Department of Physical and Environmental Sciences

PHYD38

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Chaos in Economics and Stock Market

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Background

As a general subject, economics can be split into two main categories microeconomics and macroeconomics. In this section of the paper, we will focus of the former. Microeconomics is a subject that deals with models regarding the supply and demand of goods and services. The supply and demand of goods and services are represented as functions of price with respect to quantity. In general, when graphing these curves on a price versus quantity graph, the supply curve tends to be upwards sloping, and the demand curve tends to be downwards sloping. When the supply and demand curves are found, the equilibrium price and quantity can then be found. For example, one of the simplest models is when demand and supply are both linear as shown in figure 1.

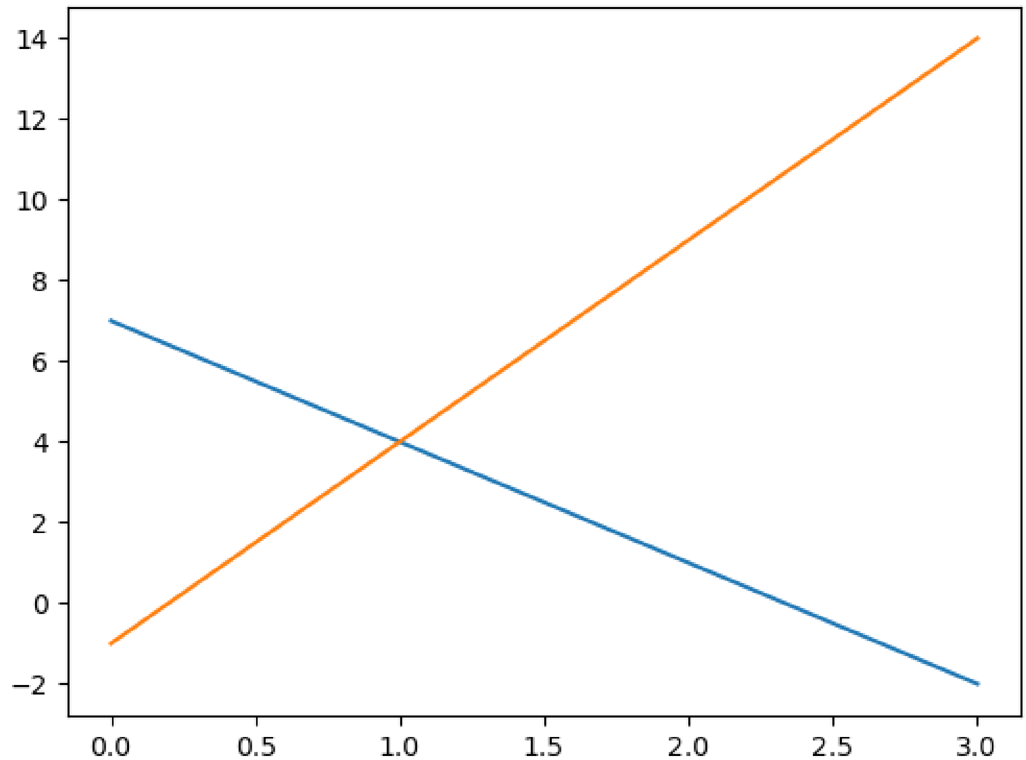


Figure 1. Supply (Orange) and Demand (Blue) graph plotted as P vs Q.

On figure 1, the supply curve is the function $P_S(Q) = 5Q - 1$, and the demand curve is $P_D(Q) = -3Q + 7$. With these two curves found, we can then see (either from the graph or algebraically by setting $P_S = P_B$) that the equilibrium price is \$4 at an equilibrium quantity of 1 where the two curves intersect. Although this simple model is very intuitive, the demand and supply curves in real life tend to be non-linear and are harder to find. This is due to the supply and demand curves being determined by multiple factors such as the number of producers and consumers, costs of production, need for such a good or service and so on. Focusing on one of the factors that affects the supply curves, there are types of markets that producers and consumers can face.

On one end of the extreme, there is perfect competition where there are many producers making the same good or service. And on the other end, there is monopoly where there is only one producer making the good or service. Some examples of goods that have perfect competition are soda drinks or computers. There are many producers of each good such that a single producer cannot influence the market in a substantial way, therefore, every producer is a price-taker. On the other hand, there are goods such as electricity or hydro where the firms who provide such goods face little to no competition and therefore have a monopoly over the good or service. There also exists an intermediate between perfect competition and monopolies called oligopolies. Oligopolies are markets where there exists competition with only a few number of firms producing the same good or service, and a special case is when there are only two firms who produce a good or service is called a duopoly. One of the topics that economists study are these monopolists and oligopolists markets and how these producers can manipulate the market to yield the greatest amount of profit. According to Puu (2000), “The monopolist must thus know at least the entire demand curve of the market” (217) to be able to take advantage of their power to gain the most amount of profit. However, because the market is so dynamic, “the monopolist does not know more than a few points on the demand function.” (222) Therefore, it will be hard to determine how much of the good the producer should make to maximize profits. To address this, we first must build a model that can account for the non-linearity of the real-world market.

Monopolies

According to Puu (2000), the demand curve could be represented as a truncated Taylor series where:

$$P_D(Q) = A - Bx + Cx^2 - Dx^3$$

In the equation, x represents the quantity demanded and A, B, C, D are positive constants. And because the demand curve is downwards sloping, it must be the case that $C^2 < 3BD$. From the demand curve, the monopolist can then calculate the marginal revenue as the derivative of the total revenue which equals to the total price multiplied by quantity. From this, the marginal revenue is determined as:

$$MR = A - 2Bx + 3Cx^2 - 4Dx^3$$

The marginal cost is calculated as the derivative of the total cost which can be assumed to also be a truncated Taylor series:

$$MC = E - Fx + Gx^2$$

From these two curves, we can then find the maximum profit by equating the marginal revenue with the marginal costs:

$$(A-E) - 2(B-F)x + 3(C-G)x^2 - 4Dx^3 = 0$$

And solve for the roots and find the maxima (using the second derivative of the function) to determine the quantity of maximum profit. However, because the demand curve is not well-defined, Puu (2000) suggests that we use a search algorithm to determine the maxima of the profit function. Given that the profit is equal to the revenue minus cost, the profit function can be defined as:

$$\Pi(Q) = (A-E)Q - (B-F)Q^2 + (C-G)Q^3 - DQ^4$$

By setting the derivative equal to zero, we can solve for the roots to find the maxima. Puu (2000) gives the search algorithm as the map:

$$x_{t+1} = f(x_t, y_t)$$
$$y_{t+1} = g(x_t, y_t)$$

$$f(x, y) = y$$
$$g(x, y) = y + \delta P(x, y)$$

$$P(x, y) = \frac{\Pi(y) - \Pi(x)}{y - x} = (A-E) - (B-F)(x+y) + (C-G)(x^2 + 2xy + y^2) - D(x^3 + x^2 y + xy^2 + y^3)$$

Based on f(x, y) and g(x, y), we can then find out the stability by calculating the Jacobian and setting it to 1. After setting x=y and substituting the maxima for x, the resulting value of delta is then the point when there stability is loss:

$$\begin{vmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{vmatrix} = -\delta(\frac{\partial P}{\partial x}) = \delta((B-F)-(C-G)(2x+2y)+D(3x^2+2xy+y^2))=1$$

$$\delta = \frac{1}{((B-F)-(C-G)(4x)+D(6x^2))}$$

As delta increases, the period of the oscillations increases, and eventually we can observe chaotic bands throughout the bifurcation diagram. To see this more clearly, we can set some values to the constants and see the emerging chaos. Let A = 5.6, B = 2.7, C = 0.62, D = 0.05, E = 2, F = 0.3, and G = 0.02. We can then plot the following curves:

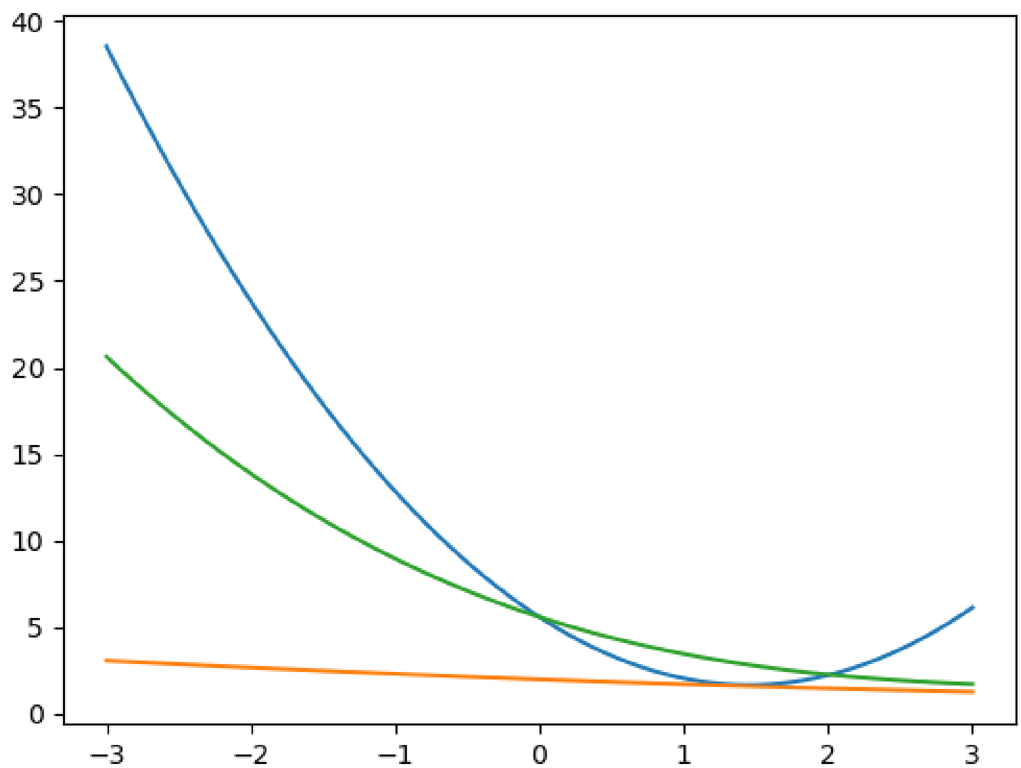


Figure 2. MR (Blue), MC (Orange), and Demand curves (Green) plotted as P vs Q.

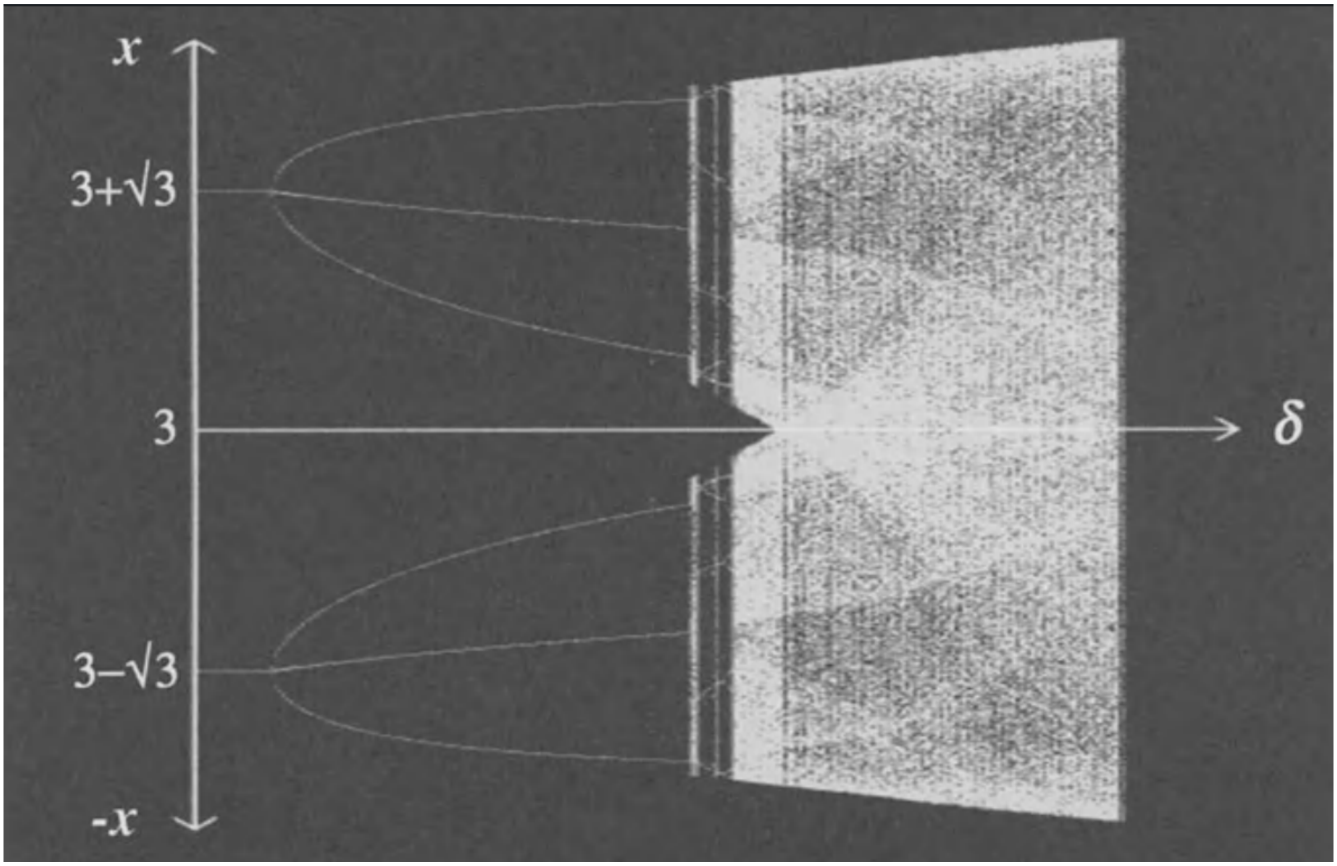


Figure 3. Bifurcation diagram of monopoly model from Puu (2000, pg 223).

We can see on figure 2 that the demand, marginal revenue and marginal cost curves are plotted with the intersections between the marginal revenue and marginal costs corresponding to the profit minima or maxima. In this case, $x=3\pm\sqrt{3}$ are the maxima. In figure 3, we can see the bifurcation diagram for such points. When $\delta=\frac{5}{3}$, the two fixed points at $x=3\pm\sqrt{3}$ becomes cycles with 2 period and as delta increases, the periodicity also increases and then chaos follows.

Duopolies with the Cournot Model

Moving on to duopolies, there are now two competing firms who are able to produce a certain good or service. Because of this, the firms aren't as free as the monopolist to set whatever production quantity they want, they must consider each other's actions and each set a certain quantity to produce such that both will try to maximize their profits. Because of this additional complication, we must consider the firm's behaviour using either the Cournot Model and Stackelberg Model. The Cournot Model assumes that both firms will decide their production simultaneously and that what they choose to produce will be final. On the other hand the Stackelberg Model requires a leader and a follower where the firm that chooses a quantity first is the leader firm and has the first mover advantage, while the other firm is the follower who responds to the leader firm's actions. In this paper, we will focus on the Cournot Model and analyze how adjustments towards the Cournot equilibrium can be chaotic.

According to Puu (2000), we assume an isoelastic demand function, and because the price is inversely proportional to quantity produced, we can define the demand function for a duopoly as:

$$P_D=\frac{1}{x+y}$$

Where x is the quantity firm 1 produces and y is the quantity firm 2 produces. We also assume a constant marginal cost for both firms, therefore the firms profit functions should look like:

$$Profit_{\text{Firm n}} = Revenue_{\text{Firm n}} - Cost_{\text{Firm n}} = (P_D)(Q_n) - c(Q_n)$$

$$\Pi_1 = P_D x - ax = \frac{x}{x+y} - ax$$

$$\Pi_2 = P_D y - by = \frac{x}{x+y} - by$$

Where a and b are both constants corresponding to the constant marginal cost for the firms. Now that we have both firms profit functions we can find the maximum profit by taking the partial derivative of each firm's profits with respect to their quantity and setting them equal to zero:

Recall: Maximum profit is when MC = MR where $MR = \frac{\partial Revenue}{\partial Q}$ and $MC = \frac{\partial Cost}{\partial Q}$

$$\frac{\partial \Pi_1}{\partial x} = -ax^2 - 2ayx - ay^2 + y = 0$$

$$\frac{\partial \Pi_2}{\partial y} = -by^2 - 2bxy - bx^2 + x = 0$$

Solving for the roots of x for firm 1 and y for firm 2, we can then get the reaction curve of each firm:

$$x = \sqrt{\frac{y}{a}} - y$$

$$y = \sqrt{\frac{x}{b}} - x$$

We can then substitute both equations with each other and solve for x and y to get the output quantities of both firms.

$$x = \frac{b}{(a+b)^2}$$

$$y = \frac{a}{(a+b)^2}$$

These quantities are the Cournot equilibrium quantities for each firm, and they can be seen at the intersection between the two relation curves such as shown in figure 4:

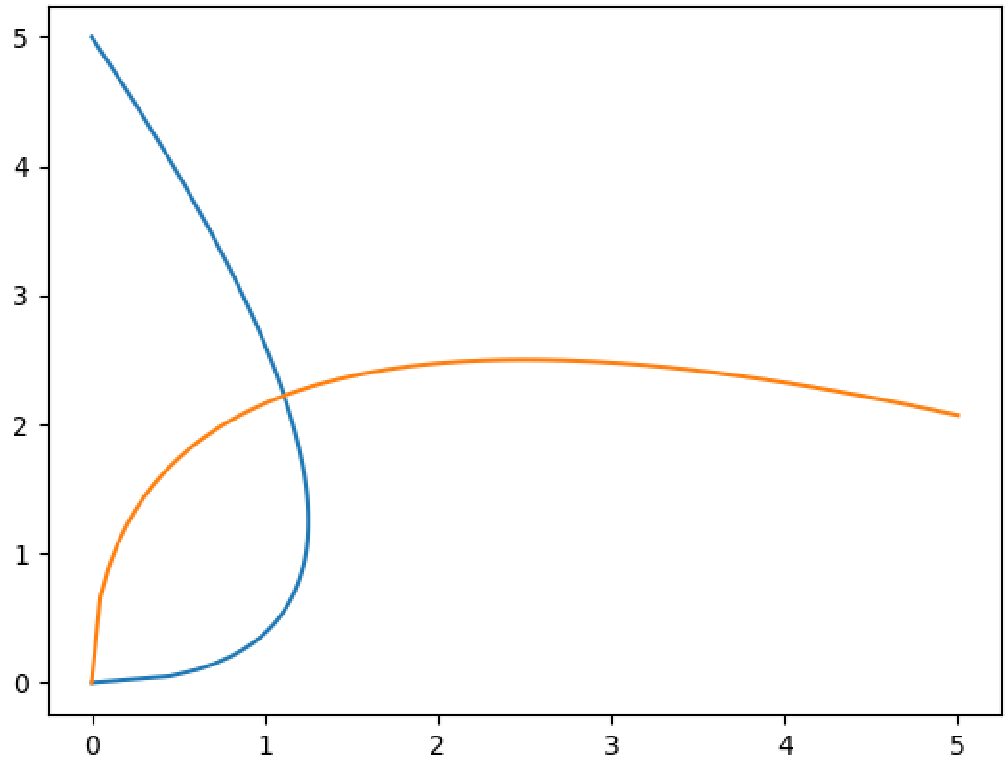


Figure 4. Reaction curves between of firm 1 (Blue) and firm 2 (Orange) plotted as y vs x (a=0.2, b=0.1).

Using these equations, we can analyze the process of adjustment towards the Cournot equilibrium point and see if it is stable by using the following mapping of the reaction curve:

$$x_{t+1}=\sqrt{\frac{y_t}{a}}-y_t$$

$$y_{t+1}=\sqrt{\frac{x_t}{b}}-x_t$$

According to Puu (2000), we can take the absolute value of the derivatives of x and y (where x and y are the reaction curves) and set it equal to 1 to find the where loss of stability occurs:

$$\left|\frac{dx}{dy}\frac{dy}{dx}\right|=1$$

$$\left|\left(\frac{1}{2}\sqrt{\frac{1}{ay}}-1\right)\left(\frac{1}{2}\sqrt{\frac{1}{bx}}-1\right)\right|=1$$

After distributing and plugging in the x and y output quantities of both firms we get:

$$(a-b)^2=4ab$$

Then solving for the roots by simplifying and normalizing the roots to $\frac{a}{b}$ (the same results follow for $\frac{b}{a}$), we get:

$$\frac{a}{b}=3\pm 2\sqrt{2}$$

Therefore, if the ratio of marginal costs between the two firms are greater than $3+2\sqrt{2}$ or less than $3-2\sqrt{2}$, then the Cournot point will not be stable. We can see this more clearly in the bifurcation graph shown in figure 5:

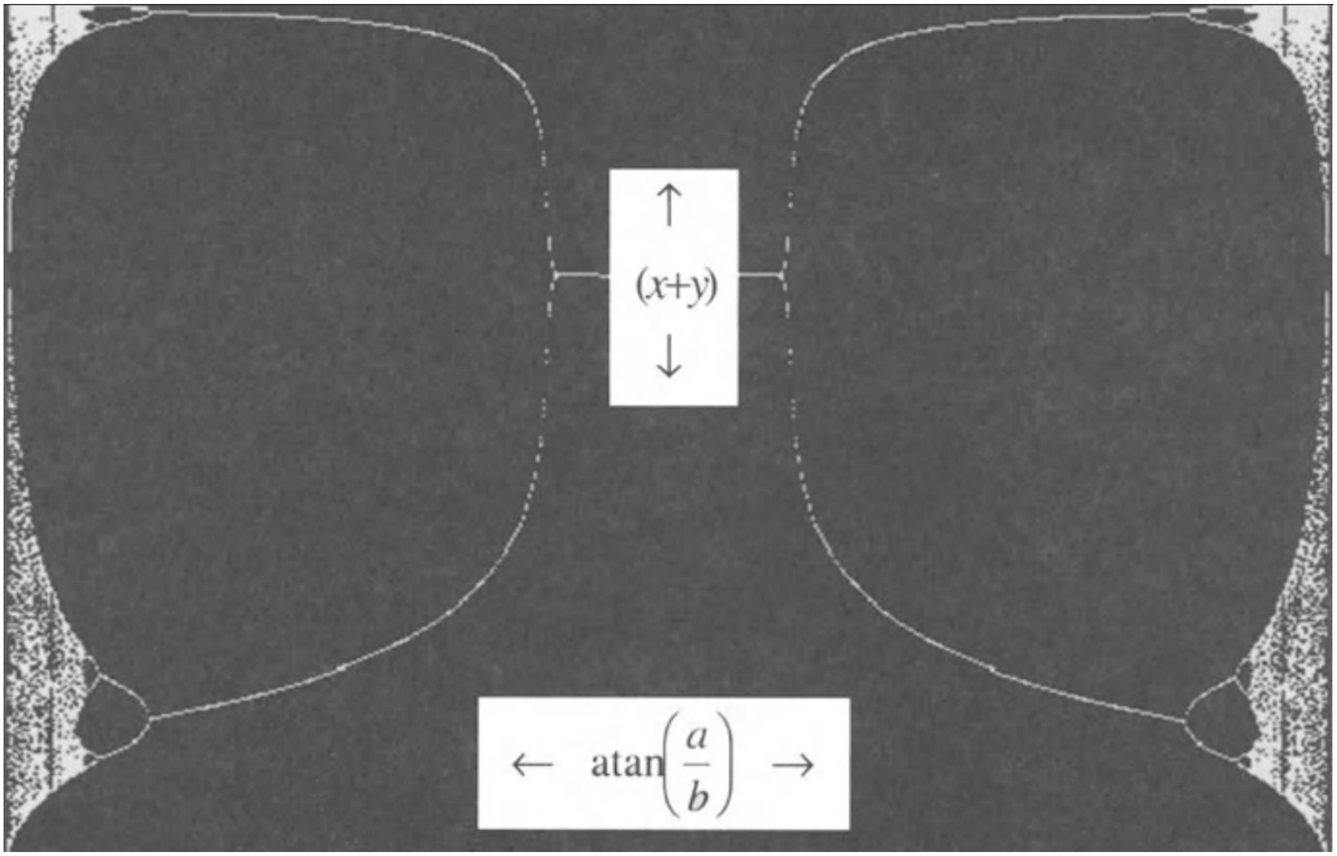


Figure 5. Bifurcation diagram of the duopolist Cournot model from Puu (2000, pg 247)

In figure 5, it is shown when $\arctan(3-2\sqrt{2}) > \arctan(\frac{a}{b}) > \arctan(3+2\sqrt{2})$ then the stable points becomes a two period cycle and as $\arctan(\frac{a}{b})$ increases from $\arctan(3+2\sqrt{2})$ or decreases from $\arctan(3+2\sqrt{2})$ then we will start to see a 4 period cycle up to infinity where it will approach chaos and no longer become periodic.

Oligopolies with n-firms with the Cournot model

Using the same type of analysis shown above for duopolies, this can be extended to oligopolies of n firms by making the demand function equal to the reciprocal of the sum of n firms, Finding the profit functions of each firm, finding their response curves and solving for their Cournot equilibrium quantities. We can notice that for a duopoly their response curves are 2-dimensional due to two individual firm quantities. When we extend this to n firms we will get an n dimension curve. From their response curve equations, we can analogously find the mapping for the process of adjustment, then find if there is loss of stability by finding the roots (eigenvalues) of the characteristic equation of an nxn Jacobian matrix. According to Puu (2000), if the “real eigenvalue can get an absolute value larger than unity [or] a pair of conjugate complex eigenvalues can cross the unit circle in the complex plane” (259) then higher order systems can be unstable.

Business cycle in continuous time

Business cycle is a major part of chaos in economics, a business cycle refers to the pattern for economic activities over time. It can show fluctuations and productions and much other useful information. A business cycle in continuous time shows the fluctuations in economic activities over time. Rather than being at a specific point in time, the continuous time model allows for analysis over a much longer period of time.

Some histories on business cycles involve one major significant event that occurred in economic theory is in 1939 when Samulson invented the business cycle machine, where the multiplier and accelerator concepts were combined. This model showed how simple forces such as the consumer spending and producer having investments can produce cyclical changes within the economy. This cyclical behavior is explained very simply by the model and is extremely convincing. In 1948, Harrod formulated the process in continuous time as a differential equation to focus on the growth rather than the cycle which showed the balance between growth and instability. When formulating business cycles, there are two types of modeling, one being continuous and one being discrete, at higher orders of continuous modeling, there are more chaotic behaviors occurring. Which results in continuous modeling being the preferred method as Phillips realizing the continuous time model can process a second order model.

The original model for the business cycle has income as Y and saving as S, K as capital stock and s being the proportion of income and the ratio denoted as v, investments is denoted as I which results in $I = v\dot{Y}$ and $S = sY$, which in equilibrium I = S. These equations are later on adjusted when Phillips assumed different speeds for the two processes which the equations became $\dot{Y} = I - sY$ and $\dot{I} = v\dot{Y} - I$, with further differentiation for the first equation, the basic equation for the original model is $\ddot{Y} - (v - 1 - s)\dot{Y} + sY = 0$. This equation is capable of producing oscillations of either damped or explosive. The problem with the original model is that it clearly shows the limitations of linear analysis, the model can only produce exponentially damped or explosive but nothing else. In the bound case that produces a standing cycle, the probability for it to vanish is small so it represents a case that is structurally unstable. The damped case does not generate any movements, it only maintains what is originally there, and the explosive case produces unbound change which has the problem of scientific procedure. Hicks rearranged the multiplier accelerator model to become nonlinear so that the process of change would be limited by the floor and the ceiling to produce a movement that is cyclic that has finite bounds. In 1951 Richard Goodwin introduced a smooth nonlinearity for the investment function of the model and now with two nonlinearity in the investment functions, using the power series, a resulting model is then complete $\ddot{Y} + sY = (v - 1 - s)\dot{Y} - \frac{v}{3}\dot{Y}^3$ which is the limit cycle. The left side of the equation represents a simple harmonic oscillator and the right side is the cubic characteristics for the business cycle. The existence of this limit cycle is analyzed by finding out the behaviour of the differential equation in phase space, focusing on the relationship between the rate of change of income and the second derivative. Which came to an observation that for certain parameter values the differential equation has elliptic paths in the phase space which shows periodic behavior.

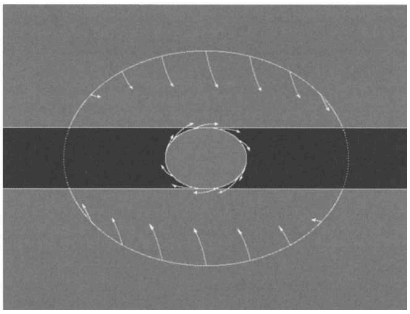


Figure #6: The punctuated elliptic annulus, and the strip of energy gain.

The proof to the existence is by differentiating the original model equation $\ddot{Y} + sY = (v-1-s)\dot{Y} - \frac{v}{3}\dot{Y}^3$ and setting $X = \dot{Y}$ so that the equation can be rewritten and results in $\ddot{X} + sX - ((v-1-s) - vX^2)\dot{X} = 0$, which is the van der Pol equation that has a graphical method of constructing the solution. For the approximation of this limit cycle, we have a special case of $\ddot{Y} + Y = \varepsilon(\dot{Y} - \dot{Y}^3)$ and by finding a solution in the form of a power series for ε then substituting it into the differential equation, the resulting equation is $\ddot{Y}_2 + Y_2 = -2\omega_1\ddot{Y}_1 - (\omega_1^2 + 2\omega_2)\ddot{Y}_0 + \dot{Y}_1 + \omega_1\dot{Y}_0 - 3\dot{Y}_0^2\dot{Y}_1 - 3\omega_1\dot{Y}_0^3$ and $\ddot{Y}_1 + Y_1 = -2\omega_1\ddot{Y}_0 - \dot{Y}_0^3$ and $\ddot{Y}_0 + Y_0 = 0$ which these equations can be solved in sequence and have an approximate solution. Then by changing the initial conditions of the equations and having fully solving the equations, the general solution is $Y_1 = A_1\cos\tau + B_1\sin\tau + \frac{1}{12\sqrt{3}}\sin 3\tau$. With this general solution an approximation is made with $Y_0 = 1.155\cos\tau$ $Y_1 = -0.144\sin\tau + 0.048\sin 3\tau$ and $Y_2 = 0.081\cos\tau + 0.036\cos 3\tau - 0.005\cos 5\tau$ and using the resulting approximate solutions, a simulation with the four point Runge-Kutta method is produced shown in figure #7 which shows how the rate of convergence of the approximated cycles is not really fast.

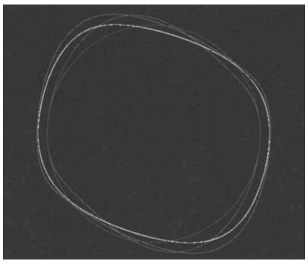


Figure #7 Three successive perturbation series compared with simulated solution.

Another model in the business cycle is the two-region model, this model represents the interactions between two regions of economies. This model has two coupled regions where both regions are second order processes, which then the total system has a total order of four which guarantees chaotic outcomes. The coupling for this model is done by a linear trade multiplier, with nonlinearity. The coupling regions are identified by firstly index all the variables, using i or j as identification index and then when both of the index are used in the same equation, they both represent two different regions, such as the equation $\dot{Y}_i = I_i - s_i Y_i$ and $\dot{I}_i = v_i \dot{Y}_i - I_i$ with i ranging through 1 and 2. Then linear propensity m is imported to form the following equation $X_i - M_i = m_j Y_j - m_i Y_i$ which represents exports minus imports for the i:th region. In the equation the index j represents other regions which are not the same as the index i. The idea is that the exports are proportional to the income in one of the regions.

Business cycle in continuous time offers insights into the dynamic nature of fluctuations within the economy. With many mathematical models and formulations, such as the basics models and two region models, it enables simulations being made to describe the chaos within the economy and have a better understanding of the economy. The continuous-time model also shows complex interactions between each variable representing economy values and showing how small changes in any variable can create chaos and massive changes.

Business cycle---interregional trade

In the examination of interregional trade within a continuum of spatial and temporal dimensions, the integration of economic dynamics with mathematical principles elucidates the complexity inherent in the modeling of economic processes. This discourse advances understanding by employing partial differential equations, notably the Laplacian operator, to articulate the nuanced interactions within import-export dynamics. Such modeling, predicated on a linear import-export multiplier approach, posits a direct correlation between imports at any given locus and local income, now a function of both temporal and spatial coordinates. This premise, aligned with Keynesian economic theory, unveils layers of complexity within what initially appears to be a straightforward linear model.

Export Surplus with the Laplacian

A fundamental component of this analysis is the Laplacian operator $((\Delta))$, which quantifies the concept of export surplus $((X - M))$as a function of income disparities across spatial dimensions, scaled by a constant import propensity $((m))$. The Laplacian in two dimensions is given by:

$$\Delta Y = \frac{\partial^2 Y}{\partial x^2} + \frac{\partial^2 Y}{\partial y^2}$$

leading to the formulation for export surplus, delineating the impact of spatial income variations on trade dynamics:

$$X - M = m\Delta Y$$

The propagation of economic effects through spatial dimensions in a linear framework is captured using partial differential equations, utilizing the Laplacian to encapsulate spatial interactions. An exemplar equation for the evolution of income (Y)over time$(t)$and space is:

$$\frac{\partial Y}{\partial t} = D\Delta Y + f(Y)$$

where(D)symbolizes the diffusion coefficients or import propensity, and $(f(Y))$ denotes linear growth factors dependent on income.

The incorporation of nonlinearity, particularly through the investment function, augments the model's complexity. A prototypical nonlinear investment function may be represented as:

$$I = vY - \frac{aY^3}{3}$$

where (v)reflects the rate of investment relative to income changes, and (a) introduces nonlinearity, capturing the effects of diminishing returns at elevated income levels.

Perturbation Analysis for System Stability

Perturbation analysis examines the system's response to infinitesimal changes $((\delta Y))$, offering insights into stability or tendencies towards chaos based on:

$$\frac{\partial(Y + \delta Y)}{\partial t} = D\Delta(Y + \delta Y) + f(Y + \delta Y)$$

The transition from discrete models, exemplified by the Samuelson-Hicks multiplier-accelerator model, to continuous spatial models marks an evolution in the complexity and applicability of economic theory. The discrete Samuelson-Hicks model is formulated as:

$$Y_t = cY_{t-1} + I_t + G_t$$

whereas in continuous models, dependencies of investment and consumption evolve through differential equations, reflecting instantaneous changes across time and space. The synthesis of discrete and continuous economic models through rigorous mathematical formulations enriches our comprehension of interregional trade and economic cycles, simultaneously illuminating paths for nuanced policy formulation and theoretical development in economics. These mathematical instruments—from the Laplacian to nonlinear investment functions and perturbation analysis—serve as foundational elements for constructing sophisticated models. These models adeptly capture the intricate interplay of economic variables across the multifaceted canvas of space and time, representing a significant advancement in economic modeling. This comprehensive approach offers profound insights for economists, policymakers, and scholars, guiding them through the complexities of global economic interdependencies with precision and analytical depth.

Digression on Order and Disorder

Inspired by the "Powers of Ten" film, this segment explores the alternating patterns of order and disorder across the universe, drawing parallels with economic models where similar patterns emerge from synergetic self-organization and deterministic chaos. The analysis underscores the capacity of simple iterative relations to generate complex fractal sets, laying the groundwork for a deeper understanding of economic fluctuations. The general model introduces a fractional approach to savings, departing from the binary conditions of previous analyses. By defining $(p = (v - s))$ and $(w = (1 - e)s)$, the model transitions to a coupled system, characterized by:

$$Y = Y_{t-1} + Z_{t-1}$$

where (Y)represents income and (Z) denotes the dynamic interplay between savings and investments over discrete time periods.

$$Z_t = \mu Z_{t-1} - (\mu + 1)Z_{t-1}^3 - \sigma Y_{t-1}$$

Chaotic nature

Lyapunov exponents (L) and fractal dimensions (N) serve as central tools in quantifying the chaotic behavior and geometric complexity of the model's attractors. The Lyapunov exponent is
given by:

$$J_t \cdot \mathbf{v}_t = \begin{bmatrix} \cos(\theta_t) + \sin(\theta_t) \\ -\sigma \cos(\theta_t) + (\mu - 3(\mu + 1)Z_{t-1}^2) \sin(\theta_t) \end{bmatrix}$$

$$L = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \ln \|J_i \cdot \mathbf{v}_i\|$$

This is a measure of the average speed of divergence of trajectories in an economic model, and a positive value indicates that the system behaves chaotically, signaling that the economic system is extremely sensitive to initial conditions. In real-world economic models, this means that even very small market shocks or policy changes can lead to large deviations in the trajectory of the economy, increasing the difficulty of predicting economic change. This is particularly common in financial markets, such as stock markets or exchange rate markets, where a small movement can trigger large fluctuations.

The fractal dimension, reflecting the geometric nature of attractors, is assessed through the relationship:

$$N = \lim_{\epsilon \to 0} \frac{\log(N(\epsilon))}{\log(1/\epsilon)}$$

The fractal dimension is used to measure the complexity of the geometric structure of the attractor, which reveals how well the economic behavior fills the state space. For complex attractors that are not precisely points, lines, or planes, the fractal dimension is a non-integer value. In economics, this means that the behavioral patterns of an economic system may not be simply linear or planar but rather exhibit more complexity, somewhere between one and two dimensions. This may be reflected in certain market behaviors, such as the time series of asset prices, where a simple linear model cannot capture the complex pattern of price movements.

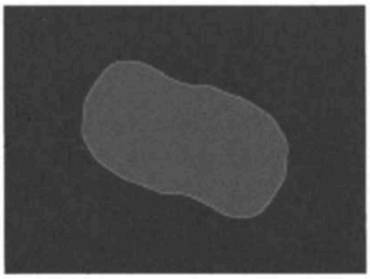


Fig. 10.27. Attractor F=1, *L*<0
μ = 1.25 σ = 0.225

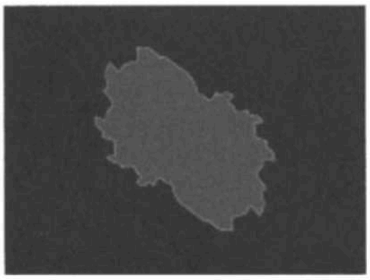


Fig. 10.28. Attractor F=1, *L*<0
μ = 1.25 σ = 0.375

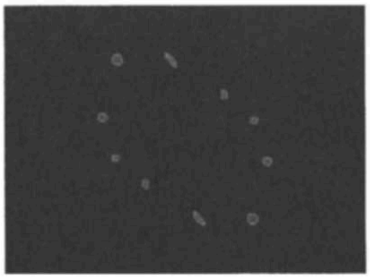


Fig. 10.29. Attractor F=1, *L*<0
μ = 1.25 σ = 0.435

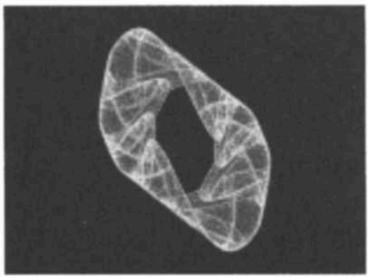


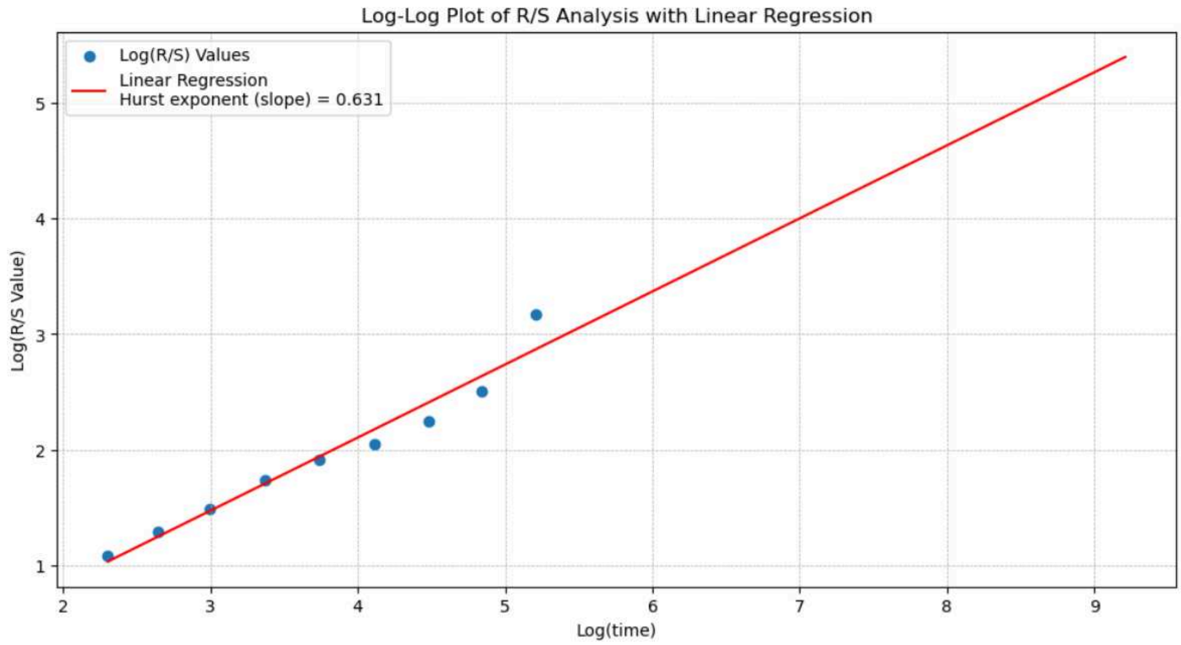
Fig. 10.30. Attractor F=1.3, *L*=0.12
μ = 1.25 σ = 0.525

These figures（10.27–10.30）are associated with different values of the Lyapunov exponent (denoted by L) and the fractal dimension (denoted by F). Changes in the Liapunov exponent and fractal dimension alter the shape and density of the attractor, indicating different degrees of predictability and stability of the economic cycle.

Numerical simulations reveal the transformation of chaotic attractors with varying savings rates, illustrating the model's sensitivity to economic parameters. The critical lines method further elucidates the emergence of attractors, employing cubic map folding to trace the evolution of economic dynamics.

Fractal Market Analysis

Chaotic behavior is a kind of deterministic behavior of a dynamical system. Although the future state of the system cannot be precisely predicted, it is determined by the initial conditions and does not depend on random factors. In economics, deterministic chaos theory is used to explain the complex phenomena observed in economic cycles, such as why identical or similar economic cycles are never completely repeated, why minor events can sometimes have major effects, and why economies can shift from a stable state to turbulence. The theory posits that these are intrinsic properties of the nonlinear dynamical systems of the economy and do not necessarily require external shocks. The mathematical foundation of this theory includes the tools used to measure chaotic dynamics, such as the largest Lyapunov exponent, which reflects the rate at which the system diverges from initial conditions, and fractal dimensions, which reflect the geometric complexity of strange attractors

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Hurst Exponent (slope): 0.6309864523202094
Intercept: -0.4174779208051036
R-squared: 0.9534291232686218
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From Figure Data from MarketWatch. (2024), a red linear regression line through these points shows the trend in the relationship between log-transformed window size and log-transformed R/S values. The slope of this line is estimated to be 0.631, which can be used as an estimate of the Hurst exponent (H).Values of H greater than 0.5 (as observed with the H values) indicate that there is a persistent long-term trend in the time series data. This means that increments tend to be followed by increments and decreases tend to be followed by decreases, indicating that there may be a trend rather than mean-reverting behavior.

The quality of the regression fit, as measured by the R-squared value, is very high at about 0.953. This suggests that more than 95% of the variance in the log-transformed R/S value is computed from the log-transformed window sizes through the regression model. The negative intercept of about -0.417 suggests that the strength of the trend diminishes as the size of the observation window decreases.

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