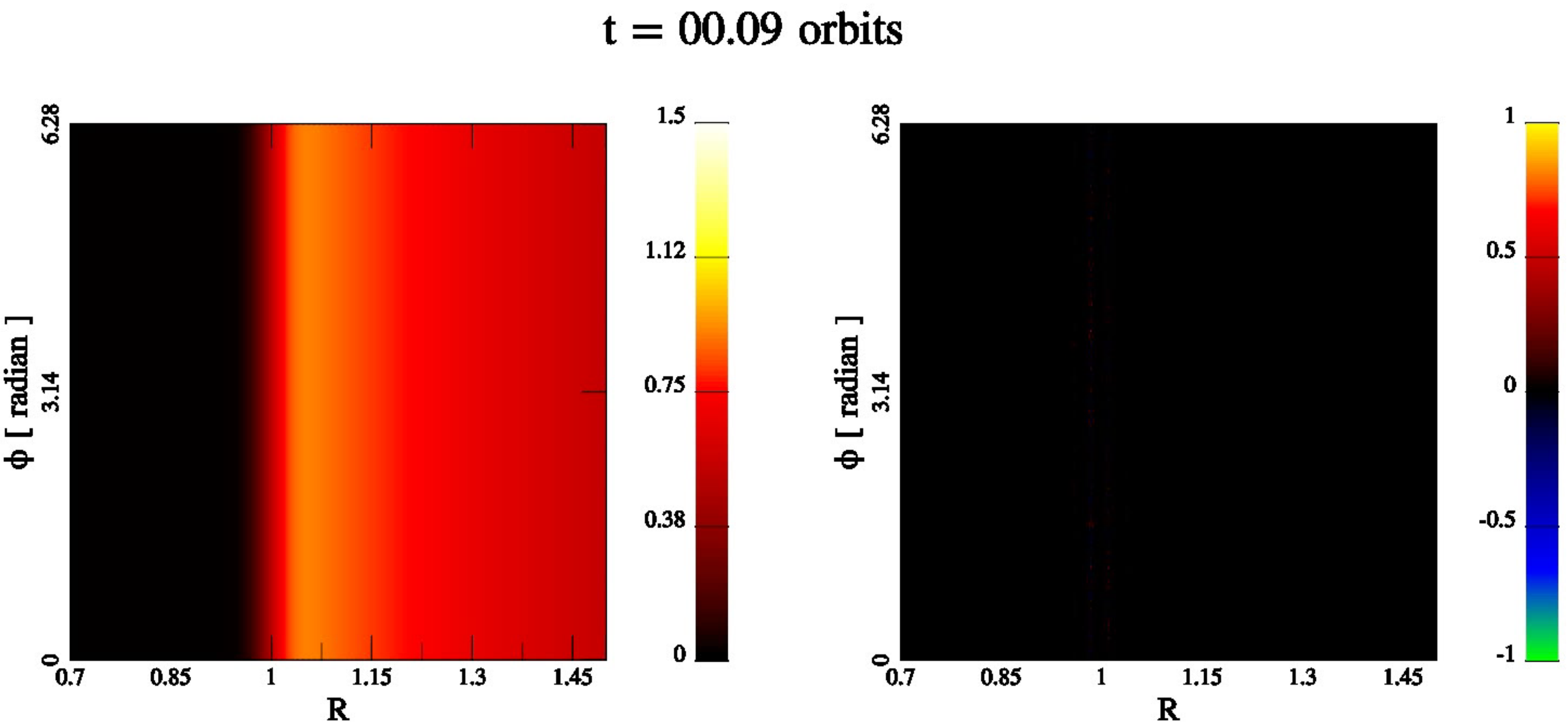
Free particles casting shadows video **$\tau = 4$, $\beta = 0.2$**





Jeffrey Fung's calculation

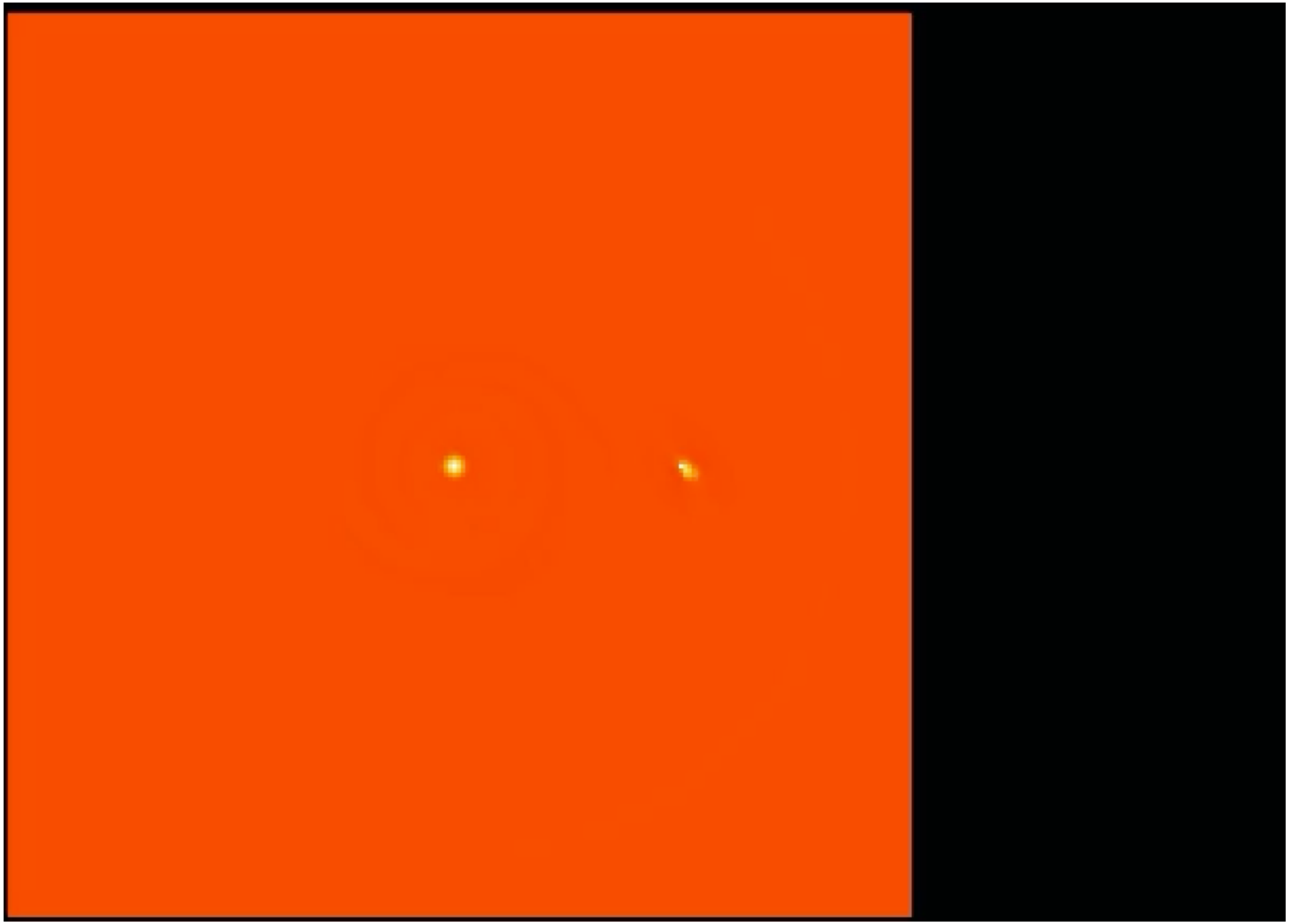
GAS DISK HYDRODYNAMICAL SIMULATION (PPM method)
The r.h.s. shows a background-removed picture of density variations in growing modes. They are predicted analytically, and their growth rates are in agreement with calculations.



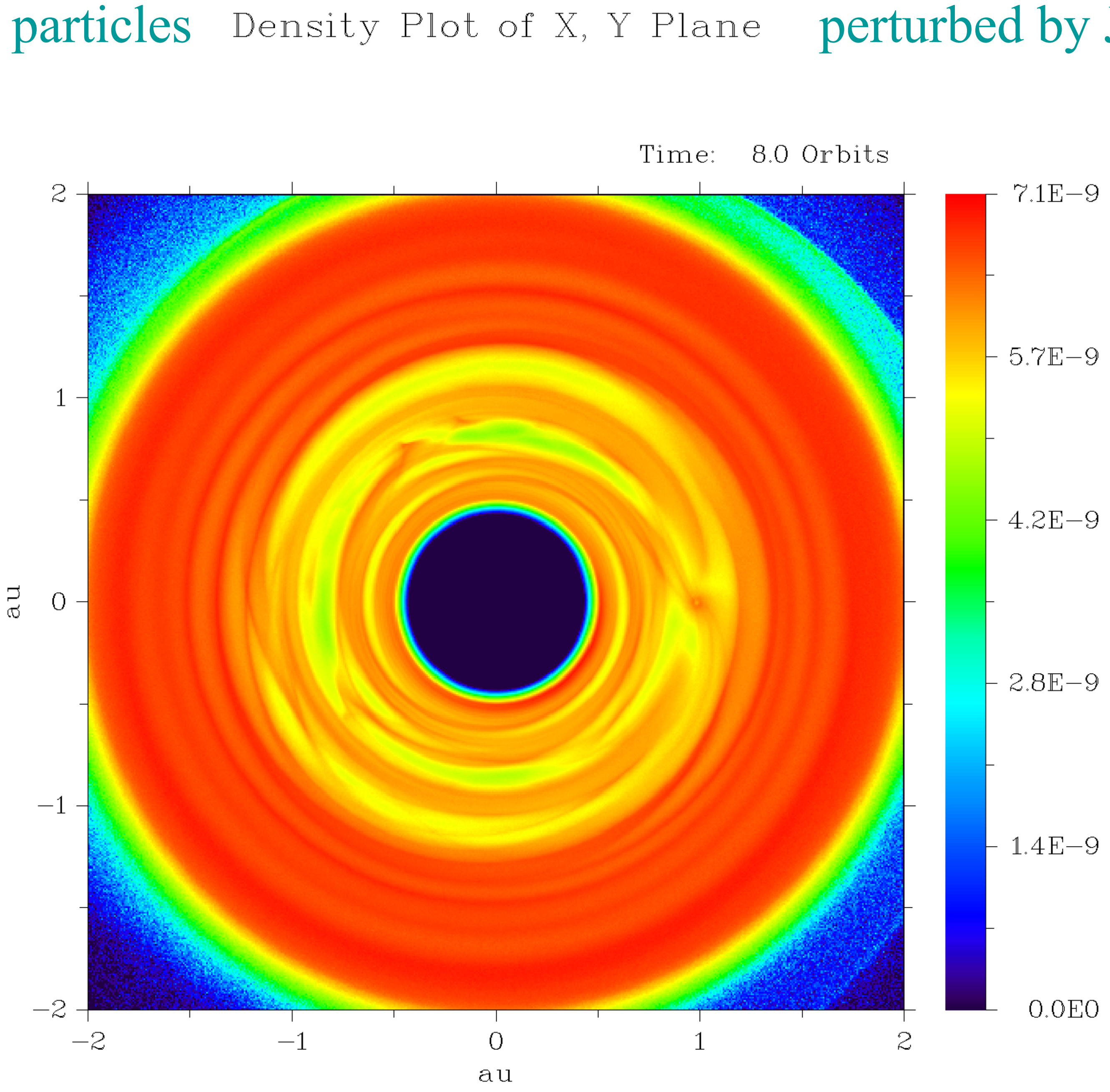
Thus opaque disks are unstable under illumination by the central object

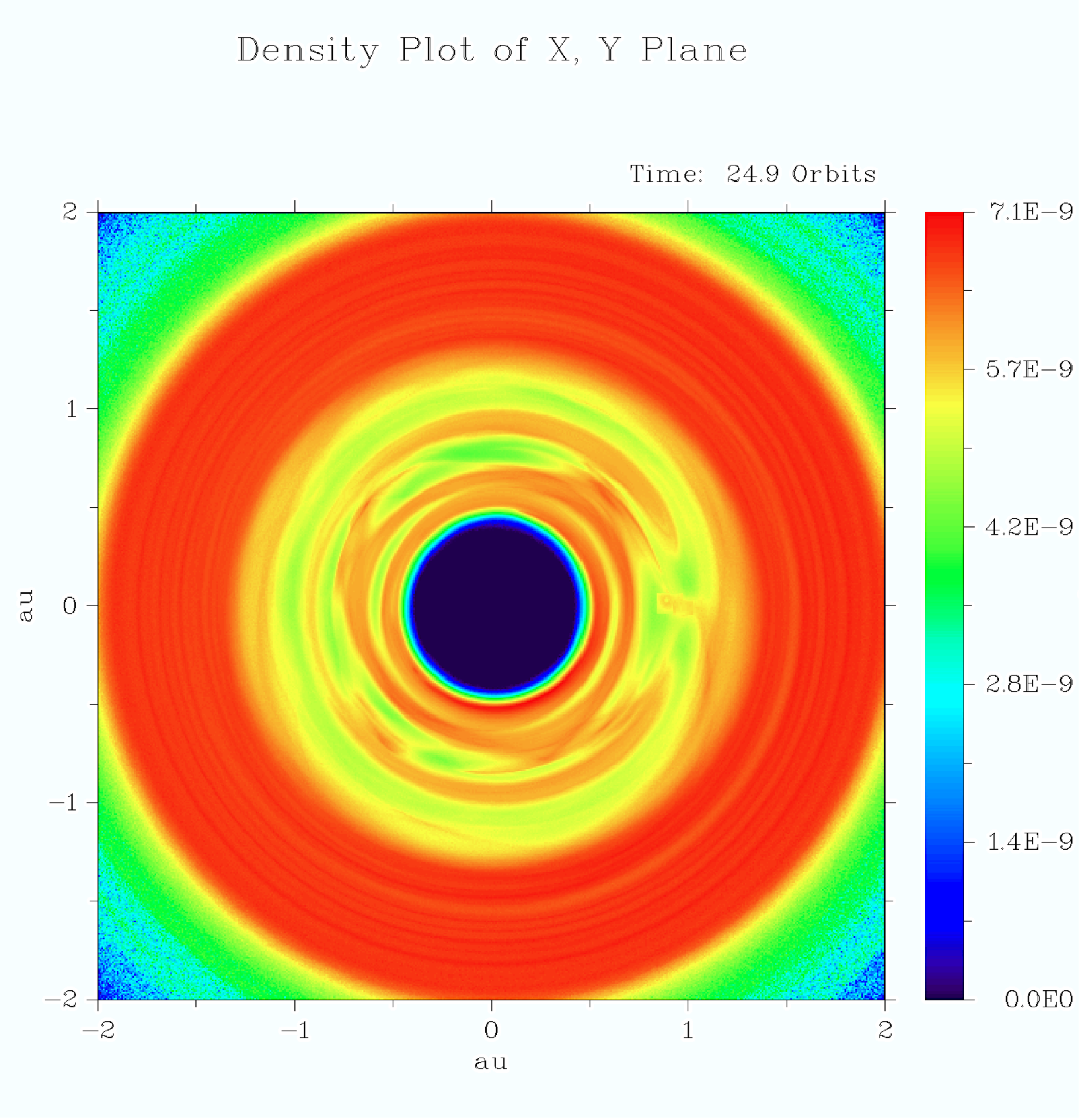
MIGRATION OF PROTOPLANETS

Artymowicz (2000) - protojupiter migrating inward in protoplanetary disk



Fergus Horrobin (2017). Simulation of collisionless disk of 1 billion particles perturbed by Jupiter

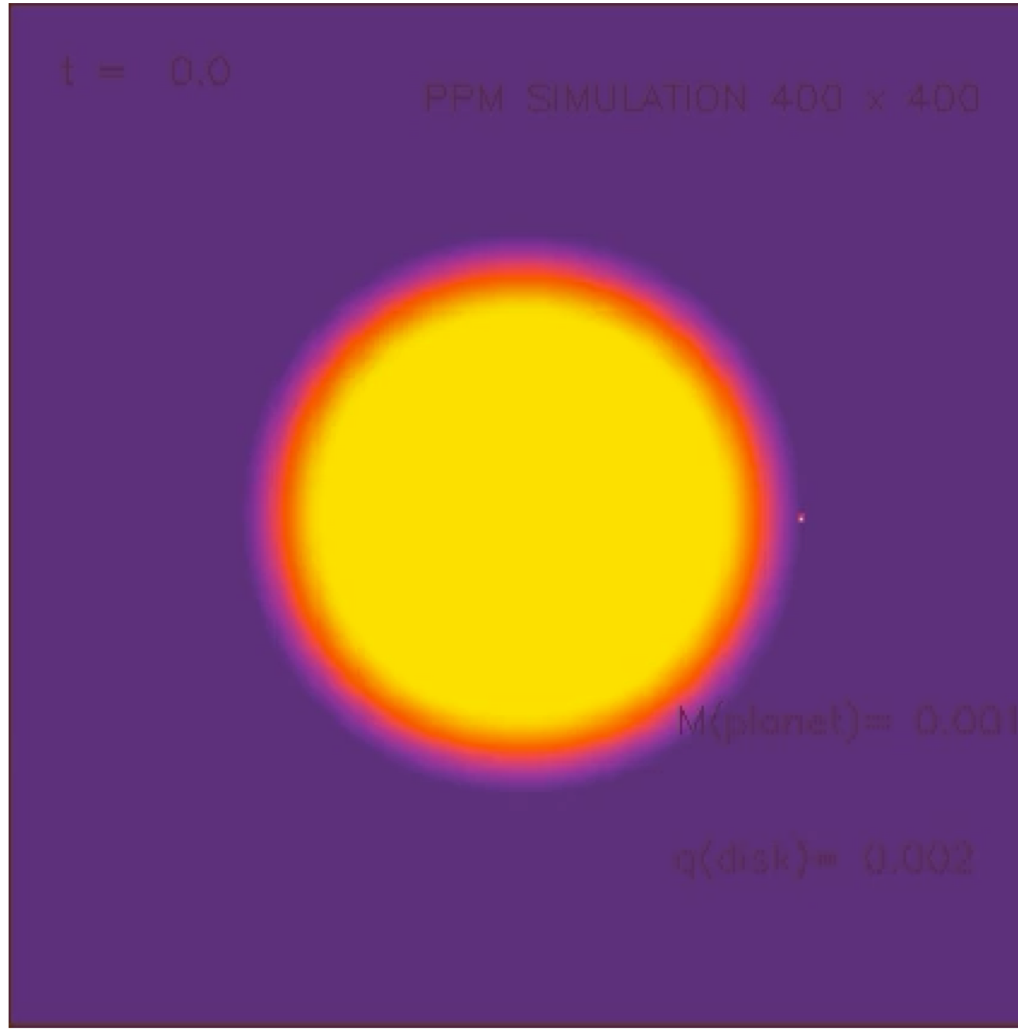



Density Plot of X, Y Plane
Time: 24.9 Orbits
2
1
0
−1
−2
au
−2
−1
0
1
2
au
7.1E−9
5.7E−9
4.2E−9
2.8E−9
1.4E−9
0.0E0

Initially inner disk only. The rapid **inward migration** is OPPOSITE of the expectation based on shepherding & tidal waves (due to mean-motion resonances, a.k.a. Lindblad resonances.)

colors show gas surface density $\Sigma(x,y)$

migration type III

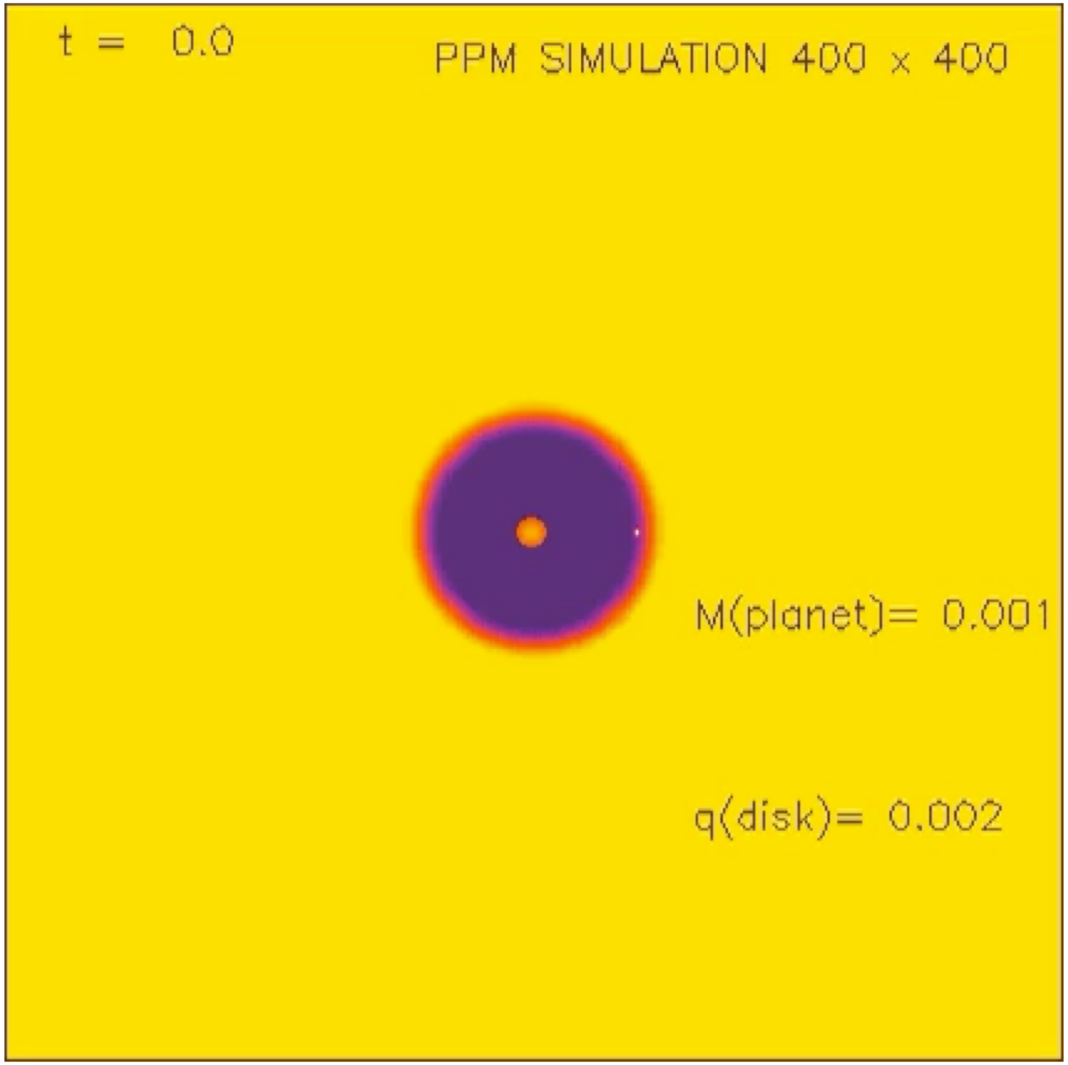


Corotational torques cause rapid inward sinking of a planet.
(Gas is transferred outward from orbits inside to those outside the protoplanet, along horseshoe orbits. To conserve angular momentum, satellite moves in.)

Now consider the opposite case of an inner hole in the disk (initially)

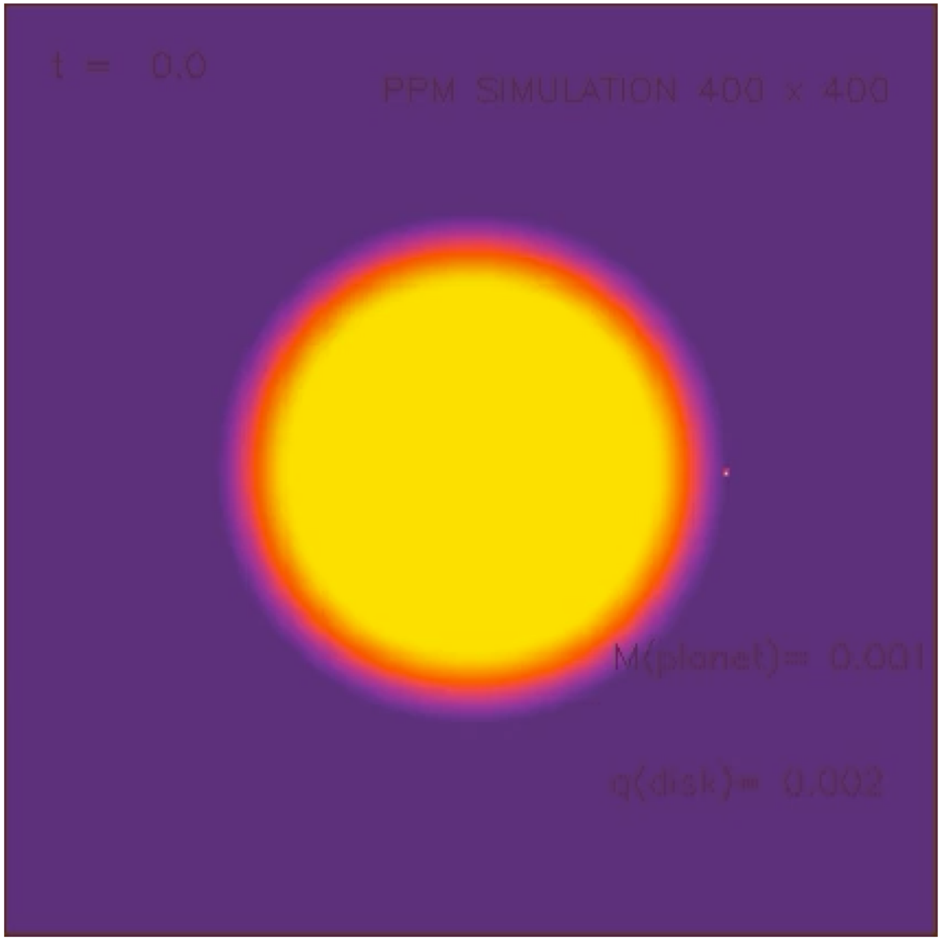
Against the prediction of shepherding satellite theory, the planet rapidly migrates outwards, toward the disk.

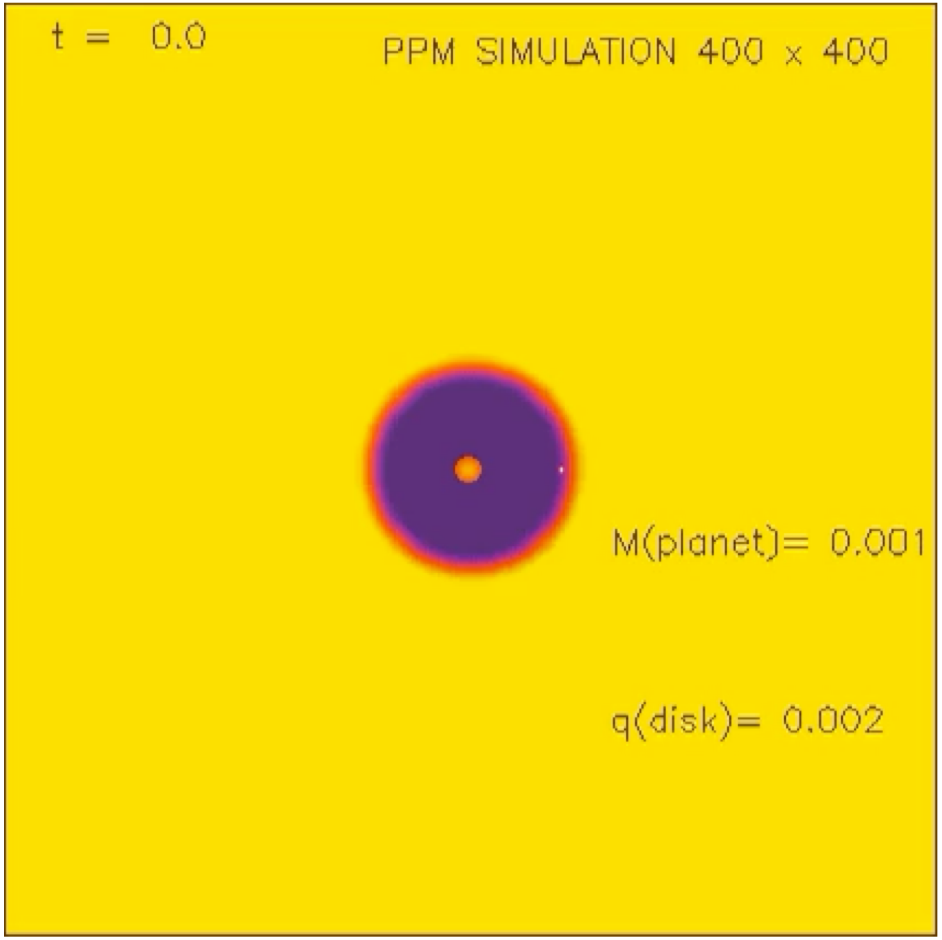
migration
type III



Here, the situation is an inverse of the previous slide: gas is initially outside the planet. Disk gas traveling on hairpin/horseshoe orbits fills the inner void. By moving in, the gas rapidly drives the planet out. Lindblad resonances in the outer disk produce the spiral waves and try to move the planet in, but are too weak compared with the CR torques.

The rapid inward/outward migration in the direction opposite to standard theory of the tidal disk-planet interaction (via Lindblad resonances)





Initially a jupiter-mass planet outside the disk
Migrates inward

Initially inside a disk gap
Migrates outward

The so-called type III migration is very rapid and can create hot jupiters

Simulation of **outward migration** (type III) of a Jupiter-class protoplanet

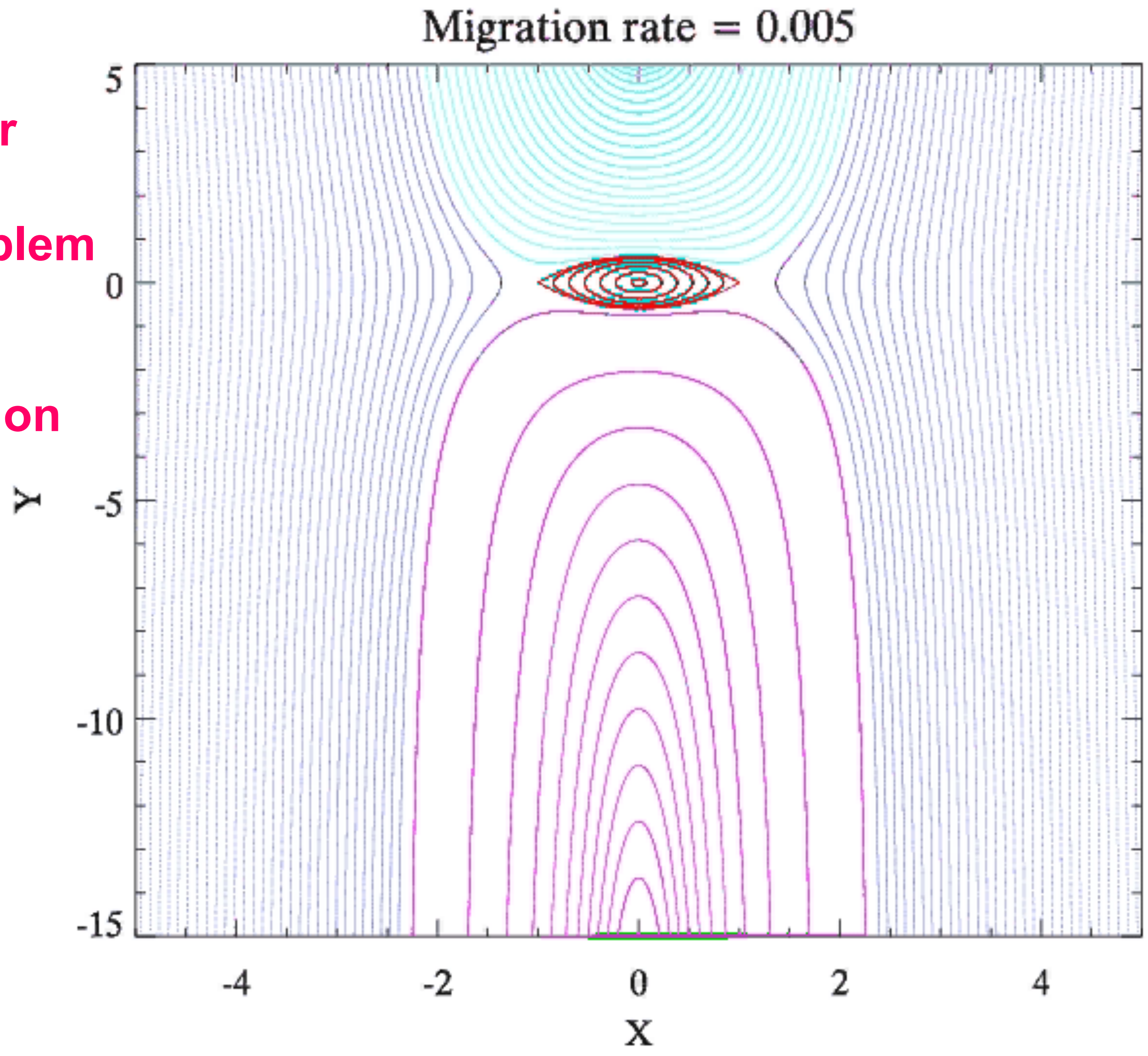
Variable-resolution, high-order adaptive grid code (following the planet).

Horizontal axis shows the radius in the range (0.5-5)a

Full range of azimuth
$\theta = 0^o$-360^o
is shown on the vertical axis
Time is displayed in units of initial orbital period.

Peplinski, Mellema and Artymowicz (2007, 2008)

Guiding center trajectories in the Hill problem can explain features of type III migration



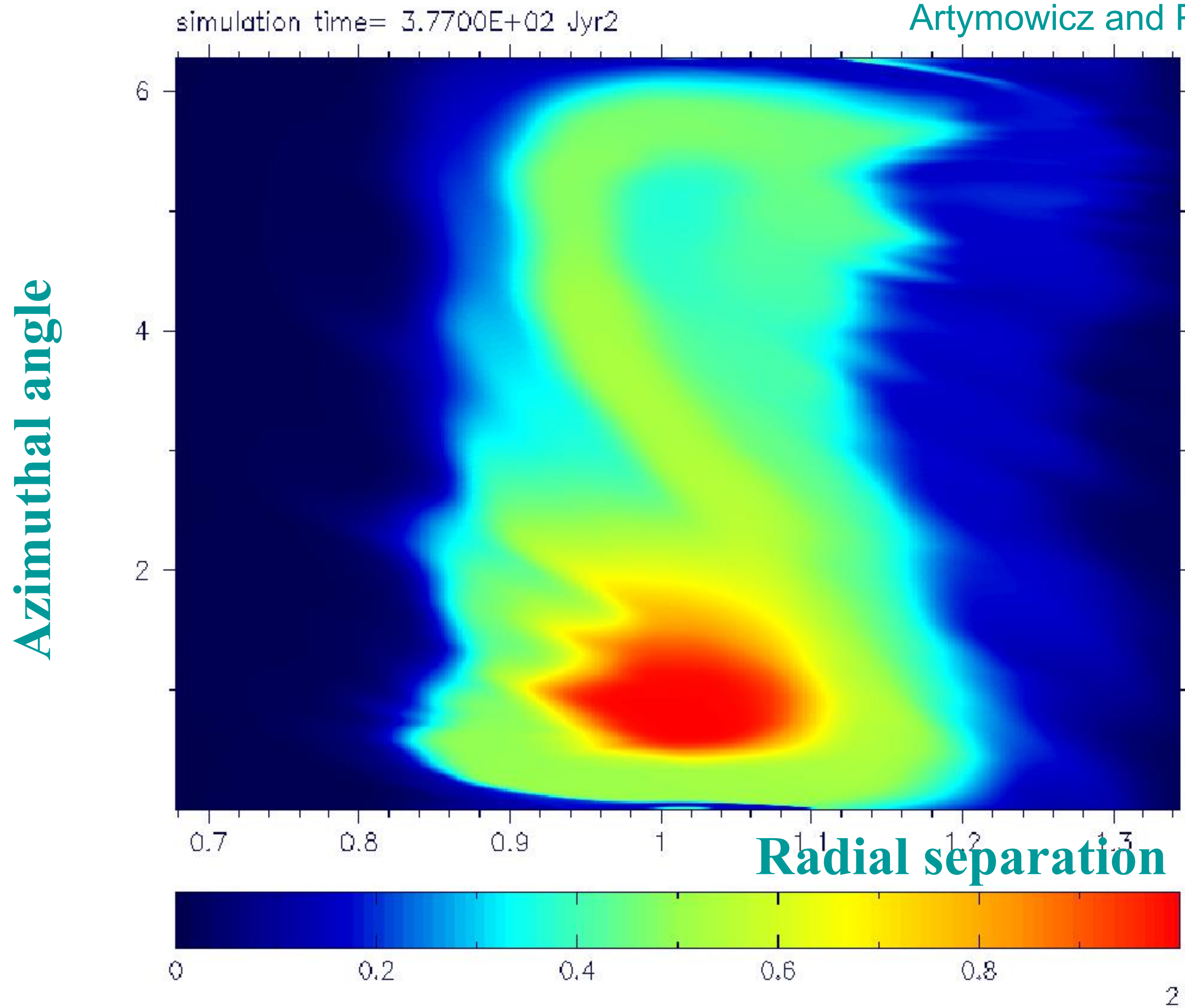
Unit of length
= Hill sphere

Unit of da/dt =
Hill sphere
radius per
dynamical time.

Animation:
Eduardo Delgado

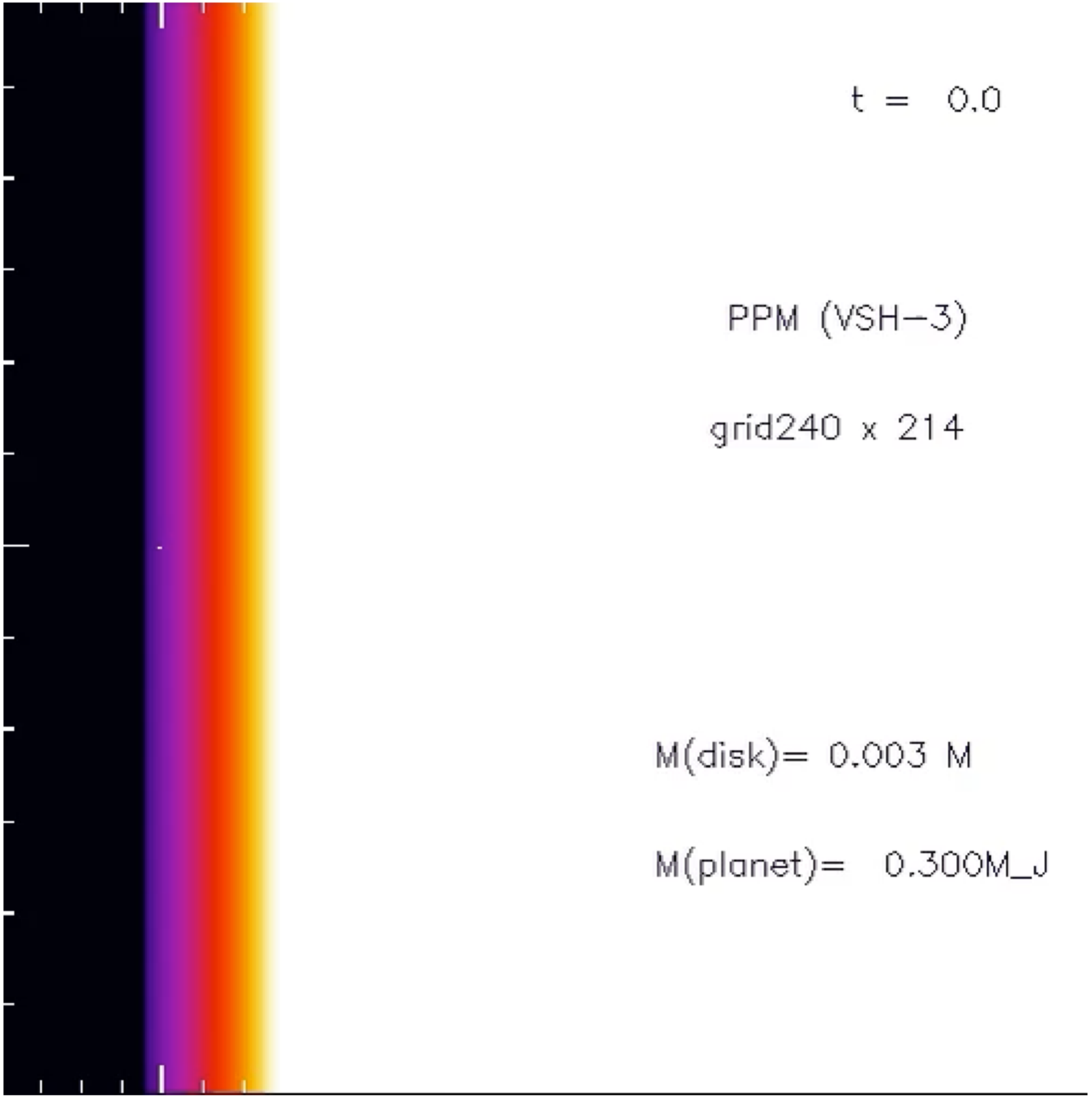
Slow migration, density of gas (PPM code FLASH on a CPU cluster)

Artymowicz and Peplinski (2012)



The annulus of circulating flow is called Corotational Region (CR)

In fluid disks, it spans $\pm 2.5\ r_L$ from planet's orbit



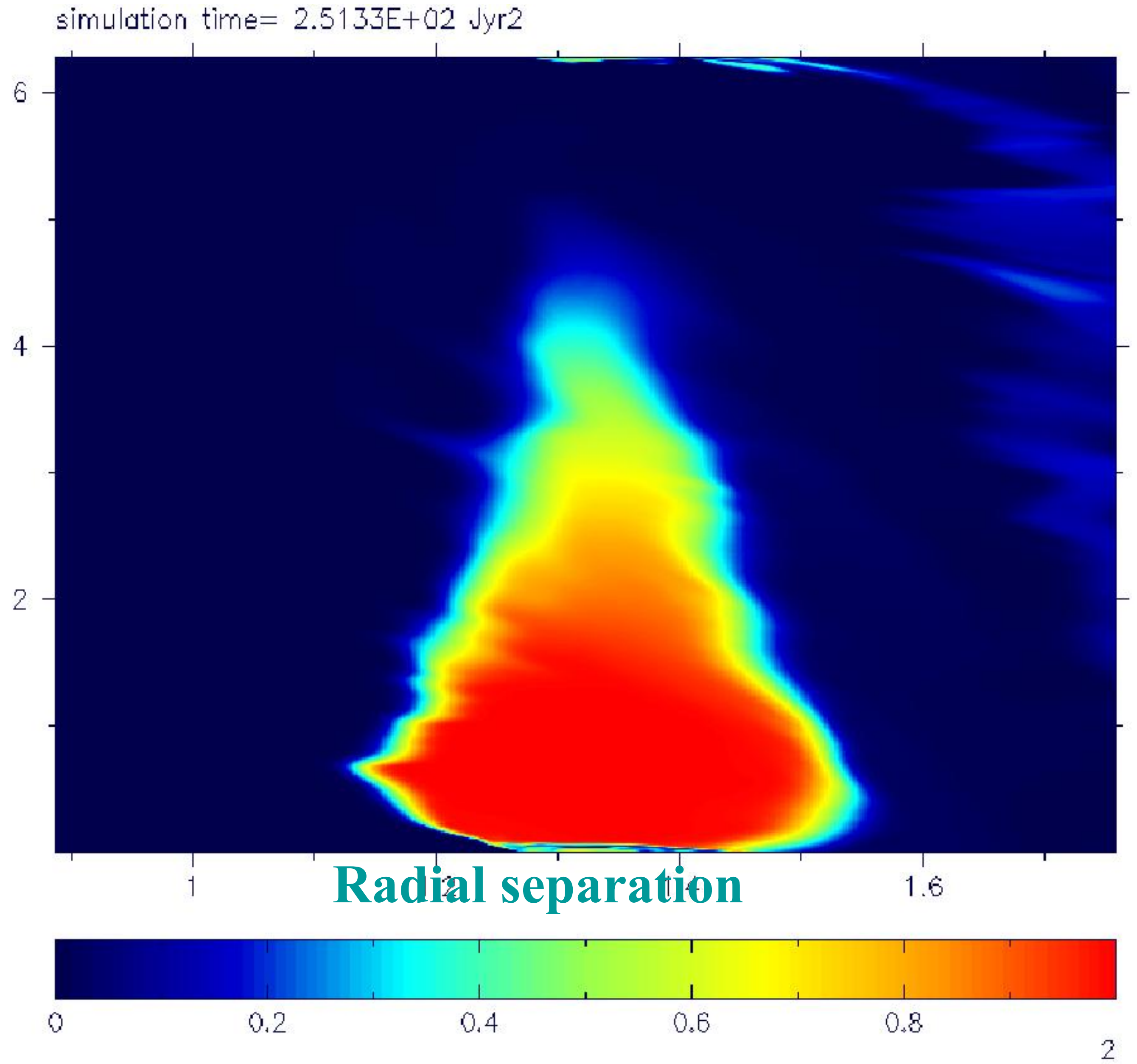
Simulation of Proto-Saturn in a primordial disk

(r,phi) view

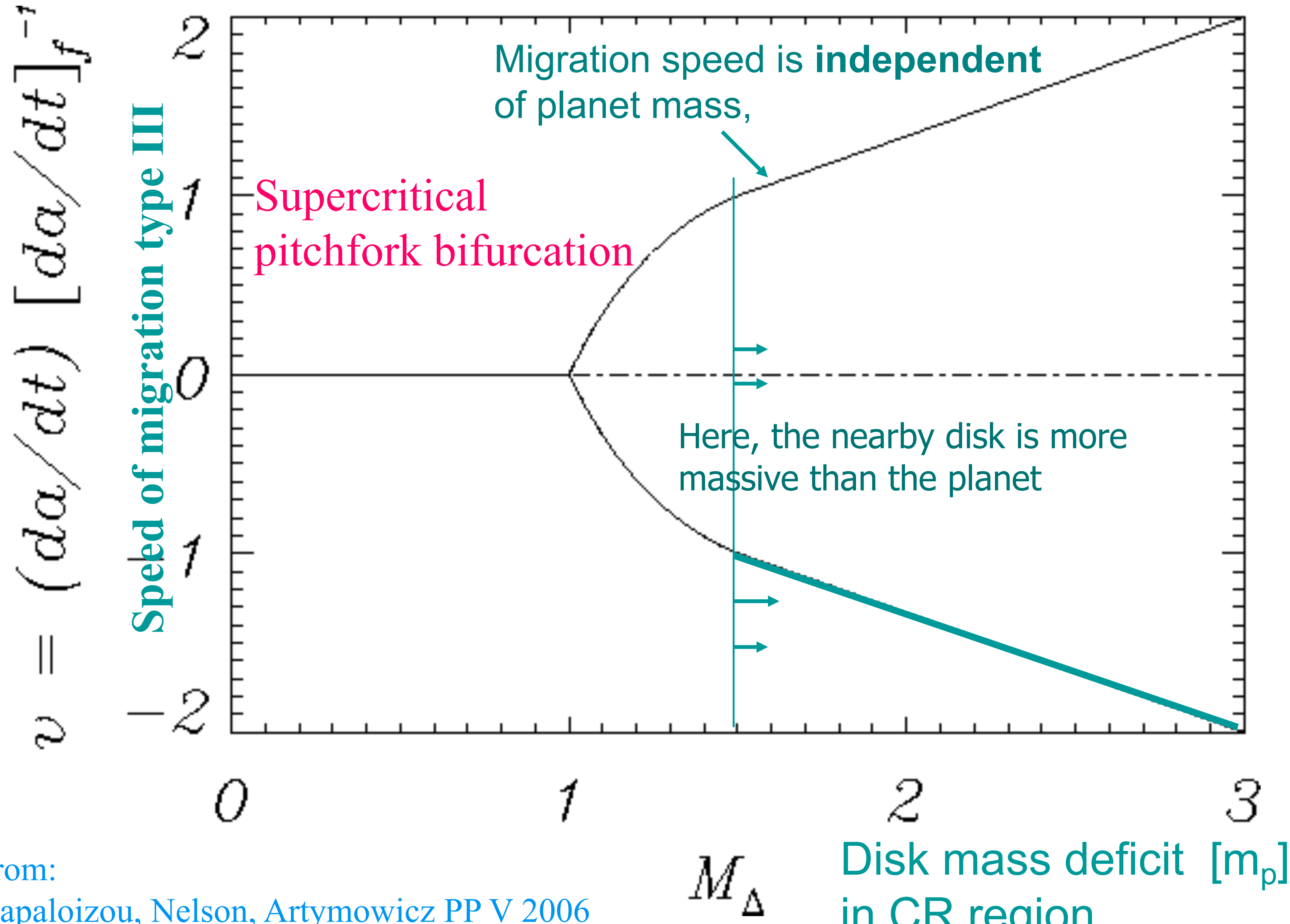
Full range (2π) of azimuth angles shown on vertical axis.
Horizontal axis shows radius from r = 1 to r = 3

Fast migration

Azimuthal angle

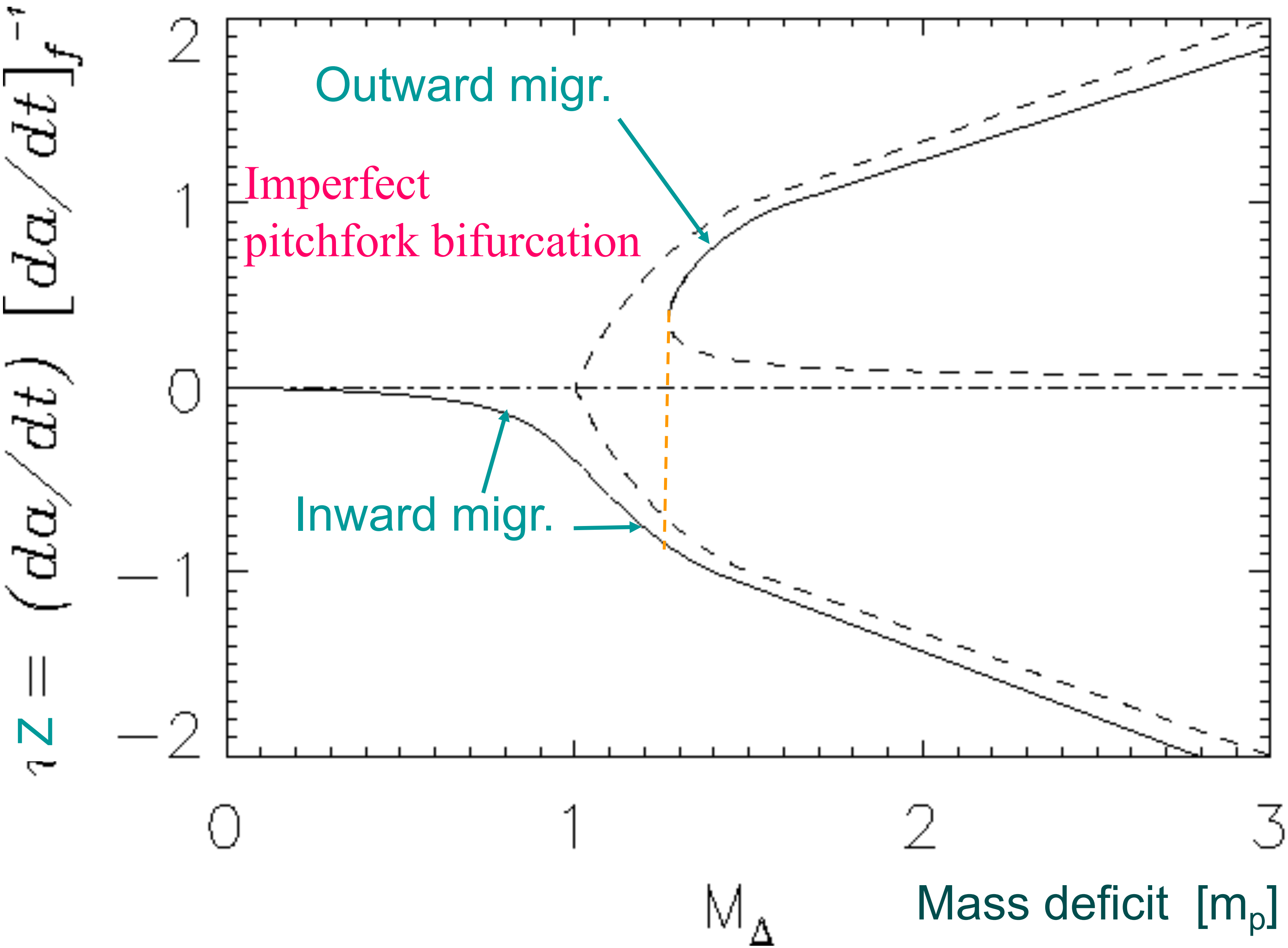


Migration type III, neglecting resonant wave generation viscosity



from:
Papaloizou, Nelson, Artymowicz PP V 2006

Migration speed including all torques (CR+LR+viscous), LRs bias toward inward migration

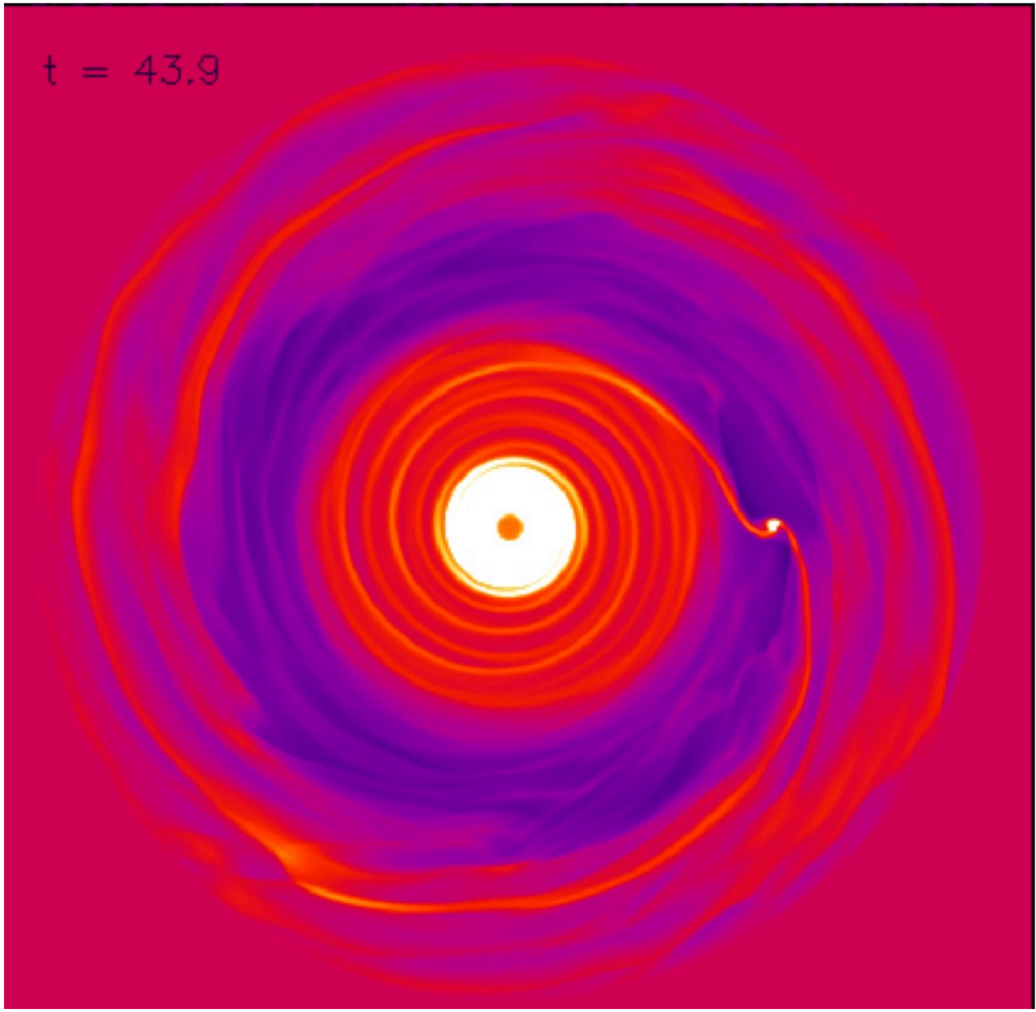


Summary of type-III migration
for those interested in astrophysics

- Rapid (timescale can be < ~100 orbits).
- Mass flow through the gap dominate wave excitation in far-away disk by mean-motion (Lindblad) resonances
- Inward migration explains 'hot jupiters' class of exoplanets (described in the next lecture)
- Its direction depends on prior history, not just on disk properties.
- Migration stops on disk features (rings, edges and/or substantial density gradients.) Such edges seem natural (dead zone boundaries, magnetospheric inner disk cavities, formation-caused radial disk structure)
- Offers possibility of survival of giant planets at intermediate distances (0.1 - 1 AU),
- and of terrestrial planets during the passage of a giant planet on its way to the star (slower migration of giant planet would destabilize Earths)

Previously, only 2-D simulations were possible.

We've recently simulated a small, embedded planet of 5 Earth masses in a protoplanetary disk in 3-D.



The results show many new phenomena, such as:

1. Columnar flow resembling Taylor-Proudman columns in rapidly rotating fluids, and
2. Wake vorticity genaration by the planet (4 counter-rotating vortices)

Fung, Artymowicz, and Wu (2015, Astroph. J.)

2-D

3-D

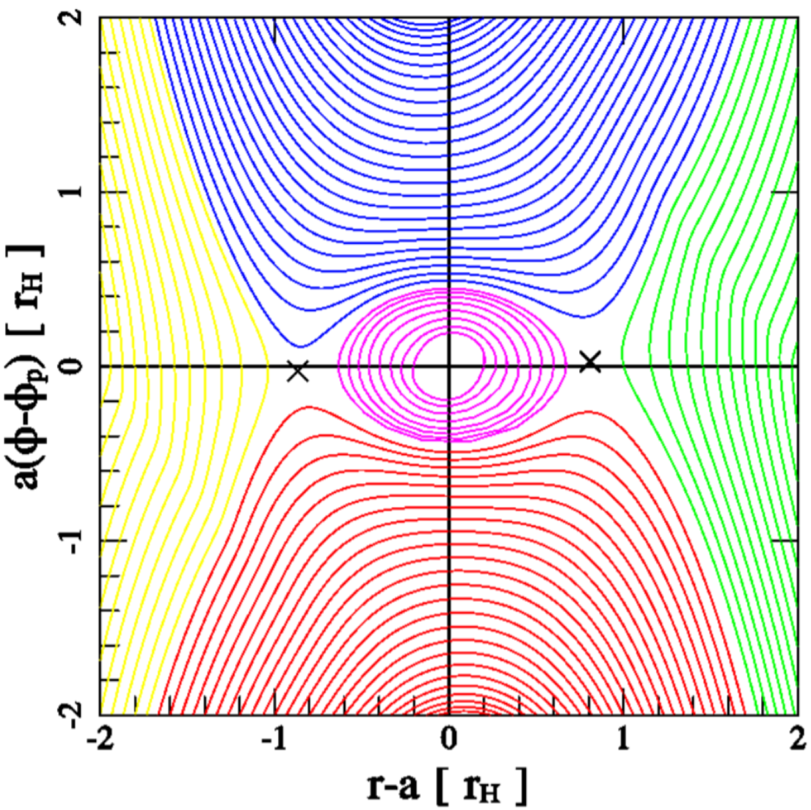


FIG. 1.— Streamlines around a planet in 2D, plotted in the corotating frame

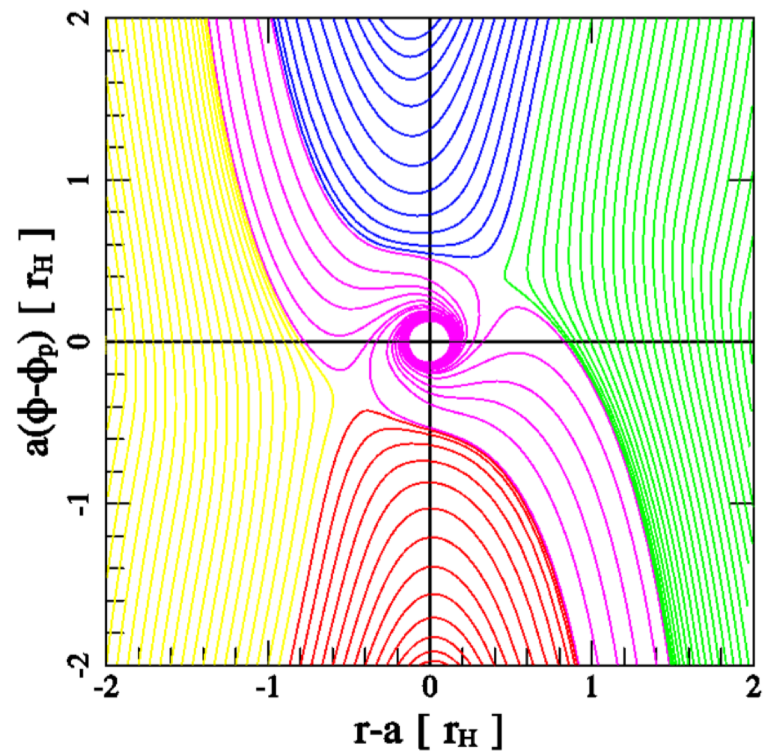
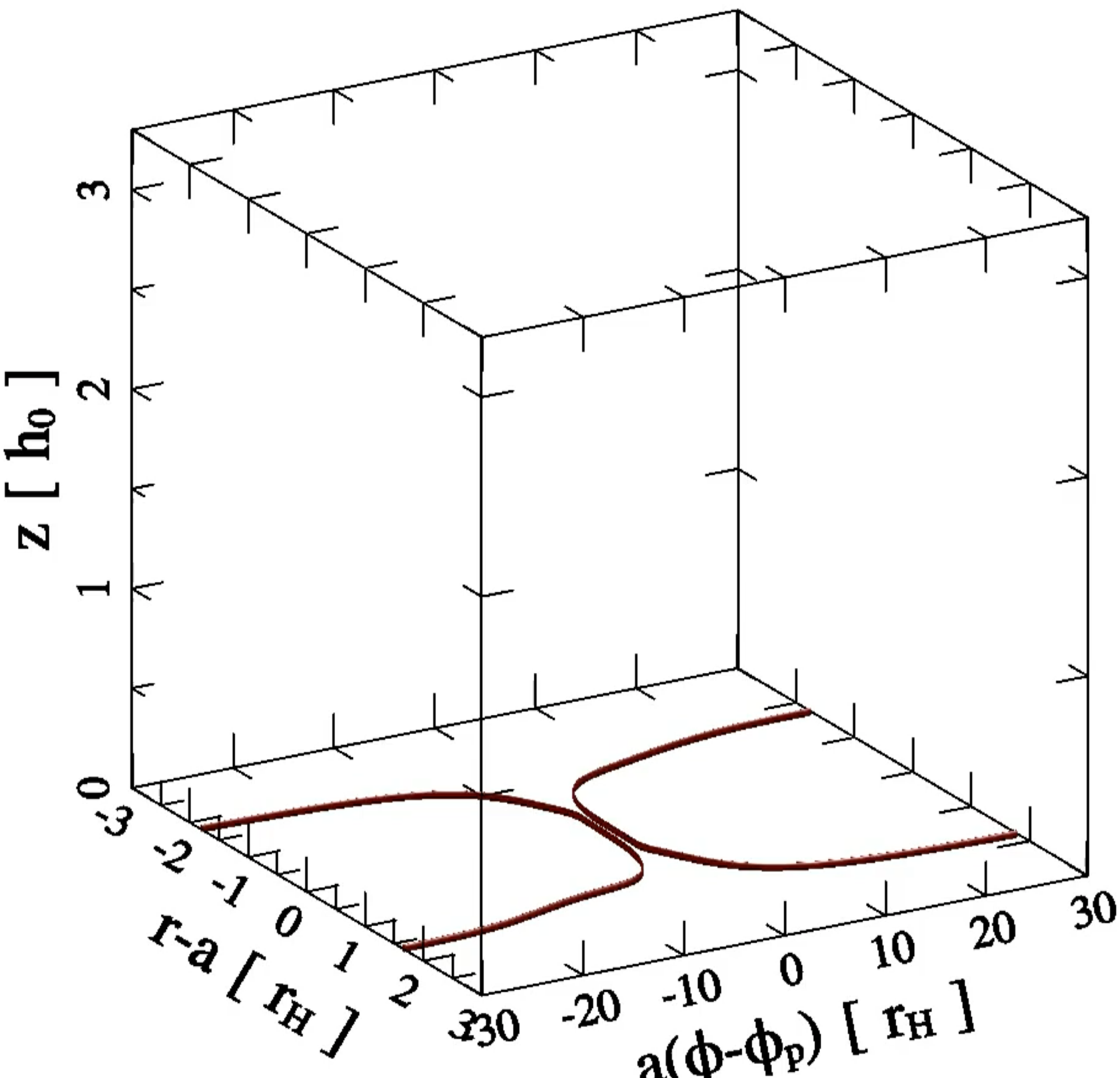
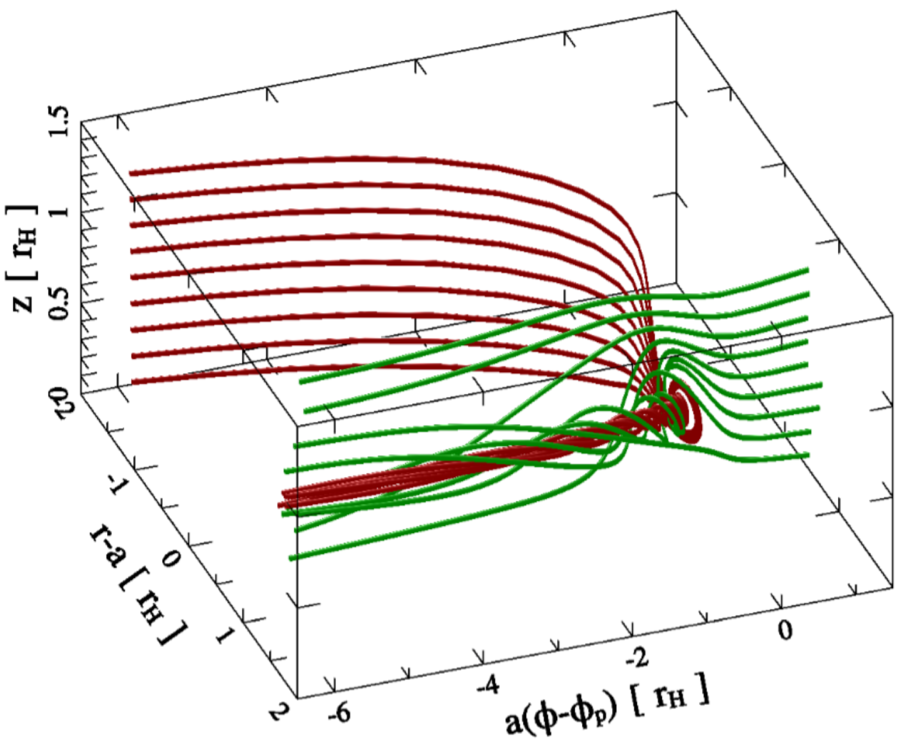


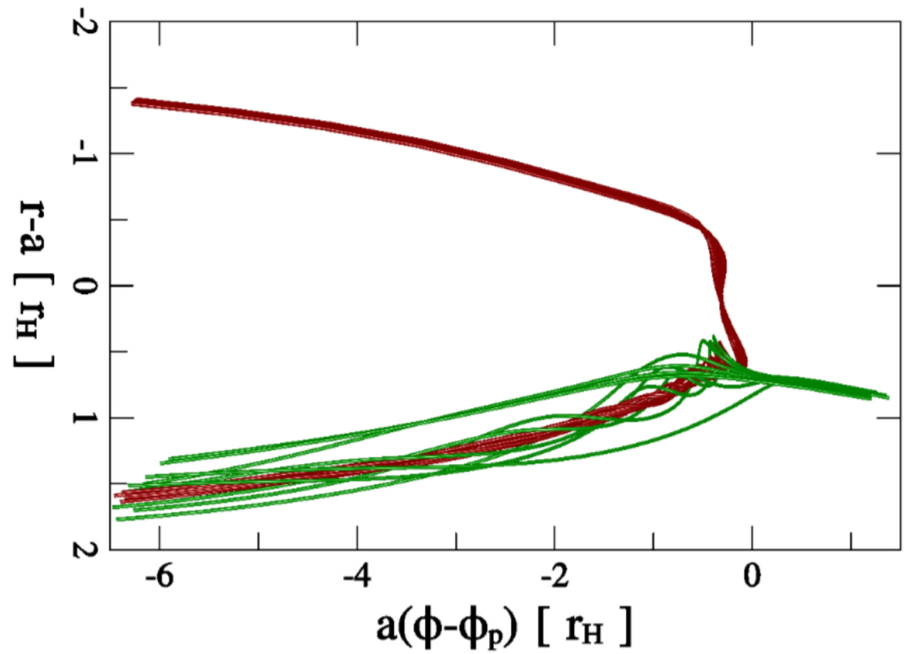
FIG. 8.— Streamlines in the disk midplane. Compare with Figure 1 for differences between 2D and 3D flow. Yellow, red, green, and blue streamlines are assigned in the same manner as Figure 1. Unlike Figure 1, magenta lines are outflows away from the planet, pulled down from initially higher altitudes. They reach as close as $1.5r_s$ from the planet and are unbound.

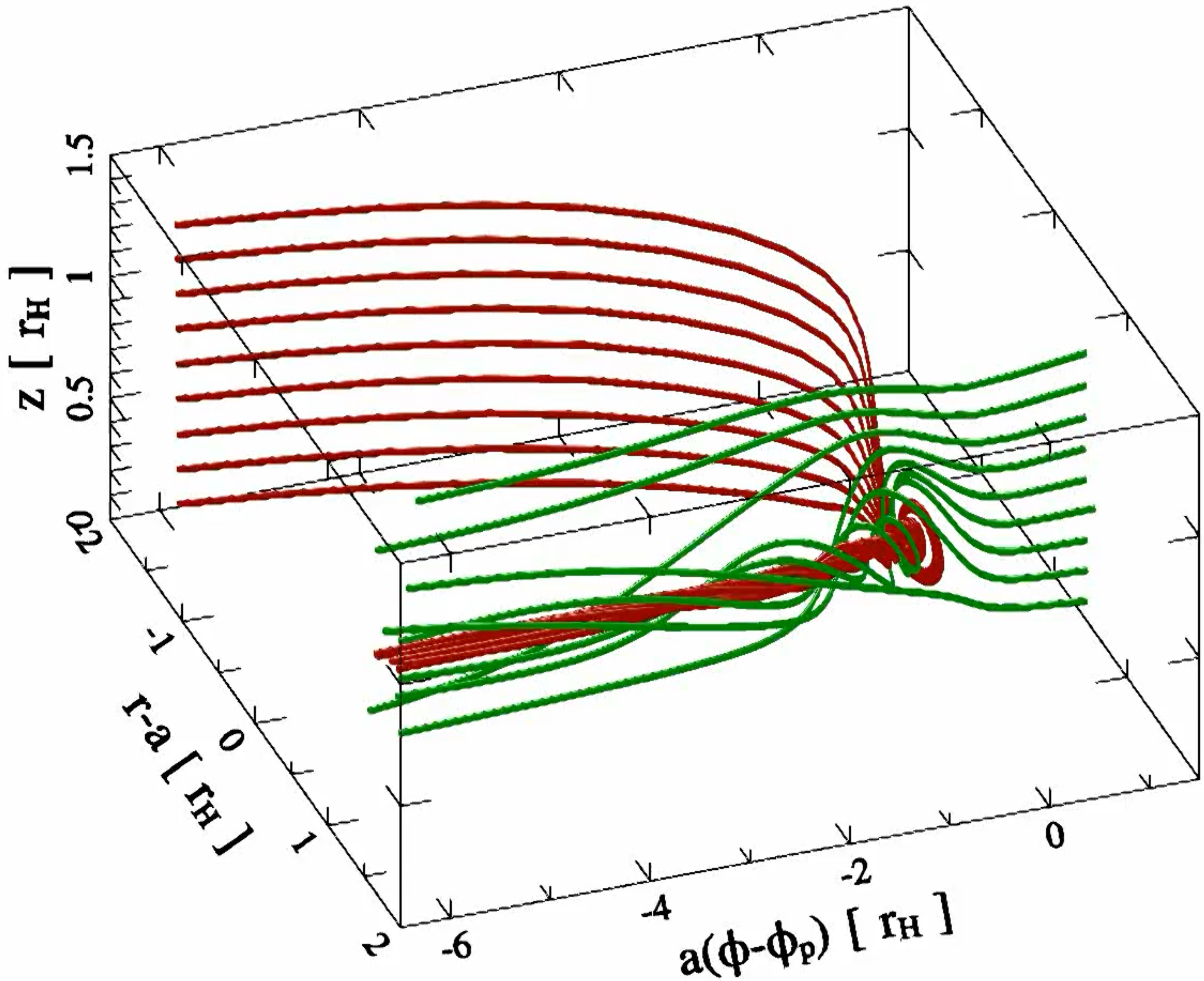


z [h0]
0
1
2
3
r-a [rH]
-3
-2
-1
0
1
2
3
a(ϕ-ϕp) [rH]
-30
-20
-10
0
10
20
30

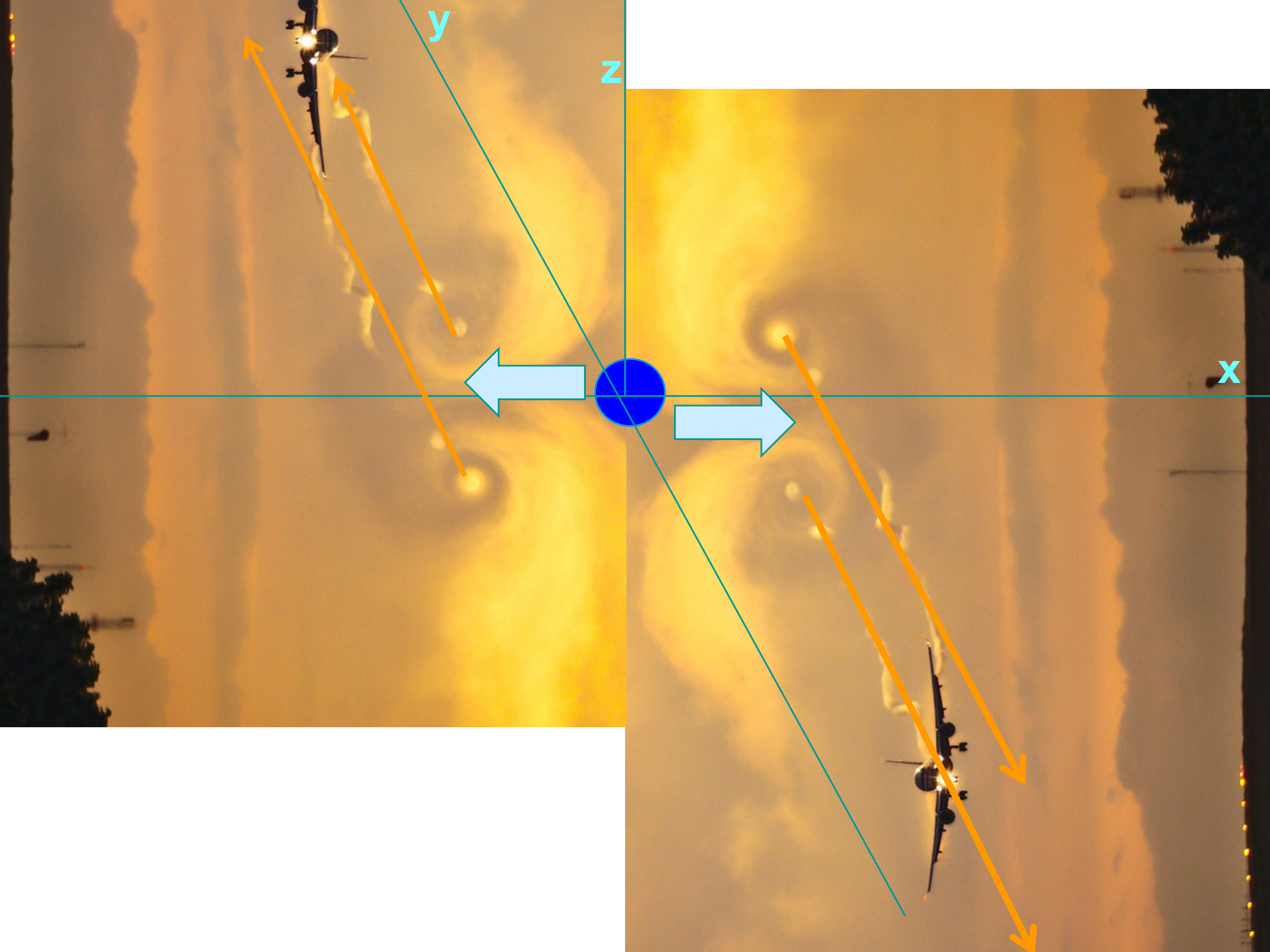
A total of 4 such vortices are shed by an embedded planet

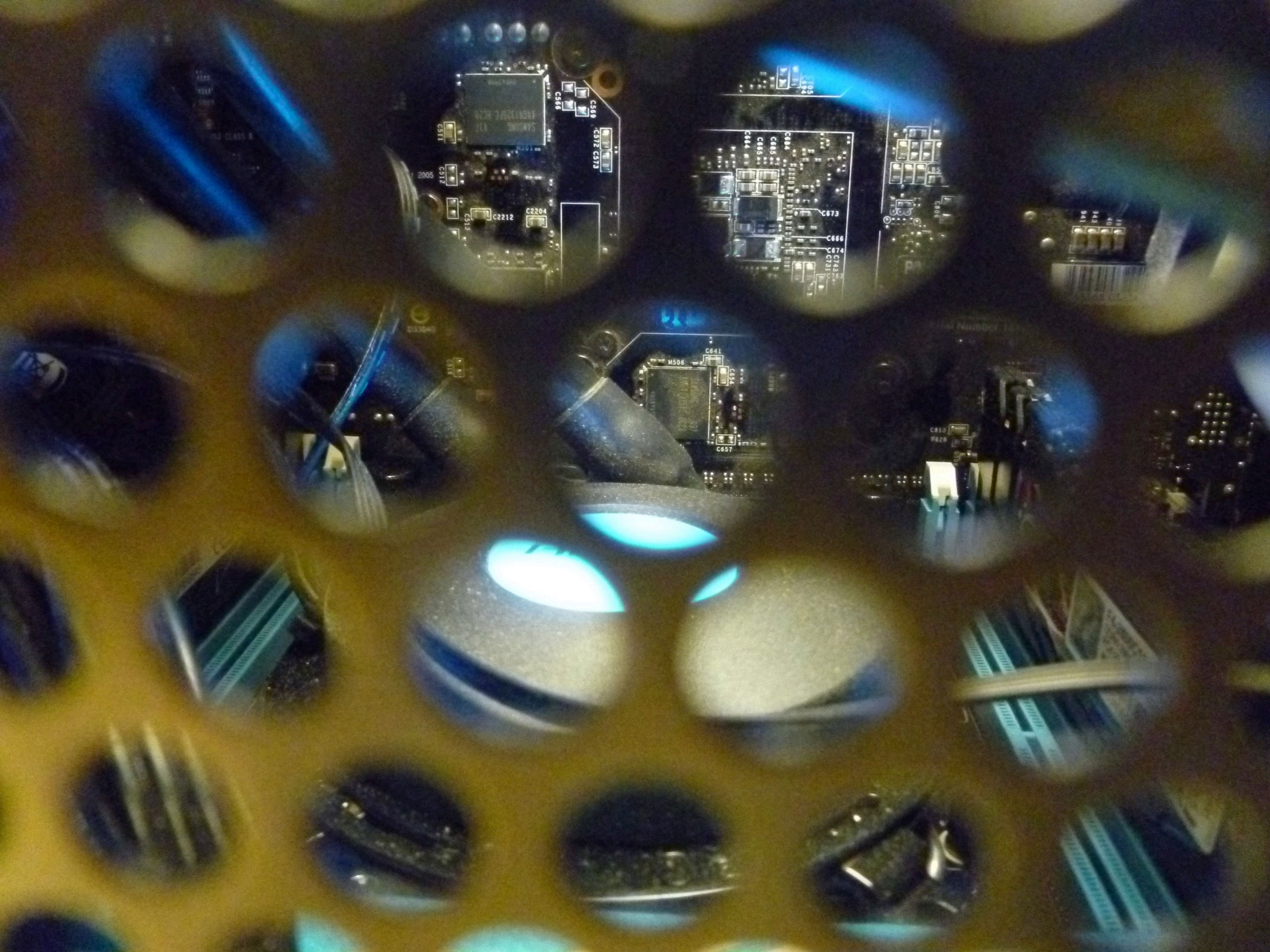


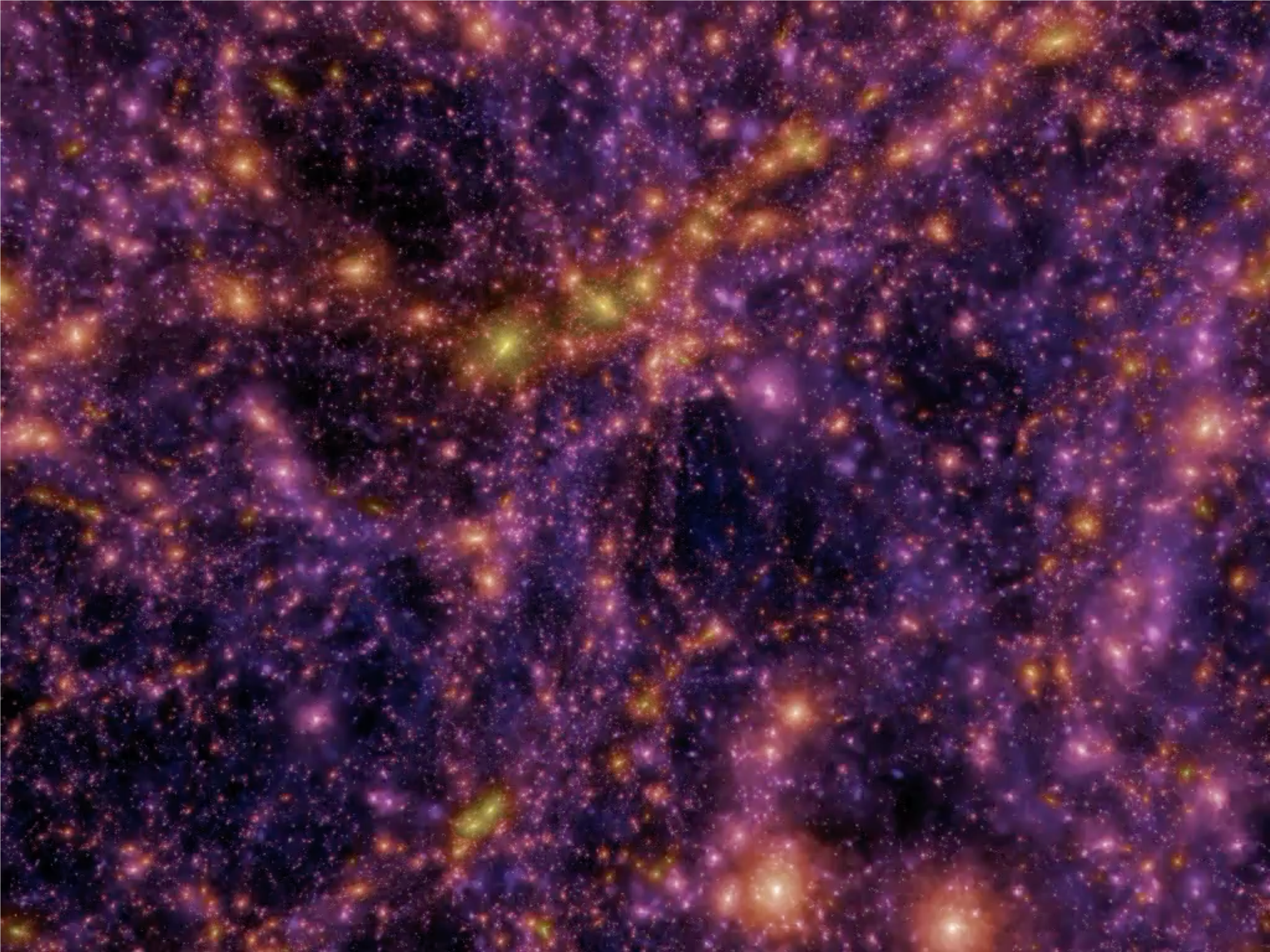




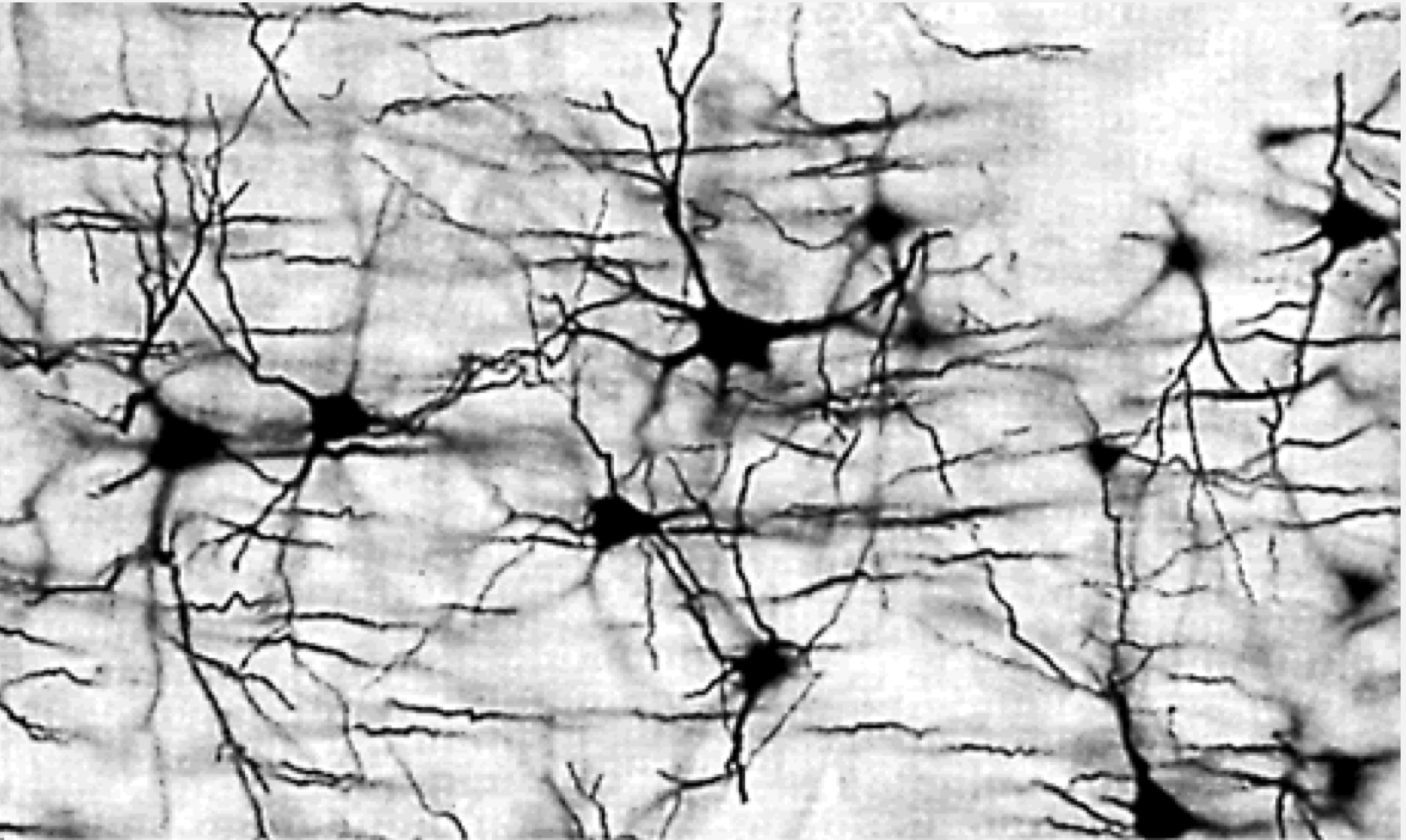
1.5
1
0.5
0
z [rH]
-2
-1
0
1
2
r-a [rH]
-6
-4
-2
0
a(ϕ-ϕp) [rH]

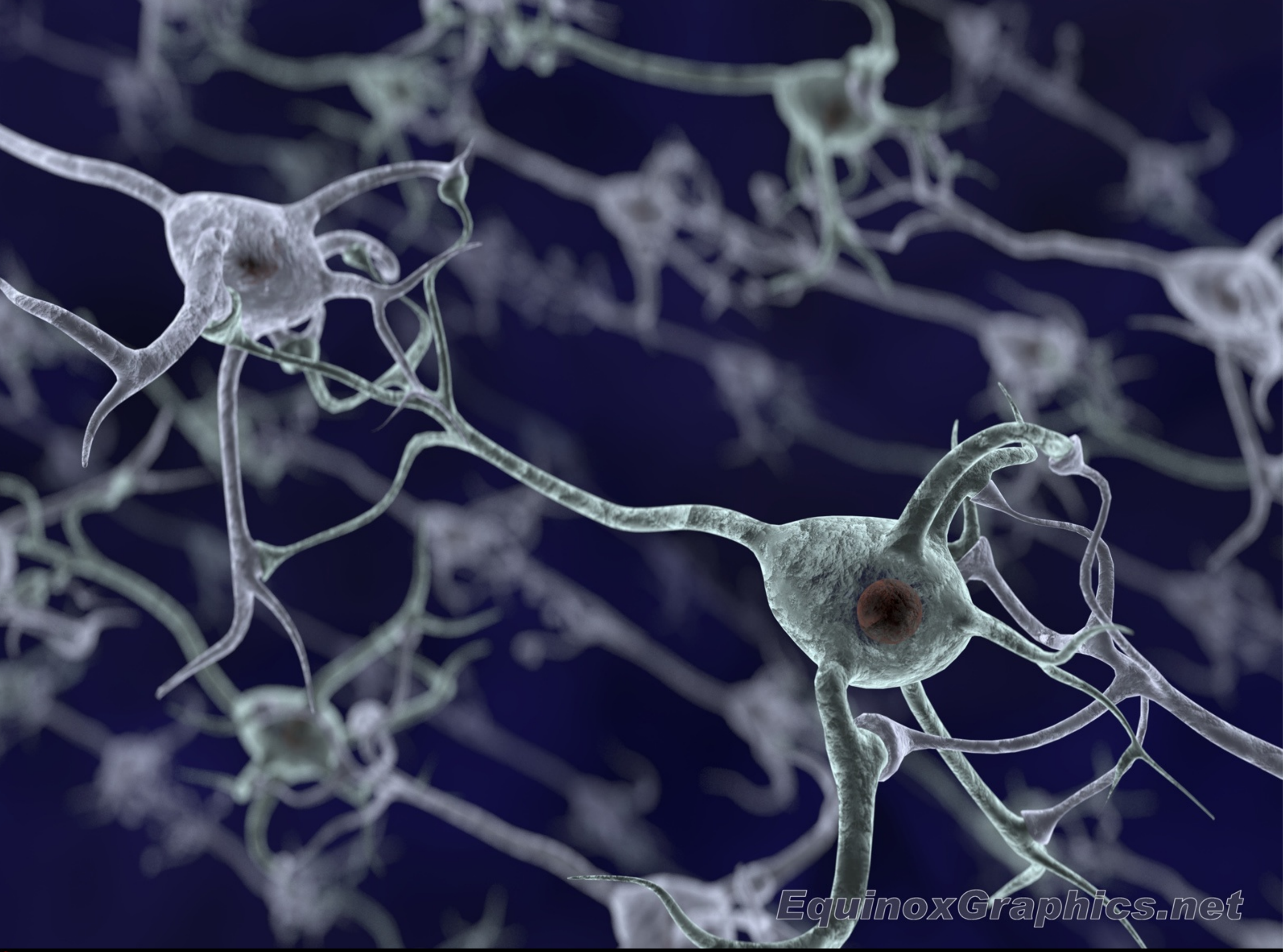

y
z
x


C2212
C2204
C673
C666
C674
C641
C657



Can machines be taught to think?





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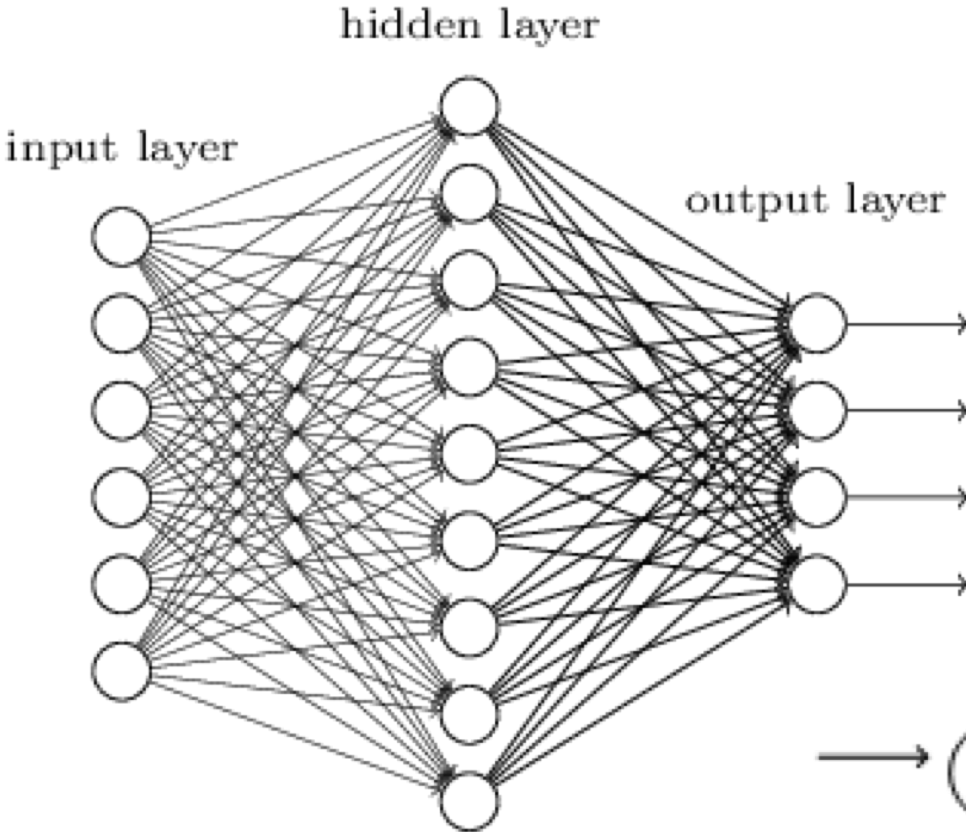
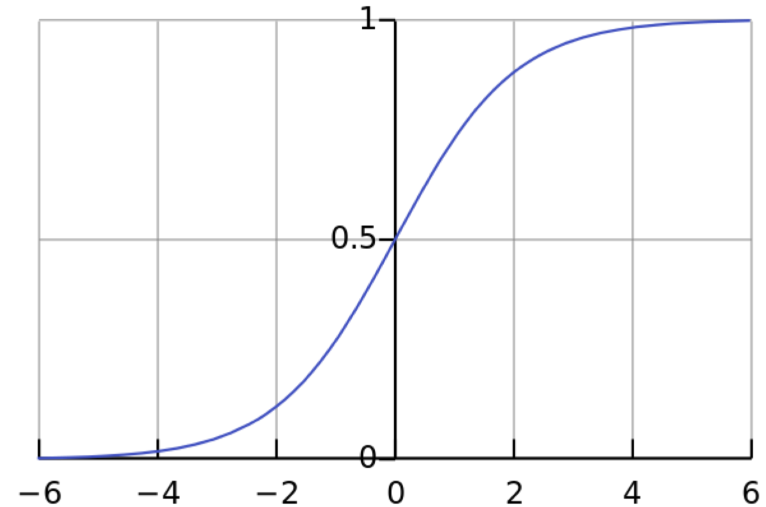
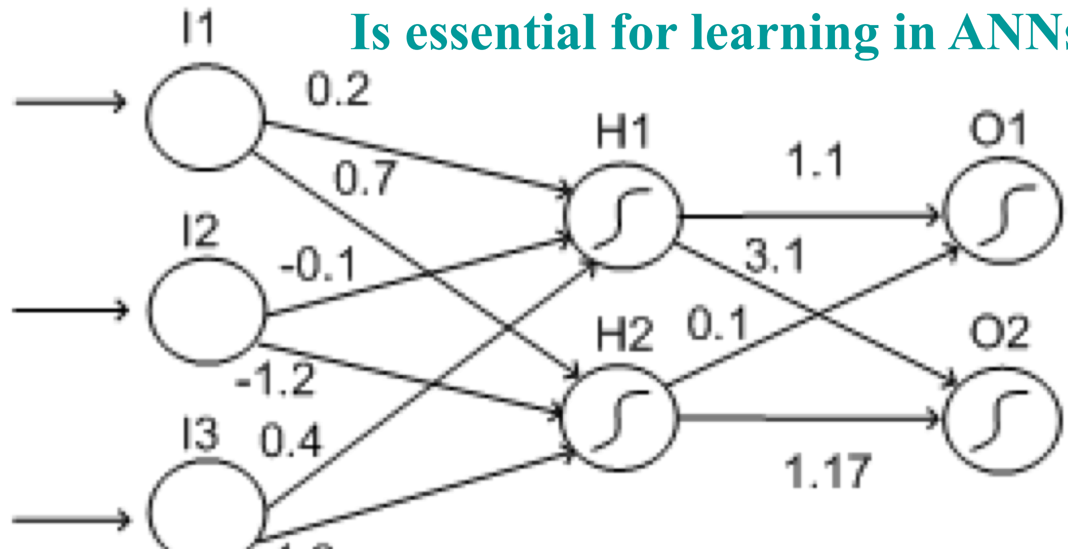
Artificial Neural Networks – computer simulates biologically-inspired layers of neurons which process information in parallel

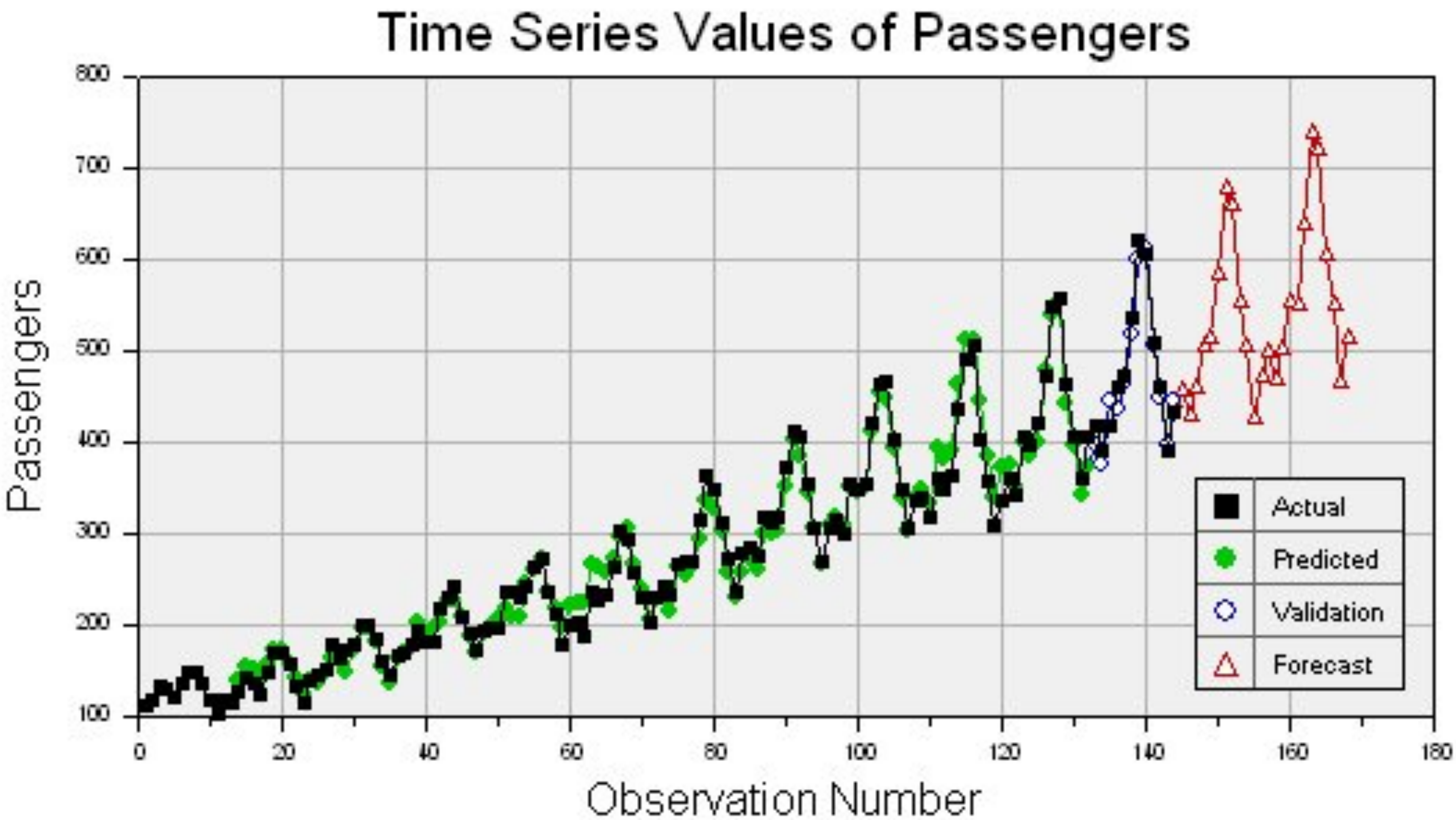




Nonlinear sigmoid function
Is essential for learning in ANNs



AI can extrapolate some regular behavior into the future



Real uses of Artificial Intelligence:

- controls robots
- does voice recognition
- trades stocks
- does data mining, e.g. Google
- designs chips
- detects fraud
- helps in scientific calculations
- image recognition, classification
- text analysis, auto-correction
- generates music
- drives taxis in San Francisco
- plays chess, Go, better than us
- may one day be in your robot friend, or your doctor
- ~~solves your assignments~~
- ~~will be a good programmer~~