# Petalling of plates under explosive and impact loading ${ }^{\boldsymbol{2}}$ 

Tomasz Wierzbicki*<br>Impact and Crashworthiness Laboratory, Department of Ocean Engineering, Room 5-218, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

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#### Abstract

Failure by petalling occurs when thin plates are struck by cylindro-conical projectiles. Similar deformation mode is produced by a localized explosion on a plate. In either case, high circumferential strains induced in the target material cause radial cracking and the subsequent rotation of the affected plate material resulting in a number of symmetric petals. A new analytical method of treating this problem is proposed by developing a simple and realistic velocity and displacement fields. The progressive tearing is quantified using the CTOD criterion while bending deformation of cylindrical petals is described by the propagating hinge line with decreasing local curvature. The above two modes are coupled through the local bent radius of the petal. Closed form solution is derived for the total energy absorbed by the system, the number of petals, and the final deformed shape of the plate as a function of plate flow stress, thickness, and parameters of the external loading. The present solution compares well with experimental results of Nurick and Radford (In: Reddy BD (editor) Recent developments in computational and applied mechanics. A volume in honour of John B. Martin. 1997: p. 276-301) and Landkof and Goldsmith (Int J Solids Struct 1993;21:245-66) as well as empirical equations published in the literature. © 1999 Elsevier Science Ltd. All rights reserved.


## 1. Introduction

Petalling of thin plates is a common failure mode of thin plates subjected to localized highintensity loading. A perfect illustration of the physical phenomenon of petalling is provided by the high speed photograph of a pierced apple, Fig. 1, Wierzbicki and Moussa [1]. Multiple symmetric petals are formed on both the entrance and exit of the projectile. Note that petals are much longer

[^0]

Fig. 1. Multiple petals in the apple skin produced by a combination of a piercing projectile and a hydrodynamic pressure wave.
than required to fit the projectile. It is clear that they are formed as a combined effect of impact and "explosion". The ogival shape projectile drives an initial hole in the apple's skin. Then the pressure generated by the shock wave and the cavity inside the apple opens up the apple's outer shell to a much larger radius.

The above example proves that petalling may occur under both impact and explosive loading. Indeed, a large volume of research has been generated over the past several decades on the perforation of thin plates by cylindro-conical projectile [2-5]. In all of the above analyses the petalling problem was described by the hole enlargement models. Such models are computationally relatively simple. The first and only available rigorous analytical treatment of the petalling problem was due to Landkof and Goldsmith [6], who also performed a thorough experimental study. Their solution was based on an energy balance in which the energy absorbed by the plate consists of that due to crack propagation, petal bending, and plate dishing. These fractional energies were assumed to be independent of each other.

A major contribution of the present paper is a development of a new model in which all of these three energies are coupled. The tearing fracture energy is related to the bending energy through the local radial curvature of the petal. In turn, the bending energy is related to the circumferential curvature of the dish. The new model predicts with great realism the spiral shape of the petal - a feature observed in most experiments with cylindro-conical projectiles and explosively loaded plates.


Fig. 2. Subsequent stages in the formation of a petalling failure of a steel plate subjected to a localized explosive loading. This is not high speed photography but a picture of final deformed shapes with increasing weight of the explosive charge (after Nurick and Radford, 1997).

Recently, Atkins et al. presented a detailed analysis of petal formation in thin plates impacted by conical and spherical projectiles [7]. By considering the energy of hoop stretching and radial cracking the number of potential necks and then radial cracks was evaluated using a relatively simple analytical expression. Their prediction compares well with test results published earlier in the literature. The present analysis carries the analysis of the initiation process further into the propagation stage of radial cracks. It captures the process of curling away the petals from the flat plate with the associated expenditure of bending energy.

The petalling failure in its purest form can be produced in explosively loaded plates. Fig. 2 illustrates the deformation and failure process of a thin, firmly clamped circular plate of a radius $R$ subjected to impulsive loading [8,9]. The explosive material was placed over a central circular


Fig. 3. Tearing fracture of a plate by a wedge resembles petal formation.
portion of the plate with a radius $a=0.25 R$. A sequence of the 10 photos show the final deformed shapes of the plate under increasing amounts of explosives. For a small weight $G$ of the explosive charge dishing of the plate is observed. At $G=4.25 \mathrm{~g}$ first necking occurs and soon after ( $G=4.75 \mathrm{~g}$ ) tensile fracture blows away a small circular plug. With increasing amounts of explosives radial cracks run outward from the central hole and a set of six to seven petals develop. At $G=10-11 \mathrm{~g}$ radial cracks reach the clamped boundary of the plate. At this point the amount of charge was two and a half times larger than that needed to initiate the initial holing process. A considerable amount of energy is then dissipated in the petalling process.
The most important element of the present theory is a proper description of the tearing process of thin metal sheets which, until recently, was lacking. An understanding of this process was gained quite unexpectedly in the unrelated study of tearing of plates by a wedge performed for the ship grounding problem [10,11]. As can be seen from Fig. 3, the tear produced in the steady-state cutting of plates has all the characteristics of fracturing and tearing in the multiple petals formation.

The new theory of tearing fracture is explained first followed by application to projectile impact and explosion.

## 2. Mechanics of the petalling process

### 2.1. Geometry and kinematics

Suppose that $n$ radial cracks develop from a point in an infinite plate dividing it into $n$ symmetric petals, Fig. 4. The central angle of the petal is denoted by $2 \theta$ so that

$$
\begin{equation*}
\theta=\frac{\pi}{n} . \tag{1}
\end{equation*}
$$

Denoting the instantaneous length of the crack by $a$, the height of a triangle OAB is

$$
\begin{equation*}
l=a \cos \theta \tag{2}
\end{equation*}
$$

Either $a$ or $l$ can be considered as the process parameter.


Fig. 4. Initial and current geometry of the plate with six radial cracks.

As the radial cracks grow, the line AB sweeps through the material leaving behind a curled triangular segment of the plate. This curled segment is referred to as the petal. The condition of kinematic continuity imposes a relation between the velocity of hinge propagation $\mathrm{d} l / \mathrm{d} t=\dot{l}$, the instantaneous rate of rotation at the hinge $\phi$ and the instantaneous bent radius $\rho$ of the petal

$$
\begin{equation*}
\phi=\frac{i}{\rho} . \tag{3}
\end{equation*}
$$

According to Eq. (3) the process of curl formation can be considered as winding-up of a triangular plate segment over a cylinder. It will be shown in the sequel that the rolling radius is an increasing function of the crack length.

### 2.2. Bending energy

It is assumed that the material is rigid - perfectly plastic characterized by the average flow stress $\sigma_{0}$. In a flat metal sheet the fully plastic bending moment per unit length is $M_{\mathrm{o}}=\frac{1}{4} \sigma_{\mathrm{o}} t^{2}$ (for the Tresca yield condition), where $t$ is the plate thickness. A curved element develops a larger bending resistance $M=\eta M_{\mathrm{o}}$, where the moment amplification factor $\eta$ depends on the circumferential curvature of the plate (see Section 3).
The rate of bending energy per one petal is defined by

$$
\begin{equation*}
\dot{E}_{\mathrm{b}}=2 M \dot{\phi} l_{\mathrm{AB}} \tag{4}
\end{equation*}
$$

where $l_{\mathrm{AB}}=2 l \tan \theta$. Using Eq. (3), the rate of bending energy is expressed as

$$
\begin{equation*}
\dot{E}_{\mathrm{b}}=4 M \frac{\dot{i}}{\rho} l \tan \theta . \tag{5}
\end{equation*}
$$

It is seen that $\dot{E}_{\mathrm{b}}$ is inversely proportional to the so far unknown rolling radius $\rho$.

### 2.3. Tearing and CTOD parameter

Consider a widening gap between two adjacent petals. In a perfectly brittle material the crack would extend all the way to the intersection point of the axes of two cylinders, Fig. 5. Let $x$ be the distance along the symmetry line between two petals measured from that point. It follows from the geometry of the problem that the gap between two neighboring petals (crack width) is given by the following function

$$
\begin{equation*}
\delta(x)=\frac{1}{3} \frac{x^{3}}{\rho^{2}} \sin \theta \cos ^{3} \theta . \tag{6}
\end{equation*}
$$

The above function is valid for small $x$ so that the bending radius can be locally treated as constant.
In a real-ductile material the ligament will hold up the point when the gap reaches the crack tip opening displacement parameter (CTOD), $\delta_{t}$ [12]. From Eq. (6) the length of the plastic zone is (see [10])

$$
\begin{equation*}
x_{\mathrm{p}}=1.44 \delta_{\mathrm{t}}^{-1 / 3} \rho^{2 / 3}(\sin \theta)^{-1 / 3}(\cos \theta)^{-1} \tag{7}
\end{equation*}
$$

By considering only circumferential stretching, the membrane rate of energy in the plastic zone near the crack tip $\dot{E}_{\mathrm{m}}$ was calculated by Wierzbicki and Thomas, [10]

$$
\begin{equation*}
\dot{E}_{\mathrm{m}}=\frac{2}{3} \sigma_{\mathrm{o}} t x_{\mathrm{p}} \dot{l}(\sin \theta)^{-1} \tag{8}
\end{equation*}
$$

or using Eq. (7) and the definition of $M_{\text {o }}$

$$
\begin{equation*}
\dot{E}_{\mathrm{m}}=3.84 M_{\mathrm{o}} t^{-1} \delta_{\mathrm{t}}^{1 / 3} \rho^{2 / 3} i(\sin \theta)^{-4 / 3}(\cos \theta)^{-1} . \tag{9}
\end{equation*}
$$

### 2.4. Total energy and bending radius

The energy dissipated per petal is the sum of bending energy and membrane energy $\dot{E}=\dot{E}_{\mathrm{b}}+\dot{E}_{\mathrm{m}}$. Thus, the total rate of energy, normalized with respect to $M_{\mathrm{o}} i$ becomes

$$
\begin{equation*}
\frac{\dot{E}}{M_{\mathrm{o}} i}=4 \eta \frac{l}{\rho} \tan \theta+3.84 \bar{\delta}^{1 / 3}\left(\frac{\rho}{t}\right)^{2 / 3}(\sin \theta)^{-4 / 3}(\cos \theta)^{-1} \tag{10}
\end{equation*}
$$

where $\bar{\delta}=\delta_{\mathrm{t}} / t$ is the dimensionless CTOD parameter. It can be seen that the bending term is inversely proportional to the instantaneous bending radius $\rho$. At the same time the membrane term increases with $\rho$. It is reasonable to expect that for a given $l$, the bending radius adjusts itself so as to minimize the total rate of energy

$$
\begin{equation*}
\frac{\mathrm{d}(\dot{E} / \dot{l})}{\mathrm{d} \rho}=0 \tag{11}
\end{equation*}
$$



Fig. 5. Illustration of cylindrical bending of a petal and plastically deforming zone near the crack tip.


Fig. 6. A minimum petal semi angle $\theta$ calculated by two different models.

Indeed, the analytical minimum exists and it is reached at

$$
\begin{equation*}
\rho_{\min }=1.3 \eta^{0.6} l^{0.6} t^{0.4} \bar{\delta}^{0.2}(\sin \theta)^{1.4} . \tag{12}
\end{equation*}
$$

Substituting this expression back in Eq. (10) the normalized rate of energy per petal becomes

$$
\begin{equation*}
\frac{\dot{E}}{\dot{i} M_{\mathrm{o}}}=7.65\left(\frac{l \eta}{t}\right)^{0.4} \bar{\delta}^{0.2}(\sin \theta)^{-0.4}(\cos \theta)^{-1} \tag{13}
\end{equation*}
$$

It is easy to see that the above function attains a minimum for $\theta \approx 30^{\circ}$, see Fig. 6. According to Eq. (1), six petals should be formed. It seems more appropriate to minimize the total rate energy (per $n$ petals).

The rate of energy per $n$ petals is

$$
\begin{equation*}
\left(\frac{\dot{E}}{\dot{i} M_{\mathrm{o}}}\right)_{n}=7.65 \pi\left(\frac{l}{t}\right)^{0.4} \bar{\delta}^{0.2} f(\theta), \tag{14}
\end{equation*}
$$

where

$$
\begin{equation*}
f(\theta)=\eta^{0.4}\left[\theta(\sin \theta)^{0.4} \cos \theta\right]^{-1} . \tag{15}
\end{equation*}
$$

Taking $\eta$ as being independent of $\theta$, the function $f(\theta)$ attains a minimum at $\theta=\cong 50^{\circ}$ giving approximately $n=4$. Thus, according to the present theory the failure process should produce four petals. However, because the minimum of the rate of energy is rather weak, a large number, i.e. five
or six petals, can be produced as well. In the experiments discussed by Atkins et al. [7], the number of experimentally measured petals varied between three and six with four being the most common. Substituting $\theta=\pi / 4$ in Eqs. (12) and (14), the minimum normalized rate of energy becomes

$$
\begin{equation*}
\left(\frac{\dot{E}}{\dot{l} M_{\mathrm{o}}}\right)=49.74\left(\frac{l \eta}{t}\right)^{0.4} \bar{\delta}^{0.2} \tag{16}
\end{equation*}
$$

and

$$
\begin{equation*}
\rho_{\min }=0.8 \eta^{0.6} l^{0.6} t^{0.4} \bar{\delta}^{-0.2} . \tag{17}
\end{equation*}
$$

It can be observed that both of the above expressions depend on the dimensionless CTOD parameter raised to the power -0.2 . For ductile material this parameter is of an order of unity. For brittle material the present theory does not strictly apply. Introducing $\bar{\delta}_{\mathrm{t}}{ }^{0.2} \cong 1.0$, Eqs. (16) and (17) can be further simplified to

$$
\begin{equation*}
\left(\frac{\dot{E}}{\dot{i} M_{\mathrm{b}}}\right)_{\text {min }}=49.74\left(\frac{l \eta}{t}\right)^{0.4} \tag{18}
\end{equation*}
$$

and

$$
\begin{equation*}
\rho_{\min }=0.8(\eta l)^{0.6} t^{0.4} . \tag{19}
\end{equation*}
$$

## 3. Bending resistance of a dished plate

A fully plastic bending moment at the root of a petal of width $2 b$ depends on the circumferential curvature of the plate. When petals are formed from a flat plate with zero circumferential curvature, the bending resistance is given by

$$
\begin{equation*}
M_{\mathrm{o}}=\frac{1}{4} \sigma_{\mathrm{o}} t^{2} 2 b . \tag{20}
\end{equation*}
$$

If dishing of the plate occurs before fracture, the bending resistance may be substantially greater. This is illustrated in Fig. 7a showing an element AB of the actually curved plate. Using a simple computational model shown in Fig. 7b, the bending moment is

$$
\begin{equation*}
M=M_{\mathrm{o}}+2 \sigma_{\mathrm{o}} t \gamma b^{2}, \tag{21}
\end{equation*}
$$

where the angle $\gamma$ is defined in Fig. 7a and $r_{\theta}$ is the circumferential radius of curvature.
The so-called bending amplification factor is

$$
\begin{equation*}
\eta=\frac{M}{M_{\mathrm{o}}}=1+2 \gamma \frac{b}{t} . \tag{22}
\end{equation*}
$$

The following geometrical relation holds (see Fig. 6b)

$$
\begin{equation*}
\theta r=r_{\theta} \gamma, \quad b=r \theta . \tag{23}
\end{equation*}
$$



Fig. 7. Effect of circumferential curvature on the bending moment magnification factor.

Substituting Eq. (23) into Eq. (22) one gets

$$
\begin{equation*}
\eta=1+2 \theta^{2} \frac{r^{2}}{t r_{\theta}} \tag{24}
\end{equation*}
$$

The circumferential curvature of the plate is defined as

$$
\begin{equation*}
K_{\theta}=\frac{1}{r_{\theta}}=\frac{w^{\prime}}{r} . \tag{25}
\end{equation*}
$$

It was shown by Wierzbicki et al. [13] that the deflection of the locally loaded plate is described by a logarithmic function

$$
\begin{equation*}
w=w_{\mathrm{o}} \frac{\ln r_{1} / r}{\ln r_{1} / r_{\mathrm{p}}}, \tag{26}
\end{equation*}
$$

where $w_{\mathrm{o}}$ is deflection amplitude and $r_{1}$ the reference radius at which the transverse deflection vanishes.

Calculating the slope and the maximum slope at $r=r_{\mathrm{p}}$

$$
\begin{equation*}
w^{\prime}=\frac{-w_{\mathrm{o}}}{\ln r_{1} / r_{\mathrm{p}}} \frac{1}{r},\left.\quad w^{\prime}\right|_{r=r_{\mathrm{p}}}=\frac{-w_{\mathrm{o}}}{r_{\mathrm{p}} \ln r_{1} / r_{\mathrm{p}}} . \tag{27}
\end{equation*}
$$

In order to calculate an upper bound on the amount of dishing, consider a different failure mode without petalling but with a circumferential crack running along the supported edge $r=r_{1}$. The fracture will then be caused by a radial strain $\varepsilon_{\mathrm{r}}$. Assuming that the plate fractures when the maximum radial strain equals to the fracture strain

$$
\begin{equation*}
\varepsilon_{\mathrm{r}}=\frac{1}{2}\left[w_{\max }^{\prime}\right]^{2}=\varepsilon_{\mathrm{f}} . \tag{28}
\end{equation*}
$$

Eqs. (27) and (28) yield

$$
w^{\prime}=\sqrt{2 \varepsilon_{\mathrm{f}}} \frac{r_{\mathrm{p}}}{r} .
$$

Thus, the change of the maximum circumferential curvature with the plate radius is described by a simple relation

$$
\begin{equation*}
\frac{1}{r_{\theta}}=\sqrt{2 \varepsilon_{\mathrm{f}}} r_{\mathrm{p}} . \tag{29}
\end{equation*}
$$

Eliminating $r_{\theta}$ from Eqs. (24) and (29) gives

$$
\begin{equation*}
\eta=1+2 \sqrt{2 \varepsilon_{\mathrm{f}}} \theta^{2} \frac{r_{\mathrm{p}}}{t} . \tag{30}
\end{equation*}
$$

The moment amplification parameter $\eta$ is seen to depend on the petal angle $\theta$ and therefore should participate in the minimization procedure (see Eqs. (14) and (15)). One would expect that more petals should be formed with larger bending resistance. However, it was found the function $\eta(\theta)$ does not substantially change the optimum value of $\theta$, which is still equal approximately to $\theta_{\text {opt }}=\pi / 4$. With this value Eq. (30) is further simplified to

$$
\begin{equation*}
\eta=1+1.74 \sqrt{\varepsilon_{\mathrm{f}}} \frac{r_{\mathrm{p}}}{t} . \tag{31}
\end{equation*}
$$

The local loading radius $r_{\mathrm{p}}$ should be interpreted differently depending on the problem. Because radial cracks will most certainly appear before the circumferential cracks, the actual value of the amplification factor will be smaller than that predicted by Eq. (31).

- For impulsively loaded plate $r_{\mathrm{p}}$ is the outer radius of the explosive disk. For example, taking after Nurik and Radford [8] $r_{\mathrm{p}}=12.5 \mathrm{~mm}$ and $t=1.6 \mathrm{~mm}$ and $\varepsilon_{\mathrm{f}}=0.3$, one gets $\eta=8.4$. In subsequent calculations, the value $\eta=8$ will be taken.
- In the projectile impact problem with a flat or hemi-spherical nose $r_{\mathrm{p}}$ should be understood as the radius of the projectile $R$.
- In the case of a cylindro-conical projectile, the initial radius of the tip of the projectile is zero so that $\eta=1$. This conclusion is confirmed by Landkof and Goldsmith [6], who found that the dishing energy is insignificant at high projectile velocity above the ballistic limit.


## 4. Determination of shape of petals

It is convenient to normalize all linear quantities with respect to the plate thickness

$$
\begin{equation*}
\bar{\rho}=\frac{\rho_{\min }}{t}, \quad \bar{l}=\frac{l}{t}, \quad \bar{s}=\frac{s}{t} . \tag{32}
\end{equation*}
$$

Equation (19) is now rewritten as

$$
\begin{equation*}
\bar{\rho}=0.8(\eta \bar{l})^{0.6} . \tag{33}
\end{equation*}
$$

For the purpose of deriving a closed-form solution, the above function is approximate by a square root function

$$
\begin{equation*}
\bar{\rho}=0.8 \sqrt{\eta \bar{l}} . \tag{34}
\end{equation*}
$$

As mentioned before, the parameter $\eta$ depends on the circumferential curvature of the plate. Usually the petalling mode is preceded by dishing of the plate. Therefore, the bending hinges propagate in a curved rather than flat plate. This problem was analyzed in the preceding section. From Eq. (31) it transpires that the bending resistance of the dished plate is much larger than that of a flat plate. For the purpose of comparing the present analysis with Nurick's experiments, it is assumed that $\eta=8$ giving

$$
\begin{equation*}
\bar{\rho}=2 \sqrt{\bar{l}} . \tag{35}
\end{equation*}
$$

Fig. 8 shows the cross-section along the symmetry plane of the petal. The local slope of the curl at point $\mathrm{A}(x, z)$ in the rectangular coordinate system is denoted by $\alpha$, while $l$ is the total length of the $\operatorname{curl}(\operatorname{arc} \mathrm{OB})$. The curvilinear coordinate of the point A , measured from the point O , is denoted by s. From the definition of curvature

$$
\begin{equation*}
\frac{l}{\rho}=\frac{\mathrm{d} \alpha}{\mathrm{ds}} . \tag{36}
\end{equation*}
$$

Combining Eq. (35) with $\bar{l} \rightarrow \bar{s}$ and Eq. (36) one gets a differential equation.

$$
\begin{equation*}
\frac{\mathrm{d} \alpha}{\mathrm{~d} \bar{s}}=\frac{1}{2 \sqrt{\bar{s}}} \tag{37}
\end{equation*}
$$

whose solution is

$$
\begin{equation*}
\alpha=\sqrt{\bar{s}}+C . \tag{38}
\end{equation*}
$$

The integration constant is found from the initial condition at $\bar{s}=\bar{l}, \alpha=0$, giving

$$
\begin{equation*}
\alpha=-(\sqrt{\bar{l}}-\sqrt{\bar{s}}) . \tag{39}
\end{equation*}
$$

Transforming into the rectangular coordinates

$$
\begin{align*}
& \mathrm{d} x=\cos \alpha \mathrm{d} s=\cos (\sqrt{\bar{l}}-\sqrt{\bar{s}}) \mathrm{d} s  \tag{40}\\
& \mathrm{~d} w=\sin \alpha \mathrm{d} s=-\sin (\sqrt{\bar{l}}-\sqrt{\bar{s}}) \mathrm{d} s \tag{41}
\end{align*}
$$



Fig. 8. Definition of geometrical parameters describing the curling process of petals.

The solution of the above system, satisfying the initial conditions at

$$
s=l, \quad x=0 \quad \text { and } w=0
$$

is

$$
\begin{align*}
& x=2[\cos (\sqrt{\bar{l}}-\sqrt{\bar{s}})-\sqrt{\bar{s}} \sin (\sqrt{\bar{l}}-\sqrt{\bar{s}})-1],  \tag{42}\\
& w=2[\sqrt{\bar{l}}-\sin (\sqrt{\bar{l}}-\sqrt{\bar{s}})-\sqrt{\bar{s}} \cos (\sqrt{\bar{l}}-\sqrt{\bar{s}})] . \tag{43}
\end{align*}
$$

The above equations define the shape function of the petals in a parametric form with $\bar{s}$ as a parameter. A plot of the shape for several values of the petal length $\bar{l}$ is shown in Fig. 9. A 3-D view of the set of four and six petals of the length $l=5,10$, and 20 t is shown in Fig. 10a and b. This should be compared with the photograph presented in Figs. 1 and 2. It can be concluded that the present theory predicts with great realism the process of petalling of thin plates.

## 5. Application to explosive loading

Consider an explosive material uniformly distributed over the disc of radius $r_{\mathrm{p}}$. Denoting by $I$ the impulse per unit area, the velocity imparted instantaneously to the plate is

$$
\begin{equation*}
V_{\mathrm{o}}=\frac{I}{m}=\frac{I}{\rho t}, \tag{44}
\end{equation*}
$$



Fig. 9. A sequence of petal formation with fully plastic tearing fracture.
where $\rho$ is the mass density of the plate. For small magnitudes of the impulse the plate undergoes dishing. This problem has been fully analyzed by Suliciu and Wierzbicki [14]. Dishing occurs until tensile necking and fracture take over at the critical velocity

$$
\begin{equation*}
\frac{V_{\mathrm{cr}}}{c}=2.83 \sqrt{\varepsilon_{\mathrm{f}}}, \tag{45}
\end{equation*}
$$

where $\varepsilon_{\mathrm{f}}$ is the uniaxial fracture strain. The detonation blows out a central cap of the radius $r_{\mathrm{p}}$. This occurs at central deflection $w_{\mathrm{f}}$

$$
\begin{equation*}
w_{\mathrm{f}}=2.47 r_{\mathrm{p}} \sqrt{\varepsilon_{\mathrm{f}}} . \tag{46}
\end{equation*}
$$

If the impulse is above the critical value, the remainder of the initial kinetic energy goes into the petalling process.

$$
\begin{equation*}
\frac{1}{2} \pi r_{\mathrm{p}}^{2} t \rho\left(V^{2}-V_{\mathrm{cr}}^{2}\right)=\int_{t_{\mathrm{c}}}^{t} \dot{E} \mathrm{~d} t . \tag{47}
\end{equation*}
$$

Substituting the expression $\dot{E}$ (Eq. (18)) and integrating, one gets

$$
\begin{equation*}
\frac{1}{2} \pi r_{\mathrm{p}}^{2} t_{\mathrm{p}} \rho\left(V^{2}-V_{\mathrm{cr}}^{2}\right)=8.9 \sigma_{\mathrm{o}} \eta^{0.6} t^{1.6}\left(l-r_{\mathrm{p}}\right)^{1.4} \tag{48}
\end{equation*}
$$

The above equation should be solved for the unknown radius of the hole as a function of a given initial velocity and other known parameters of the process.

Using the definition of the speed of plastic flexural wave $c^{2}=\sigma_{\mathrm{o}} / \rho$ (see Ref. [14]), Eq. (48) can be put into

$$
\begin{equation*}
\left(\frac{V_{\mathrm{cr}}}{c}\right)^{2}\left[\left(\frac{V}{V_{\mathrm{cr}}}\right)^{2}-1\right]=5.2 \eta^{0.6}\left(\frac{t}{r_{\mathrm{p}}}\right)^{0.6}\left(\frac{l}{r_{\mathrm{p}}}-1\right)^{1.4} . \tag{4}
\end{equation*}
$$



Fig. 10. (a) A 3-D view of the predicted final shape of petals for $n=4$ and three different values of $\bar{l}=l / t=5,10,15$; (b) A 3-D view of the predicted final shape of petals for $n=6$ and three different values of $\bar{l}=l / t=5,10,15$.

In order to validate the present theory, consider the example reported by Nurick and Radford [8] and Fig. 2. The following data are given

Plate thickness,
Radius of explosive,
Moment amplification parameter,
Strain to rupture,
Flow stress,

$$
\begin{aligned}
& t=1.6 \mathrm{~mm}, \\
& r_{\mathrm{p}}=12.5 \mathrm{~mm}, \\
& \eta=8, \\
& \varepsilon_{\mathrm{f}}=0.3, \\
& \sigma_{\mathrm{o}}=330 \mathrm{MPa} .
\end{aligned}
$$

From Eq. (49), one gets

$$
\begin{equation*}
\left(\frac{l}{r_{\mathrm{p}}}-1\right)^{1.4}=0.34\left[\left(\frac{V}{V_{\mathrm{c}}}\right)^{2}-1\right] . \tag{50}
\end{equation*}
$$

Also from tests the critical impulse to blow out the central plot was $G_{\mathrm{c}}=3.5 \mathrm{~g}$. The task is to find the size of the hole at the magnitude of charge $G=11 \mathrm{~g}$. This is the largest amount of explosive used in the reported series of experiments. The amount of impulse imparted to the plate is proportional to the mass of the explosive

$$
\begin{equation*}
\frac{G}{G_{\mathrm{c}}}=\frac{I}{I_{\mathrm{c}}}=\frac{V}{V_{\mathrm{c}}}=\frac{11.0}{3.5}=2.4 . \tag{51}
\end{equation*}
$$

Solving Eq. (50) for $l / r_{\mathrm{p}}$ one gets

$$
\begin{equation*}
l=2.41 r_{\mathrm{p}}=30.12 \mathrm{~mm} \tag{52}
\end{equation*}
$$

which gives the cracked radius $a=l \sqrt{2}=42.6 \mathrm{~mm}$.
From the photograph shown in Fig. 2, it is seen that the cracks in the petalling mode have just reached the outer radius of the plate $R=50 \mathrm{~mm}$. The agreement between the theory and experiments appears to be good.

## 6. Application to projectile impact

Consider a cylindro-conical projectile, $R$ being a radius of the cylindrical part. As discussed in Section 3, the moment amplification parameter should be set equal to unity, $\eta=0$. In order for the projectile to penetrate the plate, the radius $l$ of the inscribed circle into an $n$-side polygon should be equal to $l=R$. As seen from Fig. 8, the curved petals have to rotate by the angle $\beta$ around a stationary hinge line

$$
\begin{equation*}
\tan \beta=\frac{w^{*}}{x^{*}} . \tag{53}
\end{equation*}
$$

This would increase the energy dissipation by a small amount as follows: The coordinated ( $w^{*}, x^{*}$ ) of the mostly intruding point C can be found from the analysis presented in Section 4. The point C has a vertical slope so that $\alpha=-\pi / 2$.

Thus from Eq. (39) we find $\sqrt{l}-\sqrt{s}=\pi / 2$. Substituting this value into Eqs. (42) and (43) one finds

$$
\begin{align*}
& x^{*}=2\left[\sqrt{l}+1-\frac{\pi}{2}\right], \\
& w^{*}=2[\sqrt{l}-1] . \tag{54}
\end{align*}
$$

According to Eq. (53)

$$
\begin{equation*}
\tan \beta=\frac{\sqrt{R-1}}{\sqrt{R}-0.57} \tag{55}
\end{equation*}
$$

Thus, an upper bound for the rotation angle $\beta$ is $\pi / 4$. The additional bending energy of $n$ petals rotating by the angle $\pi / 4$ over the hinge lines of the length $2 R \tan \theta$ each is

$$
\begin{equation*}
E_{1}=\frac{\pi}{\theta} 2 R \tan \theta M_{\mathrm{o}} \frac{\pi}{4} \cong 1.23 \sigma_{\mathrm{o}} t^{2} R . \tag{56}
\end{equation*}
$$

The total energy dissipated in the perforation process is obtained by integrating Eq. (19) in time and adding the new term

$$
\begin{equation*}
E_{\text {total }}=\int_{0}^{R} \dot{E} \mathrm{~d} l+E_{1}=\sigma_{0} t^{2} R\left[1.23+8.9\left(\frac{R}{t}\right)^{0.4}\right] . \tag{57}
\end{equation*}
$$

For realistic values of the radius-to-thickness ratio $R / t>5$, the additional energy of bending about stationary hinge lines $E_{\mathrm{t}}$ is less than $10 \%$ of the total dissipation energy. The principal component of energy dissipation is thus equal to

$$
\begin{equation*}
E=3.37 \sigma_{0} t^{1.6} D^{1.4}, \tag{58}
\end{equation*}
$$

where $D=2 R$ is the projectile diameter.
The above expression compares favorably with many empirical formulas developed over the years by various research groups and critically evaluated by Corbet et al. [15]. In a dozen of expressions existing in the literature, the thickness parameter being raised to the power $m=1.4-1.7$ while the diameter being raised to the power $3-m=1.6-1.3$. The present solution falls within those ranges. It should be pointed out that the present paper does not consider the initial phase of petal formation which was mainly the subject of the recent work by Atkins et al. [7]. Therefore, the "point" angle and the "point" radius of the conical projectile does not enter the solution.

## 7. Comparison with experiments and conclusions

Landkof and Goldsmith [6] performed a series of tests on perforation of $t=3.175 \mathrm{~mm}$ thick 2024-0 aluminum plates with $D=12.7 \mathrm{~mm}$ cylindro-conical projectile of a mass of $M=29.5 \mathrm{~g}$. Measured in experiments were the initial velocity $V_{\mathrm{i}}$ and final (exit) velocity $V_{\mathrm{f}}$. The drop in the kinetic energy of the projectile must be equal to the work done on plastic deformation and fracture

$$
\begin{equation*}
\frac{1}{2} M\left[V_{\mathrm{i}}^{2}-V_{\mathrm{f}}^{2}\right]=E . \tag{59}
\end{equation*}
$$

The ballistic limit $\bar{V}_{\mathrm{b}}$ is understood as a minimum impact velocity at which perforation occurs, which occurs at $V_{\mathrm{f}}=0$.

$$
\begin{equation*}
\frac{1}{2} M V_{\mathrm{b}}^{2}=E . \tag{60}
\end{equation*}
$$

Eliminating $E$ between the above equation yields

$$
\begin{equation*}
V_{\mathrm{f}}=\sqrt{V_{\mathrm{i}}^{2}-V_{\mathrm{b}}^{2}} . \tag{61}
\end{equation*}
$$

It is seen that for initial velocities well above the ballistic limit $\left(V_{\mathrm{i}}>V_{\mathrm{b}}\right)$, the function (61) approaches the linear asymptote $V_{\mathrm{f}}=V_{\mathrm{i}}$. Defining the volume of the projectile $\Lambda$ by

$$
\begin{equation*}
\Lambda=\frac{M}{\rho} \tag{62}
\end{equation*}
$$

and substituting Eq. (58) for $E$ in Eq. (60), the ballistic limit can be put in a simple form

$$
\begin{equation*}
V_{\mathrm{b}}=c \sqrt{\frac{6.75 t^{1.6} D^{1.4}}{\Lambda}} . \tag{63}
\end{equation*}
$$

Using the experimental values, Eq. (63) yields:

$$
V_{\mathrm{b}}=0.62 c,
$$

where

$$
\begin{equation*}
c=\sqrt{\frac{\sigma_{0}}{\rho}} \tag{64}
\end{equation*}
$$

is the velocity of propagation of flexural plastic wave in plates. The parameter $c$ depends on the average flow stress of the material which in turn depends on the average strain in the deformation process. Determination of strains and strain histories in the present problem is a separate task. This task was undertaken in a related problem of shear plugging of plates under flat nose projectile impact [16]. Here a simplified approach is adopted initially developed by Wierzbicki et al. [17] where $\sigma_{\mathrm{o}}$ is calculated from

$$
\begin{equation*}
\sigma_{\mathrm{o}}=\sqrt{\frac{\sigma_{\mathrm{y}} \sigma_{\mathrm{u}}}{1+p}} . \tag{65}
\end{equation*}
$$

Here $\sigma_{\mathrm{y}}$ is the yield stress, $\sigma_{\mathrm{u}}$ is the ultimate stress, and $p$ is the exponent in the power-type stress-strain law. The aluminum alloy 2024-0 is characterized by

$$
\begin{aligned}
& \sigma_{\mathrm{y}}=100 \mathrm{MPa}, \\
& \sigma_{\mathrm{u}}=240 \mathrm{MPa}, \\
& p=0.2 .
\end{aligned}
$$

With the above values the average flow stress is calculated to be $\sigma_{\mathrm{o}}=141.4 \mathrm{MPa}$. The corresponding wave speed becomes $c=228.5 \mathrm{~m} / \mathrm{s}$. The corresponding ballistic limit is $V_{\mathrm{b}}=130 \mathrm{~m} / \mathrm{s}$ which is larger than the experimentally measured ballistic limit of $\bar{V}_{\mathrm{b}_{\mathrm{cpp}}}=90 \mathrm{~m} / \mathrm{s}$.


Fig. 11. Comparison of the predicted and measured terminal velocity as a function of initial projectile velocity. Experimental points reported by Landkof and Goldsmith (1983), denoted by

The value of the ballistic limit predicted by Landkof and Goldsmith [6] was $V_{\mathrm{b}}=70 \mathrm{~m} / \mathrm{s}$. Thus the present theory overpredicts the ballistic limit while Landkof and Goldsmith underpredicted it. The difference is in the treatment of the fracture process. In the present formulation it is assumed that fracture is preceded by considerable amounts of plastic deformation and necking. Under these conditions the approximation of the CTOD parameter $\bar{\delta}=\delta_{\mathrm{t}} / t=1$ is correct. Landkof and Goldsmith on the other hand treated the fracture process in the realm of elastic fracture mechanics. Following Atkins and Mai [12] the specific work of fracture $R$ in tearing is related to the $\delta_{\mathrm{t}}$ parameter by $R=\sqrt{3} \sigma_{\mathrm{o}} \delta_{\mathrm{t}}$ where $\sigma_{\mathrm{o}} \approx \sigma_{\mathrm{u}}$.
In the above-mentioned experiments on projectile impact fracture properties were not directly measured. Estimating $R=100 \mathrm{~kJ} / \mathrm{m}^{2}$ for aluminum, the dimensionless CTOD parameter becomes $\bar{\delta}=0.075$.

Now retaining the effect of $\bar{\delta}$ in Eq. (16) gives a modified expression for the ballistic limit

$$
V_{\mathrm{b}}=c \sqrt{6.75 \bar{\delta}^{0.2} t^{1.6} \frac{D^{1.4}}{\Lambda}} .
$$

Repeating the calculations with $\bar{\delta}=0.075$ gives an improved predicted value of the ballistic limit $V_{\mathrm{b}}=100.4 \mathrm{~m} / \mathrm{s}$. The agreement with test results is considered to be good. A plot of the entrance
velocity versus the exit velocity is shown in Fig. 11. It is seen that the experimental points lie right on the theoretical curve.

It can be concluded that the present theory provides a good qualitative and quantitative description of the plate damage in the petalling mode. New factors not reported previously in the literature include:

- prediction of very realistic intermediate and final shapes of petals.
- determination of the extent of radial cracks in the local explosion problem.
- calculation of the ballistic limit of cylindro-conical projectiles taking into account the fracture properties of the material.

The present theory was validated using experimental data on explosion and impact loading showing consistently good agreement.

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    * Corresponding author. Tel.: 001-617-253-2104; fax: 001-617-253-1962.

    E-mail address: wierz@mit.edu (T. Wierzbicki)

